PDS Based Bilateral Control System of Flexible Master-Slave Arms with Random Delay Considering Contact with Obstacle

Masaharu Yagi†, Genki Idei‡ and Yuichi Sawada‡

A PDS based bilateral control system of flexible master-slave arms with random delay is investigated in this paper. Moreover, a flexible slave arm employed in this research is affected by a contact force. A rigid master arm and a flexible slave arm which is actuated by a high-geared servomotor construct the proposed bilateral control system. Both arms are connected by a communication network. It is considered that this network occurs a random delay. Furthermore, the contact force is added to the flexible arm during its motion. Thus, the motion of this arm is restricted by the contact force. A Kalman filter is designed to estimate signals and controllers for both arms are designed as the PD- and PDS-controllers, respectively. Numerical simulations are achieved to confirm the performance of the proposed system such as the instability of its motion and the adaptivity of the Kalman filter.

1. Introduction

IoT (Internet of Things) technology is useful for human life in recent years. In the future, robots are introduced in general environments such as houses, offices and so on. Then, robots are connected to Internet for teleoperating. Thus, it is important that teleoperation technologies are realized without special equipments. Human skills can be delivered by using its technologies in distant places. Existing network such as local area networks (LAN), wide area networks (WAN), and wireless LANs are used to realize these technologies. These networks causes random delay and signals interacted each other become noisy. Therefore, it can be considered that the system becomes instability due to the communication network. In this study, a bilateral control system with a communication network is researched as one of teleoperation technologies (see Fig. 1).

Many researchers have studied teleoperation with networks. Namerikawa proposed a control strategy for bilateral systems with time-varying delay [1]. Mori et al. and Hoshino et al. studied the passivity of the bilateral system [2, 3]. Koizumi et al. researched a method for a remote diagnostic system [4]. However, these studies did not consider random delay.

A lot of researchers have investigated networked control systems with random delay. Schenato proposed the design of optimal estimators [5]. Guo et al. discussed the exponential stability for a discrete-time system [6]. Liu et al. investigated the model predictive control for networked control systems [7].

In our previous researches, a bilateral control system with the communication network was investigated [8–11]. A rigid arm and a flexible arm are employed as a master arm and a slave arm. The performance of the proposed system with time-varying delay was confirmed [8]. In this system, it was also confirmed that a contact force affected the slave arm [9]. On the other hand, the bilateral control system with random delay was researched. The performance of the proposed Kalman filter and bilateral control system were confirmed [10, 11]. However, in [11], it is not considered that the slave arm collides with an obstacle during its motion. It should be considered that the arm collides with somethings for introducing the master–slave system to general environments.

In this paper, a bilateral control system for flexible master–slave arms with random delay is discussed. Furthermore, a flexible arm is affected by a contact force during its motion. Adding the contact force differs from our previous researches. The mathematical models are derived by using Hamilton’s principle. The random delay is defined as the sum of an average time delay and a random value. A novel Kalman filter is designed as an estimator for reducing the effect of
the random delay. The PD- and PDS-controllers are designed for generating the reaction torque and the reference signal, respectively. Numerical simulations are accomplished to demonstrate the performance of the proposed bilateral control system. As seen in these simulations, it will be confirmed whether the flexible slave arm is become instable or not by using the proposed Kalman filter.

2. Mathematical Models

The proposed bilateral control system consists of the rigid master and flexible slave arms. The flexible arm is actuated by a high-geared servomotor. Both arms are connected by a communication network which occurs a random delay. In this section, mathematical models of these factors are derived.

2.1 Rigid Master Arm

A schematic drawing of the rigid master arm is shown in Fig. 2. As seen in this figure, this arm consists of an uniform rigid rod and a hub actuated by a DC motor. The inertial Cartesian coordinate system represents as $O_mX_mY_m$. $\ell_m$ is the length and $m_m$ is the mass. A human operator generate an operating force $F_h(t)$ and an operation torque $\tau_h(t)$ is generated by this force, i.e., $\tau_h(t) := F_h(t)\ell_m$. $\Theta_m(t)$ is the rotational angle and $\tau_m(t)$ is the reaction torque. $J_h$ and $J_m$ are the moments of inertia of the motor’s rotor and the master arm, respectively. $\mu_m$ is the coefficient of friction of the motor shaft.

The mathematical model is represented as

$$J_h + J_m + \frac{1}{4}m_m\ell_m^2 \dot{\theta}_m(t) = \tau_h(t) + \tau_m(t) - \mu_m\dot{\theta}_m(t) + g_m\gamma_m(t),$$

where $\gamma_m(t)$ is the system noise which is assumed as a Gaussian process and $g_m$ is the coefficient of the system noise.

2.2 High-geared Servomotor

A high-geared servomotor actuates the flexible slave arm in this research. The block diagram of the high-geared servomotor shown in Fig. 3 indicates that a high-geared DC motor and a PD-controller construct this servomotor. $\Theta_{com}(s)$ and $\Theta(s)$ are a reference and output angles, respectively. $\Theta_{com}(s)$ and $\Theta(s)$ are the Laplace transforms of $\theta_{com}(t)$ and $\theta(t)$, i.e., $\Theta_{com}(s) = \mathcal{L}[\theta_{com}(t)]$ and $\Theta(s) = \mathcal{L}[\theta(t)]$. Since the DC motor is controlled by the PD-controller, the added voltage $V(s)$ consists of the proportional and derivative factors. Moreover, the control torque $\tau(t)$ is expressed as the following equation by considering the mathematical model of the DC motor and applying the inverse Laplace transform.
\begin{align*}
\tau(t) &= \frac{k_r K_P}{R} \{ \theta_{com}(t) - \theta(t) \} \\
&+ \frac{k_r K_D}{R} \left\{ \dot{\theta}_{com}(t) - \dot{\theta}(t) \right\} - \frac{k_n k_r}{R} \ddot{\theta}(t), \quad (2)
\end{align*}

where \( K_P \) and \( K_D \) are the proportional and derivative gains; \( k_r \) is the torque constant; \( k_n \) is the back electromotive force constant; and \( R \) is the internal resistance of the coil in the DC motor. This torque drives the flexible slave arm.

### 2.3 Flexible Slave Arm

Fig. 4 depicts the parallel-structured single-link flexible arm which is used as the slave arm in this study. This flexible arm consists of a pair of uniform Euler-Bernoulli beams. A hub unit and a tip-mass are clamped on each end of the beams. In this paper, this arm is approximated as a single-link flexible arm shown in Fig. 5 because the displacement of both beams can be considered equal and the centrifugal force can be assumed to be sufficiently small [12]. The boundary conditions of the simplified arm is the same as that of the original arm.

A simplified flexible arm consists of an uniform Euler-Bernoulli beam. The inertial Cartesian and rotating coordinate systems denote \( OXY \) and \( Oxy \), respectively. \( \theta(t) \) is the rotational angle; \( \ell \) is the length; and \( u(t, x) \) is the transverse displacement from the \( x \)-axis. The physical parameters are as follows: \( \rho \) is the uniform mass density; \( S \) is the cross section; \( J_0 \) is the inertia moment of the motor shaft; \( E \) is Young’s modulus; \( I \) is the second moment of the cross-sectional area; \( \mu \) is the coefficient of friction of the motor shaft; \( c_D \) is the coefficient of Kelvin-Voigt-type damping; and \( m \) is the mass of the tip-mass which is assumed to be the point mass.

In this research, it is considered that an obstacle is contacted with the flexible arm at \( x = x_c \) \((0 < x_c \leq \ell)\). The obstacle is the rigid object and is fixed at \( x_c \). It is considered that the energy does not interact with each other due to the collision. As seen in Fig. 5, \( \phi_0 \) denotes an angle between the \( OX \)-axis and the line from \( O \) to the collision point. Then, the geometric constraint about \( \phi_0 \) is expressed as

\[
\psi(t) = u(t, x_c) - x_c \tan(\phi_0 - \theta(t)) = 0. \quad (3)
\]

Hamilton’s principle is used to derive the mathematical model of the flexible arm. The kinetic energy \( T(t) := T_h(t) + T_m(t) + T_{tm}(t) \) and the potential energy \( U(t) \) are represented as

\[
T_h(t) = \frac{1}{2} J_0 \dot{\theta}^2(t), \quad T_m(t) = \frac{1}{2} m \left\{ \ell \dot{\theta}(t) + \dot{u}(t) \right\}^2 \quad (5)
\]

\[
T_m(t) = \int_0^\ell \frac{1}{2} \rho S \left( x \dot{\theta}(t) + \dot{u}(t, x) \right)^2 \, dx, \quad (6)
\]

\[
U(t) = \int_0^\ell \frac{1}{2} EI \left\{ u''(t, x) \right\}^2 \, dx \quad (4)
\]

where \( \dot{u}(t, \ell) \) is defined as \( u(t, \ell) \). Each energies expressed as \( T_h(t), T_m(t) \) and \( T_{tm}(t) \) denote the kinetic energy of the rotation of the unit-hub, the translation of the tip-mass and the translation of the flexible beam. \( U(t) \) denotes the bending strain energy of the flexible beam. The prime is the derivative with respect to \( x \), i.e., \( \{ \cdot \}' = \partial / \partial x \).

By means of Hamilton’s principle, the following equation is obtained.

\[
\int_{t_1}^{t_2} \left\{ \delta T(t) - \delta U(t) + \delta W(t) + s \delta \psi(t) \right\} \, dt = 0, \quad (7)
\]

where \( s \) is the virtual work [8]. The Lagrange multiplier \( s \) is a force which is given by colliding with the obstacle. Therefore, this term can be defined as the collision force \( s(t) \). The mathematical model of the flexible arm including the contact force are represented as

\[
J_0 \ddot{\theta}(t) + m \ddot{u}(t, \ell) - c_D I \left\{ u''(t, 0) - u''(t, \ell) \right\} - EI \left\{ u''(t, 0) - u''(t, \ell) \right\} = \tau(t) + s(t) x_c + \int_0^\ell g_s x \gamma_s(t, x) \, dx \quad (8)
\]

\[
\rho S \ddot{u}(t, x) + c_D I \dot{u''}(t, x) + EI u'''(t, x) + \rho S x \ddot{\theta}(t) = -m \{ \ddot{\theta}(t) + \dot{u}(t) \} \delta(x - \ell) + s(t) \delta(x - x_c) + g_s \gamma_s(t, x), \quad (9)
\]
where $\gamma_s(t,x)$ denotes the system noise defined as a white Gaussian noise process and $g_s$ denotes the coefficient of $\gamma_s(t,x)$ [13].

### 3. Synthesis of Novel Kalman filter

A state estimator should be designed to reduce the effect of the random delay. The random delay is modeled by the sum of an average time delay $T$ and a random value $\gamma(t)$, i.e., $T + \gamma(t)$ [10]. Furthermore, a Kalman filter is designed as the state estimator in this study. The following time-invariant linear stochastic system including the random delay is considered.

$$\begin{align*}
    dx(t) &= Ax(t)dt + Bu(t)dt + Gdw_s(t) \quad (10) \\
    dy(t) &= Cx(t) - (T + \gamma(t))dt, \quad (11)
\end{align*}$$

where $x(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^n$ denote the state and the observation vectors, $m$ and $n$ are the dimensions of the state vector and the observation vector, respectively. $u(t) \in \mathbb{R}^p$ denotes a control input. $dw_s(t):=\gamma_s(t)dt$ denotes an increment of the Wiener process with zero-mean. $\gamma_s(t)$ denotes the system noise which is modeled by a white Gaussian noise.

The Taylor series expansion form is applied to the state vector around $t_s(t) : = t - T$. Then, an innovation process expressed by using the resultant function is defined to design the proposed Kalman filter. Therefore, the state estimate $\hat{x}(t_s|t_s)$ is given by applying the orthogonal projection [14]. Furthermore, since the control input is only get through the communication network, this signal includes the random delay. Finally, the equation of the state estimate is represented as

$$\begin{align*}
    dx(t_s|t_s) &= Ax(t_s|t_s)dt + Bu(t - (T + \gamma(t)))dt \\
    &+ P(t_s|t_s)C^T \left[ C \{ H(t_s)A + BQ_0B^T \} RC^T \right]^{-1} \\
    &\times [dy(t) - Cx(t_s|t_s)dt], \quad (12)
\end{align*}$$

where $P(t_s|t_s)$ and $H(t_s)$ are the estimation error covariance and the autocorrelation function of the state, respectively. The details of these terms are described in our previous research [11].

### 4. Synthesis of Controllers

In our previous study [11], the reaction torque is generated by the PD controller and the reference angle for the high-gearved servomotor is generated by the PDS controller [15]. These controllers are expressed as

$$\begin{align*}
    \tau_m(t) &= K_{P_m} \{ \hat{\theta}(t_s|t_s) - \theta_m(t) \} \\
    &+ K_{D_m} \{ \hat{\dot{\theta}}(t_s|t_s) - \dot{\theta}_m(t) \} \quad (13) \\
    f(t) &= \frac{R}{k_sK_D} \left[ K_{P_s} \{ \hat{\theta}_m(t_s|t_s) - \theta(t) \} \\
    &+ K_{D_s} \{ \hat{\dot{\theta}}_m(t_s|t_s) - \dot{\theta}(t) \} \\
    &+ K_S \{ c \theta''(t,0) - \theta''(t,\ell) \} \right] + \frac{k_sK_P}{R} \{ \theta_{com}(t) - \theta(t) \} + \frac{(k_e + K_D k_s)}{R} \dot{\theta}(t), \quad (14)
\end{align*}$$

where $K_{P_m}$ and $K_{D_m}$ are the proportional and derivative gains. $i$ denotes $m$ or $s$. $K_S$ is the strain gain. The control input $f(t)$ is defined as $\dot{\theta}_{com}(t)$. Hence, the reference angle for the high-gearved servomotor $\theta_{com}(t)$ is given by integrating $f(t)$.

### 5. Numerical Simulations

The numerical simulations are used to confirm the performance of the proposed bilateral control system in this study.

#### 5.1 Setup

The physical parameters of the rigid and flexible arms, and the high-gearved servomotor are listed in Table 1 to Table 3. The material of the flexible arm is phosphor bronze. The length, the thickness, and the width are $L = 0.3[m]$, $1.0 \times 10^{-3}[m]$ and $4.0 \times 10^{-2}[m]$, respectively.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_h$</td>
<td>0.70  [kg\cdot m^2]</td>
</tr>
<tr>
<td>$\mu_m$</td>
<td>$3.03 \times 10^{-2}$ [kg\cdot m^2\cdot s]</td>
</tr>
<tr>
<td>$m_m$</td>
<td>29.05 [kg]</td>
</tr>
<tr>
<td>$\ell_m$</td>
<td>0.30 [m]</td>
</tr>
<tr>
<td>$J_m$</td>
<td>$6.54 \times 10^{-4}$ [kg\cdot m^2]</td>
</tr>
</tbody>
</table>

#### Table 2 Physical parameters of the flexible slave arm

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_0$</td>
<td>0.70 [kg\cdot m^2]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$3.03 \times 10^{-2}$ [kg\cdot m^2\cdot s]</td>
</tr>
<tr>
<td>$\ell$</td>
<td>0.30 [m]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$8.8 \times 10^{3}$ [kg/m^3]</td>
</tr>
<tr>
<td>$S$</td>
<td>$4.0 \times 10^{-5}$ [m^2]</td>
</tr>
<tr>
<td>$E$</td>
<td>$1.1 \times 10^{11}$ [Pa]</td>
</tr>
<tr>
<td>$I$</td>
<td>$8.33 \times 10^{-13}$ [m^4]</td>
</tr>
<tr>
<td>$c_D$</td>
<td>$1.93 \times 10^{9}$ [N\cdot s/m^2]</td>
</tr>
<tr>
<td>$m$</td>
<td>0.245 [kg]</td>
</tr>
</tbody>
</table>

#### Table 3 Physical parameters of the high-gearved servomotor

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_P$</td>
<td>32 [V/rad]</td>
</tr>
<tr>
<td>$K_D$</td>
<td>32 [V/(rad/s)]</td>
</tr>
<tr>
<td>$k_e$</td>
<td>2.6 [V/(rad/s)]</td>
</tr>
<tr>
<td>$k_T$</td>
<td>2.6 [N/A]</td>
</tr>
<tr>
<td>$R$</td>
<td>1.73 [$\Omega$]</td>
</tr>
</tbody>
</table>

+ $EI \{ u''(t,0) - u''(t,\ell) \}$
5.2 Sample Process

Fig. 6 shows the sample processes of the proposed bilateral control system. In this case, the flexible slave arm does not contact with the obstacle. As seen in this figure, the top figure depicts the angles; the upper middle figure depicts the tip’s displacement; and the lower middle figure depicts the total energy $E(t)$ which is defined as

$$E(t) = T_h(t) + T_b(t) + T_m(t) + U(t) + T_m(t). \quad (15)$$

The under figure depicts the random delay.

The slave arm tracks the master arm well and the tip’s displacement has the large value due to the starting motion. However, the displacement converges with zero. The effect of the novel Kalman filter is confirmed in our previous research [11]. The total energy becomes zero when both arms reach desired angles. In other words, the proposed system is stable. If the average time delay becomes long, it is considered that the proposed system becomes unstable since the gains of the controllers cannot be set for theoretically stability [8].

5.3 Results

Figs. 7–10 depict the results of the numerical simulations. In each figures, the top figure shows the rotational angle; the middle figure shows the tip’s displacement; and the bottom figure shows the torques. As seen in (a) and (c) of these figures, the solid lines denote the angle of the master arm $\theta_m(t)$ and the reaction torque $\tau_m(t)$; and the dashed lines denote the estimated angle of the flexible arm $\hat{\theta}(t)$ and the torque generated by the contact force $x_c$. The estimated angle of the flexible slave arm tracks the angle of the rigid master arm well. Furthermore, it is found that the contact force constrains the motion of the slave arm. The tip’s displacement becomes the large value due to the contact force and this value is affected by the contact point $x_c$ and the contact angle $\phi_0$. On the other hand, since the tip’s displacement quickly converges to zero, it is said that the PDS controller is well-designed. The reaction torque has the large value because this torque generated by the
PD controller. As seen in each figure (c), the reaction torque includes the system noise. Thus, the operator may feel uncomfortable. When the slave arm collides with the obstacle, the reaction torque is almost equal to the contact force. Hence, the operator can feel the contact force.

Through these numerical simulation results, the proposed bilateral control system does not become unstable in spite of the effect of the contact force. Therefore, it is said that the PD and PDS controllers for the flexible slave arm and the rigid master arm and the novel Kalman filter to estimate of the signals of both arms are well-designed.

On the other hand, Fig. 11 shows the simulation result of the angles in case of collision without the Kalman filter. As seen in this figure, it is found that the slave angle $\theta(t - (T + \gamma(t)))$ become noisy due to the random delay and it is naturally considered that the tip’s displacement of the slave arm, the reaction torque and the contact force also become noisy. Thus, the slave arm cannot be controlled accurately and the operator cannot know that the slave arm reaches at a desired position or collides with obstacles. Therefore, the Kalman filter contributes to reduce the effect of the random delay.

6. Conclusions

A PDS based bilateral control system for flexible master–slave arms with random delay was researched. Moreover, the contact force affected the flexible slave arm during its motion. Hamilton’s principle was used to derive the mathematical models of the proposed system: the rigid arm, the flexible arm and the high-gearied servomotor. In the case of the flexible arm, the Lagrange multiplier in Hamilton’s principle was realized as the contact force. Thus, the mathematical model of the flexible arm included the contact force. The random delay was modeled as the sum of the average time delay and the random value. To reduce the effect of the random delay, the novel Kalman filter was designed by applying the Taylor series expansion form. The controllers of the reaction torque and the reference signal for both arms were designed as the PD and PDS controllers, respectively.

Numerical simulations were accomplished to confirm the performance of the proposed bilateral system. The Hertz contact model was employed as the contact force. The simulation results showed that the proposed Kalman filter could reduce the effect of the random delay and the PD and PDS controllers gave the reaction torque and the reference signal well. Furthermore, when the flexible slave arm collided with the obstacle, its motion did not become unstable due
Fig. 10 Numerical simulation results ($x_c = 0.15[m]$, $\phi_0 = 0.25[rad]$)

Fig. 11 Numerical simulation result without the Kalman filter ($x_c = 0.30[m]$, $\phi_0 = 0.25[rad]$)

to the proposed Kalman filter.

References


Authors

Masaharu Yagi (Member)
He received his Ph.D. degree from the Kyoto Institute of Technology, Japan, in 2014. In 2015, he joined Kyoto Prefecture. His research interests include network control systems, modeling and vibration control of flexible structures, and optimal filters. He is a member of RSJ and SICE.

Genki Idei
He received his B.S. degree from the Kyoto Institute of Technology, Japan, in 2016. He is currently an M.S. student at Kyoto Institute of Technology. His research interests include network control systems and haptic devices.

Yuichi Sawada (Member)
He received his Ph.D. degree from the Kyoto Institute of Technology, Japan, in 1994. In 1995, he joined the Kyoto Institute of Technology as an Assistant Professor in the Division of Mechanical and System Engineering. In 2015, he became a Professor in the Division of Mechanical and System Engineering. His research interests include modeling and vibration control of flexible structures, and optimal filters. He is a member of IEEE, IMechE, SICE, RSJ, and JSME.