Environment-Adaptive Genetic Algorithm-based Nesting Scheduling for Sheet-Metal Processing

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Sheet metal processing is a popular machining technique. In sheet metal processing, as many parts as possible are cut from a metal sheet to effectively use the metal without waste. The parts cut from the sheet metal are processed by a specified due-date. To meet the due-date, scheduling is important. The optimizations of the cutting layout and schedule are called nesting and scheduling, respectively. The relation between them sometimes exhibits a trade-off. To enhance the efficiency of the entire manufacturing process, nesting and scheduling should be considered simultaneously. Therefore, in this study, we proposed an environment-adaptive genetic-algorithm-based nesting scheduling method for the simultaneous consideration of two related problems with different optimization targets. We treated the problems as different environments, and the cutting layout and processing order of the parts evolved in each environment using the genetic algorithm.

1. Introduction

Sheet-metal processing is a popular machining technique. During this process, as many parts as possible are cut from a metal sheet to reduce wasted material. Thus, the design of the cutting layout is particularly important for the reduction of waste. The parts cut from sheet metal are processed by bending, welding, and assembly processes. Each product has a different due-date, and, to maintain strict timing, the optimization of the operation sequence is critical. The design of the cutting layout and the choice of the operation sequence are called nesting and scheduling, respectively. The objectives of these tasks are essentially different. They are considered separately at an actual manufacturing site. However, cutting layout and production schedule are closely related, and they often exhibit a trade-off relationship. To enhance the efficiency of the entire manufacturing process, they should be considered simultaneously.

As mentioned above, the cutting layout and operation sequence are closely related. The punching process, the first process of the sheet metal processing, is executed based on the cutting layout determined by nesting. The optimization of the schedule is performed under the constraint of the optimization of the cutting layout. Thus, the nesting–scheduling problem of sheet-metal processing can be regarded as a bilevel programming problem. In our previous studies, a scheduling method based on nesting results and nesting methods based on scheduling results were proposed. Moreover, we proposed a coevolutionary genetic algorithm (GA)-based approach. However, in these studies, the nesting and scheduling results were fed back to each other, and their trade-off relationship was not fully considered.

In this study, we propose an environment-adaptive GA-based nesting-scheduling method. Different groups of individuals are created in the scheduling and nesting environments. Then, they evolve in each environment under their respective objective functions. Moreover, some individuals are exchanged between environments to consider other objective functions. Finally, to validate the effectiveness of the proposed method, we implement a prototype system and perform computational experiments.
2. Problem Description

2.1 Sheet-metal Processing

In sheet-metal processing, parts are cut out (punched) from a sheet of metal. Afterwards, they are processed via bending, welding, and assembling. Fig. 3 shows an example of a bill of materials. In this example, P1 and P2 represent products. P1 comprises four parts (1–4). P2 also comprises 4 parts (5–8). Parts 1 and 2 in product 1 are welded, then they are assembled with parts 3 and 4.

Fig. 4 shows the relationship between a cutting layout and a schedule. Fig. 4 (a) represents an example of cutting layout, and (b) represents an example of a Gantt chart. In this example, parts 1–4 in product 1 are assigned to two metal sheets separately. Parts 5–8 in product 2 are also assigned to two metal sheets. Metal sheets are processed according to the Gantt chart. Scheduling the sheet-metal process typically involves a flexible-flow shop-scheduling problem[3]. However, some parts can skip processes based on a predetermined plan. In this example, all parts are required to be processed in the bending process. Then parts 1, 2, 5 and 8 are processed in the welding process, but parts 3, 4, 6 and 7 skip this process. Finally, all parts traverse an assembly process. Afterwards, a final product is completed.

Clearly, the relationship between nesting and scheduling requires trade-offs, because the parts of different products having different due dates could be assigned to the same metal sheet. For example, let us assume that the assignment of parts 6 and 8 exchanges. Part 8 is able to assign to sheet 1, but part 6 is not able to assign to sheet 2 because no enough space is left in sheet 2. Therefore, we need one more metal sheet for assigning part 6. This causes an increase in material waste.

On the other hand, the operation of part 8 in the bending process can start immediately after the operation of part 2. The operations of parts 5 and 8 in the welding and assembly processes can also start earlier. Finally, the completion time of product 2 becomes short. This schedule may meet the due date even if product 2 has a strict due date.

Therefore, in this study, we determine the cutting layout and the schedule considering that trade-off relationship.

2.2 Nesting Condition

To reduce waste material, as many parts as possible should be punched from one sheet. The parts are typically cut into polygonal shapes, but they are treated as solid rectangles in this study to simplify the analysis. Thus, we can regard the nesting problem of sheet-metal processing as a two-dimensional bin-packing problem[1].

In this study, when the set of parts $I$ and the set of metal sheets $K$ are given, we determine the metal sheet to which the part is assigned and the locations of parts on the sheet with the following constraints:

$$\sum_{k \in K} \gamma_{i,k} = 1, \forall i \in I,$$
the parts are placed within a metal sheet. Eq.(5) represents the constraint that the parts cannot be arranged when placed on a metal sheet. Eq.(2) represents the constraint that the parts are arranged in metal sheet k, \( \gamma_{i,k} = 1 \). Otherwise, \( \gamma_{i,k} = 0 \).

\[
\delta_k \geq \gamma_{i,k}, \forall i \in I, \forall k \in K, \quad (2)
\]
\[
0 \leq x_{i,j,k} \leq (W - w_i)\gamma_{i,k}, \forall i \in I, \forall k \in K, \quad (3)
\]
\[
0 \leq y_{i,j,k} \leq (L - l_i)\gamma_{i,k}, \forall i \in I, \forall k \in K, \quad (4)
\]
\[
\gamma_{i,k} = \gamma_{i',k}, \forall k \in K,
\]

where 
- \( k \): Metal sheet \( k \) (\( \forall k \in K \)),
- \( \delta_k \): varies between 0 and 1. If metal sheet \( k \) is used, \( \delta_k = 1 \). Otherwise, \( \delta_k = 0 \).
- \( i \): Part \( i \) (\( \forall i \in I \)),
- \( \gamma_{i,k} \): varies between 0 and 1. If part \( i \) is arranged in metal sheet \( k \), \( \gamma_{i,k} = 1 \). Otherwise, \( \gamma_{i,k} = 0 \).
- \( l_i \): Length of part \( i \),
- \( w_i \): Width of part \( i \),
- \( L \): Length of sheet metal,
- \( W \): Width of sheet metal,
- \( x_i \): \( x \)-coordinate of the lower left corner of part \( i \),
- \( y_i \): \( y \)-coordinate of the lower left corner of part \( i \).

Eq.(1) represents the constraint that the parts are placed on a metal sheet. Eq.(2) represents the constraint that the parts cannot be arranged when a sheet metal is not used. Eq.(3) and Eq.(4) ensure that the parts are placed within a metal sheet. Eq.(5) represents a constraint to prevent overlapping of parts.

### 2.3 Scheduling Problem

As mentioned in section 2.1, scheduling the sheet-metal process typically involves a flexible-flow shop-scheduling problem. However, some parts can skip processes based on a predetermined plan. We focus on each process. In the first punching process, the parts are cut from the sheet metal using a turret punch press or a laser-cutting machine. In the subsequent bending process, operators process parts one-by-one using a press brake. Afterwards, multiple parts are processed simultaneously via welding and assembly.

In our study, we determine an operational sequence of processes, except for a punching process, under the following conditions[8]:

1. Products consist of one or more parts.
2. All products have different due dates.
3. The required processes for each part are determined.
4. One or more pieces of equipment exist in each process, but the equipment assignment for each part is predetermined and fixed, by the process plan.
5. The processing time for each operation is predetermined.
6. The processing time of the punching operation is constant, regardless of the cutting layout.
7. During the bending process, each part is processed individually.
8. During the punching, welding, and assembly processes, several parts are processed simultaneously.
9. The operations of the welding and assembly processes must wait until the previous operations for all the parts in the same product are finished.

The following equations represent the constraints of our scheduling problem.

**[Constraints]**

\[
st_{i,m} + pt_{i,m} \leq st_{i,m+1}, \forall i \in I, \forall m \in M, \quad (6)
\]
\[
st_{m,n,v} + pt_{m,n,v} \leq st_{m,n,v+1}, \forall m \in M, \forall n \in N, \forall v = 1,\ldots,V, \quad (7)
\]

where 
- \( M \): Set of processes,
- \( m \): Process \( m \), \( \forall m \in M \),
- \( N \): Set of machines,
- \( n \): Machine \( n \), \( \forall n \in N \),
- \( V \): Number of operations,
- \( st_{i,m} \): Starting time of process \( m \) of part \( i \),
- \( pt_{i,m} \): Processing time of process \( m \) of part \( i \),
- \( st_{m,n,v} \): Starting time of \( v \)th operation in machine \( n \) at process \( m \),
- \( pt_{m,n,v} \): Processing time of \( v \)th operation in machine \( n \) at process \( m \).

Eq.(6) represents the constraint of process sequence. Eq.(7) represents a constraint in which multiple operations cannot be performed simultaneously at each machine. During the punching, welding, and assembly processes, operations executed on several parts simultaneously are regarded as one operation.

### 2.4 Objective Function and Fitness

In this study, we regard the problems of nesting and scheduling as different environments, and search for solutions by applying a GA. Solutions evolve under each environment while repetitively migrating. To ensure smooth migration between the two environments, we define cost-based objective functions for the nesting and scheduling problems[9]. We define the nesting and scheduling costs as follows:

\[
NC = mc \cdot am + dc \cdot as - rc \cdot ar + \sum_{p \in P} (oc_p \cdot tP_p + ic_p \cdot lP_p), \quad (8)
\]
\[
SC = pc \cdot tt + fc \cdot ms + \sum_{m \in M} (oc_m \cdot tot_m + ic_m \cdot ltm_m), \quad (9)
\]

where 
- \( NC \): Nesting cost,
- \( SC \): Scheduling cost,
- \( mc \): Material cost per unit area,
- \( dc \): Disposal cost of scrap metal per unit area,
- \( rc \): Recycle cost of reusable metal per unit area,
- \( am \): Gross area of sheet metal,
- \( as \): Gross area of scrap metal,
ar : Gross area of reusable metal,
acp : Operating cost of punching process per unit time,
ic : Idle cost of punching process per unit time,
tp : Total operating time of the punching process,
tit : Total idle time of the punching process,
pe : Penalty cost of tardiness per unit time,
fcc : Facility cost per unit time,
acm : Operating cost of machine m per unit time,
icm : Idle cost of machine m per unit time,
tt : Total tardiness,
ms : Makespan,
totm : Total operating time of machine m,
titm : Total idle time of machine m.

The first term of the right-hand side of eq. (8) represents the material cost, calculated as the product of the gross area of the sheet metal and the unit material cost. The gross area of the sheet metal can be calculated with the following equation:

\[ am = \sum_{k \in K} \delta_k LW. \]  \hspace{1cm} (10)

The second term is the disposal cost, calculated as the product of the gross area of the scrap metal and the unit disposal cost. The gross area of the scrap metal can be calculated with the following equation:

\[ as = \sum_{k \in K} \delta_k LW - \sum_{i \in I} l_i w_i - \sum_{k \in K} \delta_k (L - \max_{i \in I} \{ \gamma_{i,k} \cdot (y_i + l_i) \}) W. \]  \hspace{1cm} (11)

The third term represents the recycling cost, calculated as the product of the gross area and value of the reusable metal. In this study, we define the reusable metal as scrap metal with width \( W \), which can be calculated using the following equation:

\[ ar = \sum_{k \in K} \delta_k \cdot (L - \max_{i \in I} \{ \gamma_{i,k} \cdot (y_i + l_i) \}) W. \]  \hspace{1cm} (12)

The fourth term is the cost related to the punching process. The punching process operations depend on the cutting layout. Thus, we include this cost in the nesting cost. It can be calculated based on the Gantt chart data obtained from scheduling.

The scheduling cost represented by eq. (8) comprises a penalty cost for tardiness, a facility cost (a time-dependent fixed cost), and a running cost for each facility. Total tardiness and makespan can be obtained via the scheduling results. The running cost is the operating and idling costs of each resource. These costs are calculated based on scheduling results.

By using these cost functions, a fitness value, \( f \), of the GA is calculated as follows:

\[ f = w \left( \frac{NC_g \max_{g \in G} NC_g - \min_{g \in G} NC_g}{SC_g \max_{g \in G} SC_g - \min_{g \in G} SC_g} \right) + (1-w) \left( \frac{SC_g \max_{g \in G} SC_g - \min_{g \in G} SC_g}{NC_g \max_{g \in G} NC_g - \min_{g \in G} NC_g} \right), \]  \hspace{1cm} (13)

where \( w \) : weight for nesting cost,
\( g \) : individual ID in generation \( G \).

Because the nesting and scheduling costs have different scales, they are normalized to the range of 0–1, as shown in eq. (12).

3. Nesting Scheduling Algorithm

3.1 Basic Concepts

In this study, we propose an environment-adaptive GA-based search method, an extension of our previous study[13] to solve the problems of nesting and scheduling while accounting for their trade-off relationship. Fig. 5 shows the outline of the proposed method. We regard the nesting and scheduling problems as different environments (i.e., islands). Each island has a different objective function (e.g., scheduling or nesting costs), and the individual on each island evolves to adapt to each environment. They are treated as individuals with the same gene structure (i.e., an array of part names). Thus, a combination of cutting layout and schedule is created from an individual. On the scheduling island, the cutting layout and schedule are created to minimize the scheduling cost. On the nesting island, they are created to minimize the nesting cost. The initial population has an elite individual, one that increases convergence speed[14]. However, by dividing the environments, it can become difficult to obtain a Pareto solution. To avoid this, some individuals migrate to the other environments at constant intervals, as shown in Fig. 6. Moreover, in this study, to further search the middle area, we consider three different environments, as shown in Fig. 7, by extending the previous method.

3.2 Entire Procedure

The cutting layout and the operation sequence are determined using an environment-adaptive GA based on the following procedure.

Step 1: Creation of initial individuals

Initial individuals are created for the scheduling, nesting, and middle environments if they exist. The initial individuals include one elite and others that are created by sorting genes of the elite with individuals randomly. The elite individual for the nesting environment is created by sorting the parts by the part areas. In the nesting environment, the parts are assigned using a bottom-left algorithm. The accuracy of the nesting solution depends on the assignment order of the parts. It is known that a descending order for the part areas is effective for finding an appropriate solution. However, the elite individual for the scheduling environment is created by sorting the parts
based on the schedule created using an earliest-due-date-based dispatch rule. The elite individual for the middle environment is obtained from a preliminary experiment conducted using the single-environment GA.

Step 2: Fitness calculation

The fitness of each individual is calculated. A schedule and cutting layout are generated from one individual. First, the individual (i.e., an array of part names) is decoded to the temporary schedule by assigning the parts to the Gantt chart based on a process plan. For example, as shown in Fig. 8, a part represented by a gene in a chromosome is assigned to the bending process. Then, the operation sequence of welding and assembly is determined according to the constraint of the process sequence. However, at this point, the cutting layout has not been determined yet. Thus, the operations of the punching process cannot be generated. Therefore, a temporary schedule is obtained. Afterwards, the cutting layout is determined by assigning parts according to the starting time of the operations in the bending process. Parts assignment is done based on a bottom-left algorithm. By this procedure, the number of required metal sheets is obtained. Finally, the schedule including the punching process is determined by moving the operations forward as earlier as possible under the precedence constraint. The fitness value is calculated using eq. (13) based on the determined cutting layout and schedule. Weight $w$ is set to a high value for the nesting environment, a low value for the scheduling environment, and a middle value for the middle environment.

Step 3: Migration

When the current generation is the predetermined migration generation, execute migration according to the following sub-steps.

Step 3-1: Selection of migration individuals

Select a predetermined number of individuals as migrants in proportion to fitness using a roulette strategy.

Step 3-2: Exchange of migration individuals

Copies of the selected individuals are inserted onto the other island. For the three-environment case, half of the selected individuals move to one island and the
rest move to the other. The number of total individuals exceeds the predetermined population size by the insertion. Thus, the individuals with lower fitnesses are eliminated.

Step 4: Genetic operations
Step 4-1: Selection
Elite genes are inherited by the next generation using a preservation policy, and a pair of parents are selected using a random strategy. Immediately after the migrant individuals move to the other environment, the fitness value is relatively low. We therefore chose a random strategy to select migrant individuals with a certain probability.

Step 4-2: Crossover
Perform a two-point partially mapped crossover[4] for a given crossover rate.

Step 4-3: Mutation
Perform an inverse mutation for a given mutation rate.

Step 5: Termination
When the generation count reaches a predetermined number, the process is terminated. Otherwise, the process returns to Step 2.

To calculate fitness in Step 2, the individual is first decoded to the schedule, and then to the cutting layout. In the scheduling problem of sheet-metal processing, the bending process often becomes a bottleneck. A change of gene sequences affects the bending schedule very much. However, the position of parts in a metal sheet does not affect the nesting cost as much. Therefore, with our method, a sequence of genes influences the scheduling cost more than the nesting cost. In the scheduling environment, we need to control the operation sequence of the bending process. Thus, generating the schedule first is effective. On the other hand, even if the schedule is generated first, it is temporary, because the cutting layout is not determined yet. Therefore, in the nesting environment, even if the schedule is generated first, individuals evolve to minimize the nesting cost.

4. Numerical Experiments
To validate the effectiveness of the proposed method, we implemented a prototype of the nesting scheduling system using the object-oriented programming language, Smalltalk, and performed numerical experiments. In the experiments, we compared the nesting and scheduling costs obtained by the proposed method with those obtained by the single environment, two environments, and three environments. In the single environment, the weight, w, of the fitness was set to 0.5. We prepared a case study for sheet-metal processing using the conditions shown in Tables 1, 2. The GA parameters are listed in Table 3 based on preliminary experiments. The GA requires a probabilistic approach. Hence, we performed 20 trials. Figs. 9–11 show the typical results of the solutions in the last generation for the single environment, two environments, and three environments. Furthermore, Figs. 12–14 show the Pareto solutions. The horizontal axis shows the scheduling cost, and the vertical axis shows the nesting cost. As shown in Fig. 9, for a single environment, we obtained the solutions around the middle of the nesting and scheduling costs. However, in the case of two or three environments, we were able to obtain the solutions over a wide area, as shown in Figs. 10, 11. The round, square, and triangle markers in Figs. 10, 11 represent the solutions for scheduling, nesting, and middle environments, respectively. The large triangular dot in Fig. 11 represents the elite solution that was inserted in the initial population of the middle environment. Moreover, as shown in Figs. 12–14, a widespread Pareto solution was obtained with the proposed method, unlike the conventional method that used only a single environment.

To evaluate the accuracy and spread of the so-
Fig. 9 Solutions in last generation (single environment)

Fig. 10 Solutions in last generation (two environments)

Fig. 11 Solutions in last generation (three environments)

Fig. 12 Pareto solutions (single environment)

Fig. 13 Pareto solutions (two environments)

Fig. 14 Pareto solutions (three environments)


Lutions, we calculated the hyper-volume (HV)[7] and spread[6]. HV is the area of the polygon connecting the reference points and the solutions to become parallel to x and y axes, as shown in Fig. 15. The spread is the sum of the differences between the maximum and minimum values of the respective costs, as shown in Fig. 16. The results are summarized in Figs. 17, 18. We compared the HV and spread obtained by the single environment, two environments, and three environments. Case 1 is the same case shown in Figs. 12–14. Cases 2 and 3 are different cases under the conditions shown in Tables 1, 2. The branch number of the case indicates that only the processing time of the punching operation is differ-
The longer the processing time for the punching operation, the stronger the trade-off relation becomes. The difference between cases 1 and 2 is the number of products. Two times as much product volume is produced in case 3 compared to case 2. Higher values of HV and spread are better results. Compared to the results from the single environment, two and three environments obtained a higher value of HV and spread on average. Thus, widespread and accurate solutions can be obtained using the proposed method. Moreover, for three environments, we obtained a higher HV and a lower spread, on average, compared to those of the two-environment case. This result indicates that the solutions of the middle environment induce the solutions of other environments to their environment in the proposed method. This is effective when a decision-maker wants to obtain diverse solutions around the middle area.
We also validated the effectiveness of the migration. We compared the results obtained by the proposed method with and without migration. Figs. 19, 20 show the results of the proposed method without migration. Fig. 19 shows the distribution of solutions in the last generation, and Fig. 20 shows the Pareto solutions. Compared to Figs. 11, 14, the solutions obtained by the proposed method without migration were separated into both corners of the cost plot. From these results, we can see that the migrant individuals induced the other individuals in the direction of their original environment appropriately.

5. Conclusion

To optimize the cutting layout and operation sequence while accounting for the trade-off relationship between nesting and scheduling sheet-metal processing, we proposed an environment-adaptive GA-based nesting-scheduling method. With this method, the nesting and scheduling solutions evolve under different environments while sometimes exchanging their solutions. By preparing the different environments with different objective functions and including elite solutions (individuals) in each environment (island), the search ability is improved. Furthermore, widespread solutions can be obtained by allowing migration between environments. The effectiveness of our method was validated with numerical experiments.

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