Left-right Polynomial Matrix Factorization for MIMO Pole/Zero Cancellation with Application to FEL*

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Motivated by a biological motion control model called feedback error learning (FEL), feedforward learning control schemes have been extensively studied with emphasis on their implementation by linear filters and tuning parameters. Its multi-input multi-output (MIMO) generalization, however, has been redundant in terms of pole/zero cancellation structure. The objective of this paper is to propose an exact feedforward controller by introducing the notion of left-right polynomial matrix factorization.

1. Introduction

Pole/zero cancellation plays an important role in various two-degrees-of-freedom (2DOF) control system design schemes such as model reference adaptive control and feedback error learning (FEL). The former is further generalized to VRFT scheme [1,2]. In the latter, fixed feedback (FB) controller stabilizes the closed-loop while feedforward (FF) controller is tuned on-line to attain a good response by canceling the stable zeros of the plant; see [3–6], just to name a few. This “role-sharing” in 2DOF structure is promising in practice since we can concentrate FB control on robust stability, while tuning FF control gives much better transient property than other adaptive control schemes that also require stabilization.

Application of FEL to mechanical systems can be found say in [7–9], but they have been confined to single-input single-output (SISO) plant so far. The authors’ group have tried to generalize this scheme to the multi-input multi-output (MIMO) case [10–12]. Most of these works, however, resorted to a certain shortcut to address non-commutativity of matrix multiplication, and hence suffers from a redundancy.

The objective of this paper is to refine the MIMO tuning FF controller by introducing the novel concept of left-right factorization by polynomial matrices,

thereby giving an exact parameterization for pole/zero cancellation. It is interesting to note that *left* and *right* factorizations of polynomial matrices have been extensively used for representing MIMO systems [13,14], but *left-right* matrix factorization has never been used, though its inverse form appeared half a century ago [15].

This paper is organized as follows. After mentioning preliminaries in Section 2, we discuss parametric issues of polynomial matrices in Subsection 3.1 and attain pole/zero cancellation in Subsection 3.2. Subsection 3.3 is devoted to some examples. Then we develop a tuning law for FF controller in Subsection 3.4. This is done by a recursive least square algorithm. Numerical simulation is carried out in Section 4 to show the effectiveness of the proposed method.

Notation: We often use upper cases for polynomial matrices, and lower cases with a single (double) subscript for their row vectors (for their entries, respectively); e.g.,

$$N(s) = \begin{pmatrix} n_1(s) \\ \vdots \\ n_p(s) \end{pmatrix} = \begin{pmatrix} n_{11}(s) & \cdots & n_{1p}(s) \\ \vdots & \ddots & \vdots \\ n_{p1}(s) & \cdots & n_{pp}(s) \end{pmatrix}.$$

We further add superscript for their coefficients; e.g.,

$$n_{ij}(s) = \sum_{k=0}^{\mu} n_{ij}^k s^{\mu-k}, \text{ where } n_{ij}^k \text{ does not mean the } k\text{-th power of } n_{ij}.$$

I_p denotes the p -th order identity but we may omit the suffix when the size is irrelevant or obvious. $\mathbb{N}$ and $\mathbb{C}$ respectively denote the set of positive integers and the set of complex numbers.

All signals are described in the frequency domain up to Section 3.2. In Section 3.4 time domain signals are also treated since we tune the parameter in FF controller on-line.

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Fig. 1 2DOF structure with tuning FF controller

2. Preliminaries

2.1 Control Objective

Consider the configuration of Fig. 1, where $P(s)$ is a plant to be controlled, whose coefficients are uncertain. Assume that FB controller $K(s)$ stabilizes the closed-loop, either by a model-free design such as PID control or a robust control design, but it gives a poor response to the reference signal r . We improve the response by tuning FF controller $Q(s)$ rather than by $K(s)$, thereby avoiding the conflict between sensitivity reduction and robust stability, which we often confront in usual FB control design.

To be more specific, let us assume that $P(s)$ is a $p \times p$ strictly proper rational matrix of minimum phase; i.e., $\det P(s) \neq 0$ for all $s \in \mathbb{C}$ such that $\text{Re } s \geq 0$. We further assume that

$$\det E \neq 0, \quad E := \begin{pmatrix} c_1 A^{\rho_1-1} B \\ \vdots \\ c_p A^{\rho_p-1} B \end{pmatrix}, \quad C = \begin{pmatrix} c_1 \\ \vdots \\ c_p \end{pmatrix}, \quad (1)$$

where (A, B, C) is a minimal realization of $P(s)$, and

$$\rho_i := \min\{\rho \in \mathbb{N}; c_i A^{\rho-1} B \neq 0\}, \quad i = 1, \dots, p.$$

We call ρ_i the i -th row relative degree. Then we can take a polynomial matrix

$$L(s) = \text{diag}(\ell_1(s), \dots, \ell_p(s)), \quad (2)$$

$$\ell_j(s) = \ell_j^0 s^{\rho_j} + \dots + \ell_j^{\rho_j-1} s + 1, \quad j = 1, \dots, p,$$

where $\ell_j(s)$ are Hurwitz polynomials. In adaptive control literature $L(s)$ is called interactor. It is easily verified that $L(s)P(s)$ is a biproper rational matrix.

Now we can formally state our control objective as follows. Take $W(s) = L(s)^{-1}$, which can be regarded as a response model. (Taking (2) requires that the signals are decoupled and the steady gain is unity for each output.) We want the feedback error $e = w - y$ to vanish, where $w = W(s)r$. In this paper we attain this by tuning FF controller $Q(s)$ so that

$$P(s)Q(s) = W(s). \quad (3)$$

2.2 Discussion

Since $W(s)$ is zero-free, (3) means that the zeros of $P(s)$ must be canceled by the poles of $Q(s)$. For the SISO case it is easy to take a biproper function

$$Q(s) = \frac{d(s)}{n(s)\ell(s)}, \quad \text{where } P(s) = \frac{n(s)}{d(s)},$$

so that

$$P(s)Q(s) = \frac{n(s)}{d(s)} \cdot \frac{d(s)}{n(s)\ell(s)} = \frac{1}{\ell(s)}.$$

Our objective is to give a natural MIMO extension of the above SISO pole/zero cancellation, which is by no means trivial.

As is well known [13,14], a left (or right, respectively) coprime factorization by polynomial matrices is given by

$$P(s) = D(s)^{-1}N(s) \quad (= \tilde{N}(s)\tilde{D}(s)^{-1}), \quad (4)$$

where (D, N) (or $(\tilde{N}, \tilde{D})$) is a left (or right) coprime pair, namely

$$\text{rank}(D(s) \ N(s)) = p \quad (\text{rank} \begin{pmatrix} \tilde{D}(s) \\ \tilde{N}(s) \end{pmatrix} = p) \quad (5)$$

for all $s \in \mathbb{C}$. Then we can write

$$D(s)y = N(s)u, \quad (6)$$

where u and y are input and output vector signals of the plant respectively. It is important to note that in i/o representation (6), the coefficients appear linearly, hence the left, rather than right, factorization is suitable for on-line estimation; see Section 3.4 for a detail.

From (4) it is obvious that

$$Q(s) = N(s)^{-1}D(s)L(s)^{-1} \quad (7)$$

is biproper and satisfies (3). We call (7) left-right factorization in this paper. This is, however, not easy to directly implement as FF controller since matrix multiplication is noncommutative and we cannot pack $N(s)$ and $L(s)$ into one factor. In the configuration of Fig. 1, taking (7) is equivalent to

$$N(s)u_{ff} = D(s)w, \quad (8)$$

but this is not directly realizable, because $P(s)^{-1} = N(s)^{-1}D(s)$ is improper. We have to implement the three matrices in (7) in a way compatible with parameter tuning.

2.3 Existing Schemes

The authors' group proposed a method for the MIMO pole/zero cancellation [10,11] as follows. Write

$$P(s)^{-1} = M(s) + \frac{1}{g(s)}R(s),$$

where $g(s)$ is the least common multiple denominator polynomial of the entries of $P(s)^{-1}$. Each entries of $M(s)$ and $\frac{1}{g(s)}R(s)$ are respectively the polynomial part and the strictly proper part of the corresponding entry of $P(s)^{-1}$. Then

$$Q(s) = \{g(s)I\}^{-1}\{g(s)M(s) + R(s)\}L(s)^{-1}$$

Fig. 2 Conventional scheme for FF controller

attains (3) and can be realizable by linear filters. We further transform this into

$$Q(s) = \left(\frac{g(s)}{f(s)} \right)^{-1} \{g(s)M(s) + R(s)\} \{f(s)L(s)\}^{-1}, \quad (9)$$

where we take a Hurwitz polynomial $f(s)$ having the same degree of $g(s)$. In (9), plant-dependent parameters appear linearly and are tuned on-line (Fig. 2). The tuned blocks are improper in itself but they can be realized by using the state variables in $L(s)^{-1}$ and $f(s)^{-1}I$; see [10,11] for a detail. They thus avoid the difficulty pointed out in Subsection 2.2 via scalar polynomials $f(s), g(s)$. A drawback of (9) is that it suffers from an over-parameterization and hence tuning may be insufficient; see Sections 3.3 and 4.

Another scheme has been proposed in [12], where the numerator matrix was parameterized to address MIMO pole/zero cancellation. This is, however, implemented by using multiple filters for each of the signal channels in their approach, which is also too redundant.

3. Main Result

3.1 Parametric Structure

It is widely known that left coprime factorization (4) is not unique; see, e.g., [13,14]. We often assume that denominator matrix $D(s)$ is row proper [13], where its row degrees agree with the observability indices of (A, B, C) . In this paper, however, we are more interested in numerator matrix $N(s)$, thus we first introduce the following notion.

[Definition 1] If a polynomial matrix is written with its row vectors

$$N(s) = \begin{pmatrix} n_1(s) \\ \vdots \\ n_p(s) \end{pmatrix}, \quad n_i(s) = \sum_{k=0}^{\mu_i} n_i^k s^{\mu_i-k}, \quad (10)$$

where $n_i^0 \neq 0$ for $i = 1, \dots, p$, then μ_i is called the i -th row degree of $N(s)$. Furthermore, define the constant matrices

$$[N(s)]_r := \begin{pmatrix} n_1^0 \\ \vdots \\ n_p^0 \end{pmatrix} \quad \text{and} \quad [N(s)]_c := ([N(s)^T]_r)^T.$$

Namely, the matrix $[N(s)]_r$ (or $[N(s)]_c$, respectively) consists of the highest-degree coefficient row (column)

vectors.

[Lemma 1] Let $P(s)$ be a square rational function matrix that is invertible and strictly proper. Then it admits left coprime factorization (4) such that

$$[N(s)]_r = \begin{pmatrix} 1 & * \\ & \ddots \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad [N(s)]_c = I_p. \quad (11)$$

(Proof) It is known [13] that there exists left coprime factorization (4) such that $N(s)$ is row proper (also called row reduced by [14]); namely,

$$\det[N(s)]_r \neq 0.$$

It is also known that $(\bar{D}, \bar{N})$ gives another left coprime factorization iff

$$\bar{D}(s) = U(s)D(s), \quad \bar{N}(s) = U(s)N(s)$$

for any unimodular polynomial matrix $U(s)$, and that premultiplying the pair by such $U(s)$ is equivalent to applying a series of row transformation to it; see [13,14].

Let us derive the coprime factorization satisfying (11) by such transformations. The first condition has been already shown; see the Appendix of [12]. Furthermore, we can derive a pair such that the row degrees of $N(s)$ are non-increasing; i.e., $\mu_1 \geq \mu_2 \geq \dots \geq \mu_p$. Then it is straightforward to show that row transformations lead to the second condition of (11), along with the first condition. This completes the proof. $\square$

It is easy to show that in the above numerator matrix

$$\deg \det N(s) = \sum_{i=1}^p \mu_i,$$

which is equal to the number of zeros of $P(s)$. Furthermore, the degrees of the denominator matrix are characterized by the following lemma.

[Lemma 2] Given (4), (11), and the interactor (2), the denominator matrix is parameterized as

$$D(s) = (d_{ij}(s)), \quad d_{ij}(s) = \sum_{k=0}^{\mu_i + \rho_j} d_{ij}^k s^{\mu_i + \rho_j - k}$$

and $\det(d_{ij}^0) \neq 0$ holds.

(Proof) By the definition of interactor

$$P(s)^{-1}L(s)^{-1} = N(s)^{-1}D(s)L(s)^{-1}$$

is biproper. Furthermore,

$$\text{diag}(s^{\mu_1}, \dots, s^{\mu_p})^{-1}N(s), \quad L(s)\text{diag}(s^{\rho_1}, \dots, s^{\rho_p})^{-1}$$

is also biproper. Hence

$$\Xi(s) = \text{diag}(s^{\mu_1}, \dots, s^{\mu_p})^{-1}D(s)\text{diag}(s^{\rho_1}, \dots, s^{\rho_p})^{-1}$$

is biproper. We have $\Xi(\infty) = (d_{ij}^0)$, which is nonsin-

Fig. 3 Block diagram of FF controller $Q(s)$ for MIMO pole/zero cancellation

gular. This completes the proof. $\square$

(Remark 1) **Lemma 2** prescribes the number of coefficients in the denominator matrix. But $d_{ij}^0 \neq 0$ does not necessarily hold for all i, j ; i.e., $d_{ij}^0 = 0$ may be true for some i, j .

3.2 Pole/Zero Cancellation

Let us rewrite (8) in terms of coefficients of each row:

$$\left(\sum_{k=0}^{\mu_i} n_i^k s^{\mu_i-k} \right) u_{ff} = \sum_{j=1}^p \left(\sum_{k=0}^{\mu_i+\rho_j} d_{ij}^k s^{\mu_i+\rho_j-k} \right) w_j, \quad i=1, \dots, p. \quad (12)$$

From (11), the highest-degree coefficient vector of $N(s)$ is written as

$$n_i^0 = (0 \ \cdots \ 0 \ 1 \ \bar{n}_i^0) \quad (13)$$

where $\bar{n}_i^0$ is a $(p-i)$ -dimensional row vector.

Now we have the following theorem.

[Theorem 1] The FF controller $Q(s)$ satisfying (12) is implemented by the block diagram given by Fig. 3.

(Proof) By tracing back from the right end of the diagram, for $i=1, \dots, p$, we can easily show

$$\xi_{i1} = s \left(n_i^0 u_{ff} - \sum_{j=1}^p d_{ij}^0 s^{\rho_j} w_j \right)$$

by (13). Proceeding similarly to the left, we finally obtain

$$\begin{aligned} \xi_{i\mu_i} &= s \left(\xi_{i\mu_i-1} - \sum_{j=1}^p d_{ij}^{\mu_i-1} s^{\rho_j} w_j + n_i^{\mu_i-1} u_{ff} \right) \\ &= \cdots \\ &= \sum_{k=1}^{\mu_i} \left(n_i^{\mu_i-k} s^k u_{ff} - \sum_{j=1}^p d_{ij}^{\mu_i-k} s^{\rho_j+k} w_j \right). \end{aligned} \quad (14)$$

On the other hand, in view of the adding point at the left side of $\xi_{i\mu_i}$, we have

$$\xi_{i\mu_i} = -n_i^{\mu_i} u_{ff} + \sum_{j=1}^p \sum_{k=0}^{\rho_j} d_{ij}^{\mu_i+k} s^{\rho_j-k} w_j. \quad (15)$$

Finally we obtain (12) by equating (14) and (15). $\square$

3.3 Examples

3.3.1 Uniform Degree Case

We first consider the so-called uniform degree case:

$$\rho := \rho_1 = \cdots = \rho_p, \quad \mu := \mu_1 = \cdots = \mu_p.$$

In this case, we can write the interactor (4) and left fraction (2) as

$$\begin{aligned} L(s) &= \sum_{k=0}^{\rho} L_k s^{\rho-k}, \quad L_{\rho} = I_p. \\ D(s) &= \sum_{k=0}^{\mu+\rho} D_k s^{\mu+\rho-k}, \quad N(s) = I_p s^{\mu} + \sum_{k=1}^{\mu} N_k s^{\mu-k}. \end{aligned}$$

The above equations contain plant-dependent matrices D_k, N_k hence the number of parameters is $\#Q = (2\mu + \rho + 1)p^2$. For brevity let $\mu = 2$, and $\rho = 2$. Then $\#Q = 7p^2$ and the FF controller (7) is implemented by the block diagram in Fig. 4. The structure turns out to be rather simple in this case, but it is interesting to see that the feedback loop is separated into two parts that implement the interactor $L(s)$ and the numerator matrix $N(s)$ respectively.

In the conventional scheme [10], we take $g(s) = \det N(s)$ whose degree is $p\mu$ and entries of $g(s)M(s) + R(s)$ have degree $p\mu + \rho + 1$. Hence The total number is $\#Q = p\mu + (p\mu + \rho + 1)p^2$. In the above case we have $\#Q = 2p^3 + 3p^2 + 2p$, which becomes large if $p > 1$.

3.3.2 Non-uniform Degree Case

We then treat the non-uniform degree case. Typically, let $p = 2$ and $(\rho_1, \rho_2) = (1, 2)$, $(\mu_1, \mu_2) = (1, 0)$. Take the interactor and numerator polynomial matrices

$$L(s) = \begin{pmatrix} s+1 & 0 \\ 0 & (\frac{1}{2}s+1)^2 \end{pmatrix}, \quad N(s) = \begin{pmatrix} s+n_{11}^1 & 0 \\ 0 & 1 \end{pmatrix}$$

which is extended periodically, and

$$r_2(t) = \begin{cases} 0; & 0 \leq t < 5 \\ r_1(t-5) & 5 \leq t. \end{cases}$$

In the tuning law (19), we take $\sigma = 10000$.

Figs. 6 through 7 are simulation results for the proposed scheme. In an early stage the output is insufficient but after tuning we obtain perfect tracking (Fig. 6). Fig. 7 shows time evolution of the tuned parameter.

Figs. 8 through 10 are simulation results for the conventional scheme [10]; see (9) and Fig. 2. We observe that tuning does not work well and even after some time tracking is insufficient (Figs. 8 and 9). Parameter tuning does not converge either, at least during this period (Fig. 10).

Fig. 6 Tracking by proposed scheme; 0 [s] ~ 40 [s]

Fig. 7 Time evolution of estimated parameter by proposed scheme

Fig. 8 Tracking by conventional scheme; 0 [s] ~ 40 [s]

Fig. 9 Tracking by conventional scheme; 40 [s] ~ 60 [s]

Fig. 10 Time evolution of estimated parameter by conventional scheme

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