Scalable Optimization Approach of Optimal PV Sizing Problem for Minimizing Network Loss and Inverter Loss*

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The use of photovoltaics (PV) in electric power networks has increased because of advantages such as power loss reduction, environmental friendliness, voltage profile improvement, and postponement of system upgrades. However, using PVs of an inappropriate size leads to greater power losses due to variations in PV outputs and demand loads. This paper formulates the optimal sizing PV problem (OPSP) for minimizing inverter losses and network losses. The OPSP is a large-scale and non-convex optimization that is difficult to solve. To resolve this computational issue, this paper proposes a new approach that consists of two steps. The first step is to relax the original non-convex problem into a convex one and to prove that the convexification is exact provided that over-satisfaction of the load is allowed. The second step is to decompose the original large-scale optimization into small-scale sub-problems. Moreover, the decomposed sub-problems can be computed in parallel. Numerical simulations implementing parallel processing verify the effectiveness of our approach.

1. Introduction

The share of renewable-energy-based distributed generators (DG) in power systems, especially distribution networks, has increased significantly in the past decade. Such renewable-energy DGs are playing important roles in distribution networks such as for power loss reduction, voltage profile improvement, and postponement of system upgrades [1,2].

Most renewable energy resources (RES) are intermittent compared with conventional energy resources. Yet, investment in RES is important for the future, in particular, to meet global targets for reducing greenhouse gas emissions.

Here, photovoltaics (PV) are major resources among the variety of RESs available, and installations of PVs have increased dramatically in recent years thanks to

feed-in tariff programs promoting them[3].

One of the important aspects of using PVs in distribution networks is that placement of PVs of the optimal size close to the load can reduce the power loss. However, inappropriate selection of the size of PVs near the energy consumption location leads to greater network losses. Therefore, it is necessary to determine the optimal size of PVs close to the load's location[4].

Other studies have discussed the optimal sizing of PVs for reducing total network losses in power grids[2,5–8]. Our review of the literature revealed that although the above cited studies did not consider inverter losses in determining the optimal size of PVs, some approaches have been proposed for the optimal operation of PVs considering inverter losses[9–11]. However, to the best of our knowledge, no work in the literature has studied the optimal sizing of PVs to reduce both network losses and inverter losses. For this reason, this paper addresses the optimal PV sizing problem (OPSP) for minimizing network losses and inverter losses.

In solving the OPSP, there are two issues. One is scalability. The performance of PV systems depends upon several factors, especially meteorological conditions such as solar radiation and ambient temperature. For appropriately determining the size of PVs, a lot of profiles of PV outputs and demand load levels should be considered because of their variations. However, doing so increases the scale of the problem to the point that it becomes difficult to solve. That is why the other studies[2,5–11] used data from a limited number of representative days so as not to make the

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problem large. The other problem is that the OPSP is generally formulated as non-convex quadratically constrained quadratic programming (QCQP) problem of calculating an exact power flow. Because of the non-convexity, it is hard to solve or it is hard to guarantee its convergence to optimality. As a way of dealing with these issues, in this paper, we propose a new approach for solving large-scale and non-convex OPSPs that includes two steps (i.e., convexification and decomposition).

First, we relax the non-convex original problem by applying a second-order cone relaxation and reformulate it as a second-order cone programming (SOCP) model, which has the following benefits: 1) efficient computation with a commercial solver (i.e., the SOCP can be solved in polynomial time by using interior point methods) and 2) convergence to optimality that is guaranteed because of its convexity. Moreover, we prove that this convexification is exact provided that over-satisfaction of the load is allowed.

Second, we decompose the relaxed large-scale problem into small-scale sub-problems. Our decomposition technique includes a partial Lagrangian relaxation (PLR) approach to solving the primal problem by using Lagrange multipliers and updating them with a sub-gradient method and projected sub-gradient method. No matter how large the primal problem becomes, our technique is able to decompose it into smaller sub-problems that can be solved more easily. Furthermore, our decomposed algorithm can be computed in parallel; we executed it on a 47-bus distribution feeder model to illustrate its effectiveness.

This paper is an extended version of our previous study[12], which also uses the convexification and decomposition method for a large-scale OPSP as well as this paper, but the effect of inverter losses is not considered. Therefore, it is necessary in this paper to develop our previous method in order to deal with non-convex constraints related to inverters.

Concretely, in convexification, this paper transforms the non-convex constraints related to inverters into convex constraints by using second-order cone relaxation, and also shows that the relaxation is exact. For decomposition, we apply the PLR to the relaxed constraints related to inverters, and newly employ the projected sub-gradient method as an update algorithm of Lagrange multipliers because a part of those constraints are inequalities. Additionally, this paper illustrates the usefulness of our proposed method by implementing parallel processing in numerical simulations, focusing on that our proposed method is easy to be computed in parallel.

In summary, the major contributions of this paper are as follows:

- we formulate (OPSP) considering inverter losses and network losses and obtain new knowledge about the effect of inverter losses on the PV allocation;
- we propose a new approach that consists of two

steps (i.e., convexification and decomposition) to solve the large-scale and non-convex optimization problem;

- we prove the proposed relaxations are exact provided that over-satisfaction of load is allowed;
- we show a parallel computing implementation of decomposed sub-problems to verify the effectiveness of our approach.

The rest of this paper is organized as follows. In the next section, literature related to this study is reviewed, including exactness of convex relaxation and decomposition method. The optimization problem is formulated in section 3. The new approach to solving the problem is proposed in section 4. It is tested on a 47-bus distribution feeder model and an overall summary of simulation results are presented in section 5. Conclusions are stated in section 6.

2. Literature Review

In this section, we describe the relationship with and differences from the previous studies regarding the exactness of convex relaxation and the decomposition method which are the core of our proposed framework, and clarify the position of this study.

2.1 Exactness of Convex Relaxation

First, we review related works regarding the exactness of convex relaxation. In power system analysis and design, it is important to calculate the optimal power flow (OPF) in order to determine the optimal allocations of power equipment and the optimal operations of power systems. Accordingly, many studies have addressed the OPF problem since it was first proposed by [13] in 1962. The OPF problem seeks to optimize a certain objective function, such as power loss and generation cost subject to various electrical constraints such as power balance, voltage and current limits. In general, it is difficult to solve the OPF problem due to non-convex constraints.

There are three main approaches to resolving the issue; (1) approximating non-convex constraints, (2) searching for local solutions, and (3) relaxing non-convex constraints. In this paper, we focus on (3), which can deal with an exact power flow model and guarantees convergence to the global optimum under certain conditions.

Many studies have investigated convexification in the OPF problem so far. For instance, references [14] and [15] proposed SOCP relaxation and semidefinite programming (SDP) relaxation of non-convex OPF respectively, and conducted numerical analysis on power flow. However, they did not examine the exactness of convex relaxation.

The exactness of convex relaxation in bus-injection-model was first discussed in [16]. This study presents the necessary and sufficient conditions of exactness (i.e., the optimal value of the original problem equals the optimal value of the semidefinite relaxation problem).

On the other hand, branch-flow-model was first proposed in [17] and [18] based on dist-flow-model by Farivar et al., and the exactness of the SOCP relaxation was also proven under certain conditions. Furthermore, references [19] researches the exactness of angle relaxation that has not been taken into consideration in dist-flow-model[20]. The conditions regarding the exactness of convex relaxation in the case of radial-networks are described in detail in [21]. Finally, references [17] and [18] show that the bus-injection-model and the branch-flow-model are equivalent.

Based on the above discussion, from the viewpoint of research on the exactness of convex relaxation, this paper is positioned as an extension of [17] and [18] for the OPSP. Specifically, references [17] and [18] mainly focus on determining the optimal operation of the inverter to minimize the power loss cost given the PV capacity, and prove the exactness of the convex relaxation in that problem. However, in terms of voltage control and minimization of power loss, the PV capacity has a very large influence on the grid, and it is extremely important to determine the optimal sizing of PV capacity.

Therefore, in this paper, we propose a model that can simultaneously determine the optimal sizing of PV capacity and the optimum inverter operation for minimizing the power loss, and prove the exactness of convex relaxation for that model by developing [17] and [18].

2.2 Decomposition Method

Second, we will discuss related works on decomposition methods. Optimization problems in power systems are often complicated and large-scale. In order to efficiently solve these problems, many decomposition methods have been proposed. Typical decomposition methods include Lagrangian relaxation (LR)[22], benders decomposition (BD)[23] and alternating direction method of multipliers (ADMM)[24], and appropriate decomposition methods should be selected according to the optimization problem.

LR has been applied in many studies[25–29]. In [25] and [26], LR is employed to the SDP relaxation problem, while it is employed to the multi-area OPF in [27]. Furthermore, as a more practical applications, a solution method which combines LR and dynamic programming (DP) was proposed to unit commitment (UC) problems[29]. In [28], a new framework for a large-scale charge / discharge problem of an electric vehicle is proposed by combining the advantages of MILP and LR.

BD also has been used in many optimization problems in power systems[30–33]. In [30], the multi-area economic dispatch (ED) problem is efficiently solved by using BD. Moreover, BD is implemented for stochastic two-level programming problems to specify the optimal size of energy storage systems (ESS) in the electricity market in [32]. In determining the optimal allocation of ESS considering network recon-

figuration, the reference [31] succeeded in solving a large-scale MIQCQP by dividing it into two parts (i.e. 0-1 variable part and continuous variable part) and solving it alternately. For the power network design problem, GA-Benders' Decomposition combining GA and BD is proposed and significant reduction of calculation time is realized in [33].

Application of ADMM in power systems is considered in [11,34–37]. In [11], ADMM is applied for the SDP-relaxed formulation and while the SOCP-relaxed formulation is applied in [34], respectively. As a more practical applications, in order to obtain the optimal ESS allocation, a solution method which divides the problem into ESS allocation part and operation part and solves them alternately was proposed in [36]. In addition, a distributed algorithm based on ADMM is proposed for the large-scale transmission switching problem[38], the multi-area ED problem[35] and virtual power plant (VPP) problem[37]. In addition, other algorithms developing ADMM such as Auxiliary Problem Principle (APP)[39], Analytical Target Cascading (ATC)[40] and Optimality Condition Decomposition (OCD)[41] have been applied to optimization problems in power systems in [42,43,41], respectively.

As mentioned above, there are many application examples of decomposition methods to optimization problem in power systems, and it is necessary to determine which decomposition method is applied according to the formulation and characteristics of the optimization problems.

Thus, in this paper, to decompose a large-scale OPSP considering inverters, we apply the LR to a part of constraints (i.e., the PLR approach), focusing on the feature of the optimization problem that the PV capacity does not depend on time. By doing so, we can reduce the number of Lagrange multipliers. Furthermore, the Partial Lagrangian Dual Problem is maximized by using the sub-gradient method and projected sub-gradient method.

3. Problem Formulation

This section formulates the OPSP for minimizing network losses and inverter losses. We use the network model (DistFlow equations) first introduced in [20]. Let $G(N, E)$ be a graph representing a radial distribution circuit. Each node in N is a bus, and each link in E is a line. We index the nodes by $i = 1, \dots, |N|$. Let $V_{i,t}$ denote the complex voltage in node i at time t and $I_{ij,t}$ be the complex current flowing from node i to j , respectively. $V_{i,t}^{sqr}, I_{ij,t}^{sqr}$ is defined as follows:

$$V_{i,t}^{sqr} := |V_{i,t}|^2 \quad (1)$$

$$I_{ij,t}^{sqr} := |I_{ij,t}|^2 \quad (2)$$

We assume that the power factor of the PVs is 1.0 (i.e., direct current). Let N^{pv} denote the set of nodes with PVs, and let $P_{j,t}^{pv}, S_j^{pv,max}$, and α_t^{pv} be the real

power outputs generated by the PVs, the PV rated apparent power outputs under standard test conditions (STC), and the PV generation rate. According to [44,45], the relationship between these terms is

$$P_{i,t}^{pv} = \frac{G_t}{1000} [1 + C(T_t^{cell} - T_0)] S_i^{pv,max} \quad (3)$$

$$T_t^{cell} = T_t^{amb} + G_t \left(\frac{NOCT - 20}{800} \right) \quad (4)$$

where G_t , T_t^{cell} and T_t^{amb} respectively correspond to the solar radiation (W/m^2), cell temperature ($^{\circ}C$), and ambient temperature ($^{\circ}C$) at time t . T_0 , C and $NOCT$ are constants that stand for the standard temperature, a factor which depends on the PV panel type, and nominal operating cell-temperature conditions ($^{\circ}C$). Here, we set $T_0 = 25^{\circ}C$, $C = -0.00045/^{\circ}C$ and $NOCT = 50(^{\circ}C)$. We define the PV generation rate α_t^{pv} as follows:

$$\alpha_t^{pv} = \frac{G_t}{1000} [1 + C(T_t^{cell} - T_0)] \quad (5)$$

α_t^{pv} can be calculated using G_t and T_t^{cell} .

For node i , let $P_{i,t}^d$ and $Q_{i,t}^d$ be the real and reactive power demand, respectively, at time t . If there is no load in node i , we set $P_{i,t}^d \equiv Q_{i,t}^d \equiv 0$ for all t . Similarly, if there is no PV at node i , we set $S_j^{pv,max} \equiv 0$. Here, the real and reactive power demand $P_{i,t}^d$ and $Q_{i,t}^d$ in node i at time t are given.

In determining the optimal size of PVs for minimizing the network loss, we need to take into account lots of profiles of PV outputs $S_{j,t}^{pv}$ and demand load levels $P_{i,t}^d, Q_{i,t}^d$ because of their variations. This enlarges the OPSP.

3.1 Objective Function

As mentioned above, to minimize total power losses, our objective function considers network losses and inverter losses.

- network losses

$$\sum_{t \in T} \sum_{(i,j) \in E} r_{ij} I_{ij,t}^{sqr} \quad (6)$$

where r_{ij} is the resistance from node i to node j .

- inverter losses

$$\sum_{t \in T} \sum_{i \in N^{pv}} P_{i,t}^{loss} \quad (7)$$

$$\begin{aligned} P_{i,t}^{loss} &= c_s + c_v S_{i,t}^{pv} + c_r (S_{i,t}^{pv})^2 \\ &= c_s + c_v \sqrt{(P_{i,t}^{pv})^2 + (Q_{i,t}^{pv})^2} \\ &\quad + c_r \left\{ (P_{i,t}^{pv})^2 + (Q_{i,t}^{pv})^2 \right\} \end{aligned} \quad (8)$$

where c_s is the inverter's standby loss, c_v is the voltage-dependent loss over the power electronic components, which is proportional to the current, c_r is the ohmic loss proportional to square of the current, and $P_{j,t}^{pv}$ and $Q_{j,t}^{pv}$ are respectively the active and reactive power outputs of PVs in node i at time t .

Fig. 1 Illustrative power flows with three nodes

$S_{i,t}^{pv} = \sqrt{(P_{i,t}^{pv})^2 + (Q_{i,t}^{pv})^2}$ is the magnitude of the apparent power injection of the inverter in node i at time t .

3.2 Constraints

The constraints of the OPSP are formulated as follows.

- power balance constraints, $\forall j \in N$ and $t \in T$,

$$P_{i,j,t} = \sum_{k:(j,k) \in E} P_{j,k,t} + r_{ij} I_{ij,t}^{sqr} - P_{j,t}^{pv} + P_{j,t}^d \quad (9)$$

$$Q_{i,j,t} = \sum_{k:(j,k) \in E} Q_{j,k,t} + x_{ij} I_{ij,t}^{sqr} - Q_{j,t}^{pv} + Q_{j,t}^d \quad (10)$$

$$P_{i,j,t}^2 + Q_{i,j,t}^2 = I_{ij,t}^{sqr} V_{i,t}^{sqr} \quad (11)$$

where x_{ij} is the reactance from node i to node j , and $P_{i,j,t}$ and $Q_{i,j,t}$ are respectively active and reactive power flowing from node i to node j at time t , and $P_{i,t}^d$ and $Q_{i,t}^d$ are respectively the active and reactive power demands in node i . Illustrative power flows are shown in Fig. 1.

- branch equation constraints, $\forall i \in N$ and $\forall t \in T$,

$$V_{i,t}^{sqr} = V_{j,t}^{sqr} + 2(r_{ij} P_{i,j,t} + x_{ij} Q_{i,j,t}) - z_{ij}^2 I_{ij,t}^{sqr}, \quad \forall j \in N(i) \quad (12)$$

$$V_{i,t}^{sqr} = (V^{ss})^2, \forall i \in N^{ss} \quad (13)$$

where $z_{ij} = r_{ij} + jx_{ij}$ is the impedance from node i to node j , V^{ss} is a constant representing the substation voltage and N^{ss} represents the set of substation buses. These constraints can be derived from Ohm's law.

- voltage magnitude constraints, $\forall i \in N$ and $\forall t \in T$,

$$(\underline{V})^2 \leq V_{i,t}^{sqr} \leq (\bar{V})^2 \quad (14)$$

where $\bar{V}$ and $\underline{V}$ are respectively the minimum and maximum voltage magnitude. Note that $V_{i,t}^{sqr} \geq (\underline{V})^2 \geq 0$ holds.

- PV inverter constraints, $\forall i \in N^{pv}$ and $\forall t \in T$,

$$P_{i,t}^{pv} = \alpha_t^{pv} S_i^{pv,max} \quad (15)$$

$$-\sqrt{1 - (\alpha_t^{pv})^2} \cdot S_i^{pv,max} \leq Q_{i,t}^{pv} \quad (16)$$

Fig. 2 Illustrative PV inverter

$$Q_{i,t}^{pv} \leq \sqrt{1 - (\alpha_t^{pv})^2} \cdot S_i^{pv,max} \quad (17)$$

$$S_{i,t}^{pv} = \sqrt{(P_{i,t}^{pv})^2 + (Q_{i,t}^{pv})^2} \quad (18)$$

$$0 \leq S_i^{pv,max} \leq \overline{S_i^{pv,max}} \quad (19)$$

where $\alpha_t^{pv} \in [0, 1]$ is a scalar that represents the generation level of the PVs at time t , the variable $S_i^{pv,max}$ is the PV apparent power capacity in node i under standard test conditions (STC), and $\overline{S_i^{pv,max}}$ is the maximum PV apparent power capacity in node i . Fig. 2 is an illustration of a PV inverter.

3.3 Original Problem

From the objective function and constraints mentioned above, the original problem (OP) of the OPSP can be formulated as follows:

(OP)

$$\min \sum_{t \in T} \left(\sum_{(i,j) \in E} r_{ij} I_{ij,t}^{sqr} + \sum_{i \in N^{pv}} P_{i,t}^{loss} \right) \quad (20)$$

$$\text{s.t. eqs. (9)–(19)} \quad (21)$$

$$\text{over } \mathbf{X} := (\mathbf{P}, \mathbf{Q}, \mathbf{I}^{sqr}, \mathbf{V}^{sqr}, \mathbf{P}^{pv}, \mathbf{Q}^{pv}, \mathbf{S}^{pv}, \mathbf{S}^{pv,max}) \quad (22)$$

where $\mathbf{P}, \mathbf{Q}, \mathbf{I}^{sqr}, \mathbf{V}^{sqr}, \mathbf{P}^{pv}, \mathbf{Q}^{pv}, \mathbf{S}^{pv}$, and $\mathbf{S}^{pv,max}$ denote the vector variables

$$\mathbf{P} := \{P_{ij,1}, \dots, P_{ij,|T|}, \forall (i,j) \in E\},$$

$$\mathbf{Q} := \{Q_{ij,1}, \dots, Q_{ij,|T|}, \forall (i,j) \in E\},$$

$$\mathbf{I}^{sqr} := \{I_{ij,1}^{sqr}, \dots, I_{ij,|T|}^{sqr}, \forall (i,j) \in E\},$$

$$\mathbf{V}^{sqr} := \{V_{i,1}^{sqr}, \dots, V_{i,|T|}^{sqr}, \forall i \in N\},$$

$$\mathbf{S}^{pv} := \{S_{j,1}^{pv}, \dots, S_{j,|T|}^{pv}, \forall j \in N^{pv}\},$$

$$\mathbf{P}^{pv} := \{P_{j,1}^{pv}, \dots, P_{j,|T|}^{pv}, \forall j \in N^{pv}\},$$

$$\mathbf{Q}^{pv} := \{Q_{j,1}^{pv}, \dots, Q_{j,|T|}^{pv}, \forall j \in N^{pv}\},$$

$$\text{and } \mathbf{S}^{pv,max} := \{S_j^{pv,max}, \forall j \in N^{pv}\}.$$

It is easy to see that the objective function of (OP) is convex with respect to the state vector $\mathbf{X}$. In addition, the constraints of (OP) are convex with respect to $\mathbf{X}$ except for eqs. (11), (18). (OP) is non-convex because these nonlinear equality constraints eqs. (11), (18) are the sources of the non-convexity. Thus, (OP) is hard to solve in this form.

4. Proposed Approach

This section describes a framework of our approach to determine the optimal sizes of PVs for minimizing network losses and inverter losses. The framework consists of two steps: convexification of the original non-convex problem by using second-order cone relaxation (Section 4.1), and decomposition for the large-scale problem by using the sub-gradient method and projected sub-gradient method (Section 4.2). The framework is shown in Fig. 3.

Original Problem

(OP): QCQP model

- non-convex
- large-scale

Convexification

Relaxed Problem

(RP): SOCP model

- convex
- large-scale

Decomposition

Sub-Problems

(Sub1) (Sub2)

⋮ ⋮

(Sub1) (Sub2)

- convex
- small-scale

Fig. 3 Framework of our approach

4.1 Convexification Technique

In section 3., the OPSP is formulated as a non-convex QCQP model. However, the non-convexity of this problem makes it hard to guarantee convergence to optimality. Here, we relax the non-convex constraints eqs. (11), (18) as follows:

$$P_{i,j,t}^2 + Q_{i,j,t}^2 \leq V_{i,t}^{sqr} I_{i,j,t}^{sqr} \quad (23)$$

$$\sqrt{(P_{i,t}^{pv})^2 + (Q_{i,t}^{pv})^2} \leq S_{i,t}^{pv} \quad (24)$$

Under the condition $V_{i,t}^{sqr} \geq (V)^2 \geq 0$, these relaxed constraints can be written as second-order cone constraints eqs. (25), (26) that are convex:

$$\left\| \begin{bmatrix} 2P_{ij,t} \\ 2Q_{ij,t} \\ I_{ij,t}^{sqr} - V_{i,t}^{sqr} \end{bmatrix} \right\|_2 \leq I_{ij,t}^{sqr} + V_{i,t}^{sqr} \quad (25)$$

$$\left\| \begin{bmatrix} P_{ij,t} \\ Q_{ij,t} \end{bmatrix} \right\|_2 \leq S_{i,t}^{pv} \quad (26)$$

where $\|\cdot\|_2$ represents 2-norm. We will later prove that the inequalities are tight in any optimal solution. After making the above relaxations, the (OP) is reformulated into the following relaxed problem (RP).

(RP)

$$\min \sum_{t \in T} \left(\sum_{(i,j) \in E} r_{ij} I_{ij,t}^{sqr} + \sum_{i \in N^{pv}} P_{i,t}^{loss} \right) \quad (27)$$

s.t. eqs. (9), (10), (12)–(17), (19), (25), (26)

$$\underline{P}_{i,t}^d \leq P_{i,t}^d, \forall i \in N^d, t \in T \quad (28)$$

$$\underline{Q}_{i,t}^d \leq Q_{i,t}^d, \forall i \in N^d, t \in T \quad (29)$$

$$\text{over } \mathbf{X}' := (\mathbf{P}, \mathbf{Q}, \mathbf{I}^{sqr}, \mathbf{V}^{sqr}, \mathbf{S}^{pv}, \mathbf{S}^{pv,max}, \mathbf{P}^{pv}, \mathbf{Q}^{pv}, \mathbf{P}^d, \mathbf{Q}^d) \quad (30)$$

where $\underline{P}_{i,t}^d$ and $\underline{Q}_{i,t}^d$ are respectively the original active and reactive power demands in node i at time t . Hence, we use the power demands $P_{i,t}^d$ and $Q_{i,t}^d$ of (OP) for the parameters $\underline{P}_{i,t}^d$ and $\underline{Q}_{i,t}^d$ in (RP), respectively.

Here, (RP) can be described as a convex QCQP model, since the objective function is a convex quadratic function eq. (27) and eqs. (25), (26) are second-order cone constraints. Therefore, the convexity of (RP) guarantees convergence to optimality. In addition, a convex QCQP model can be formulated as an SOCP model by reformulating the objective function as a constraint. Thus, (RP) can be efficiently solved by commercial solvers.

In transforming (OP) into (RP), there are two kind of relaxation. First, the equalities (i.e., eqs. (11), (18)) are relaxed to inequalities in eqs. (25), (26). Second, we allow “over-satisfaction” of all active and reactive loads eqs. (28), (29). These constraints mean that it suffices to allow any extra amount of active/reactive power to be wasted (thrown away) at each node of the network (if necessary). This assumption has been considered in other papers as well [46–48], as it is expected to reach to the same optimal solution regardless of the over-satisfaction assumption.

[Theorem 1] The optimal solution of the relaxed problem (RP) is also optimal for the original problem (OP).

(Proof) A similar theorem is proved in [18]. To show that the convex relaxation is exact, an optimal solution of (RP) has equality in eqs. (25), (26). First, we assume that $\mathbf{X}'^* := (\mathbf{P}^*, \mathbf{Q}^*, \mathbf{I}^{sqr*}, \mathbf{V}^{sqr*}, \mathbf{P}^{pv*}, \mathbf{Q}^{pv*}, \mathbf{S}^{pv*}, \mathbf{S}^{pv,max*}, \mathbf{P}^{d*}, \mathbf{Q}^{d*})$ is the optimal solution and that for some ij, t at least one of eq. (31) and eq. (32) holds.

$$P_{ij,t}^{*2} + Q_{ij,t}^{*2} < V_{i,t}^{sqr*} I_{ij,t}^{sqr*} \quad (31)$$

$$\sqrt{(P_{i,t}^{pv*})^2 + (Q_{i,t}^{pv*})^2} < S_{i,t}^{pv*} \quad (32)$$

There are two cases;

(i) eq. (31) holds

Now, using $\epsilon_1 > 0$, consider another solution $\tilde{\mathbf{X}}' := (\tilde{\mathbf{P}}, \tilde{\mathbf{Q}}, \tilde{\mathbf{I}}^{sqr}, \tilde{\mathbf{V}}^{sqr}, \tilde{\mathbf{S}}^{pv}, \tilde{\mathbf{P}}^{pv}, \tilde{\mathbf{Q}}^{pv}, \tilde{\mathbf{S}}^{pv,max}, \tilde{\mathbf{P}}^d, \tilde{\mathbf{Q}}^d)$. $\tilde{\mathbf{X}}'$ is as follows:

$$\begin{aligned} \tilde{P}_{ij,t} &= P_{ij,t}^* - r_{ij}\epsilon_1/2, \tilde{P}_{-ij,t} = P_{-ij,t}^*, \\ \tilde{Q}_{ij,t} &= Q_{ij,t}^* - x_{ij}\epsilon_1/2, \tilde{Q}_{-ij,t} = Q_{-ij,t}^*, \\ \tilde{I}_{ij,t}^{sqr} &= I_{ij,t}^{sqr*} - \epsilon_1, \tilde{I}_{-ij,t}^{sqr} = I_{-ij,t}^{sqr*}, \\ \tilde{V}_{i,t}^{sqr} &= V_{i,t}^{sqr*}, \tilde{S}_{j,t}^{pv} = S_{j,t}^{pv*}, \\ \tilde{P}_{j,t}^{pv} &= P_{j,t}^{pv*}, \tilde{Q}_{j,t}^{pv} = Q_{j,t}^{pv*}, \tilde{S}_{j,t}^{pv,max} = S_{j,t}^{pv,max*}, \\ \tilde{P}_{i,t}^d &= P_{i,t}^{d*} + r_{ij}\epsilon_1/2, \tilde{P}_{j,t}^d = P_{j,t}^{d*} + r_{ij}\epsilon_1/2, \\ \tilde{Q}_{i,t}^d &= Q_{i,t}^{d*} + x_{ij}\epsilon_1/2, \tilde{Q}_{j,t}^d = Q_{j,t}^{d*} + x_{ij}\epsilon_1/2, \\ \tilde{P}_{-(i,j),t}^d &= P_{-(i,j),t}^{d*}, \tilde{Q}_{-(i,j),t}^d = Q_{-(i,j),t}^{d*}, \end{aligned}$$

where $\tilde{P}_{-ij,t}$, $\tilde{Q}_{-ij,t}$ and $\tilde{I}_{-ij,t}^{sqr}$ are respectively the variables which exclude the index ij, t from $\tilde{\mathbf{P}}, \tilde{\mathbf{Q}}$ and $\tilde{\mathbf{I}}^{sqr}$. Similarly, $\tilde{P}_{-(i,j),t}^d$ and $\tilde{Q}_{-(i,j),t}^d$ are respectively the variables which exclude the indexes i, t and j, t from $\tilde{\mathbf{P}}^d$ and $\tilde{\mathbf{Q}}^d$. Now, $\tilde{\mathbf{X}}'$ is a feasible solution, since it satisfies eqs. (9), (10), (12)–(17), (19), (25), (26), (28), (29). By substituting $\tilde{\mathbf{X}}'$ for each constraint, it can be confirmed that $\tilde{\mathbf{X}}'$ actually satisfies all constraints at all nodes (see **Appendix**). Furthermore, the objective value of $\tilde{\mathbf{X}}'$ is obviously smaller than the object value of $\mathbf{X}'$, since $\tilde{I}_{-ij,t}^{sqr} = I_{-ij,t}^{sqr*} - \epsilon_1$.

(ii) eq. (32) holds

As above, using $\epsilon_2 > 0$, consider another solution $\hat{\mathbf{X}}' := (\hat{\mathbf{P}}, \hat{\mathbf{Q}}, \hat{\mathbf{I}}^{sqr}, \hat{\mathbf{V}}^{sqr}, \hat{\mathbf{S}}^{pv}, \hat{\mathbf{P}}^{pv}, \hat{\mathbf{Q}}^{pv}, \hat{\mathbf{S}}^{pv,max}, \hat{\mathbf{P}}^d, \hat{\mathbf{Q}}^d)$. $\hat{\mathbf{X}}'$ is as follows:

$$\begin{aligned} \hat{P}_{ij,t} &= P_{ij,t}^*, \hat{P}_{-ij,t} = P_{-ij,t}^*, \\ \hat{Q}_{ij,t} &= Q_{ij,t}^*, \hat{Q}_{-ij,t} = Q_{-ij,t}^*, \\ \hat{I}_{ij,t}^{sqr} &= I_{ij,t}^{sqr*}, \hat{I}_{-ij,t}^{sqr} = I_{-ij,t}^{sqr*}, \\ \hat{V}_{i,t}^{sqr} &= V_{i,t}^{sqr*}, \hat{S}_{j,t}^{pv} = S_{j,t}^{pv*} - \epsilon_2, \\ \hat{P}_{j,t}^{pv} &= P_{j,t}^{pv*}, \hat{Q}_{j,t}^{pv} = Q_{j,t}^{pv*}, \hat{S}_{j,t}^{pv,max} = S_{j,t}^{pv,max*}, \\ \hat{P}_{i,t}^d &= P_{i,t}^{d*}, \hat{P}_{j,t}^d = P_{j,t}^{d*}, \\ \hat{Q}_{i,t}^d &= Q_{i,t}^{d*}, \hat{Q}_{j,t}^d = Q_{j,t}^{d*}, \\ \hat{P}_{-(i,j),t}^d &= P_{-(i,j),t}^{d*}, \hat{Q}_{-(i,j),t}^d = Q_{-(i,j),t}^{d*}, \end{aligned}$$

where the definition of indices and variables are the same as in the case (i). $\hat{\mathbf{X}}'$ is also a feasible solution that satisfies eqs. (9), (10), (12)–(17), (19), (25), (26), (28), (29), and the objective value of $\hat{\mathbf{X}}'$ also becomes smaller the objective value of $\mathbf{X}'$, since $\hat{S}_{j,t}^{pv} = S_{j,t}^{pv*} - \epsilon_2$.

This statement in cases (i) and (ii) can be proved for any t , and there are different optimal solutions in both cases (i) and (ii). This contradicts the assumption that $\mathbf{X}'$ is the optimal solution. Therefore, an optimal solution of the relaxed problem (RP) is also an optimal solution of the original problem (OP) $\square$

4.2 Decomposition Technique

The aim of the optimal PV sizing problem is to determine the apparent power capacity of PV ($\mathbf{S}^{pv,max}$). Moreover, a lot of load profiles and weather conditions must be considered, meaning that the problem becomes large and it becomes difficult to solve because $\mathbf{S}^{pv,max}$ does not depend on time t . In this

section, we propose a decomposition method for a large-scale OPSP that takes a PLR approach. This method decomposes the primal problem (RP) into cardinality $|N^{pv}|$ sub-problems per PV generator $j \in N^{pv}$ (i.e., Sub1_j) and T sub-problems per unit time t (i.e., Sub2_t).

4.2.1 Partial Lagrangian Dual Problem

Partial Lagrangian relaxation is a relaxation method that puts some of the constraints into the objective function with Lagrange multipliers and approximates a difficult problem by a simpler one. Partial Lagrangian relaxation is described in more detail in [49]. As mentioned above, the variable vector $\mathbf{S}^{pv,max}$ in our formulation (RP) makes the problem difficult because it does not depend on the unit time t . Therefore, to make it easier, we incorporate the constraints related to $\mathbf{S}^{pv,max}$ (i.e., eqs. (15)–(17)) into the objective function eq. (20) through the Lagrange multipliers $\lambda_{1,j,t}$, $\lambda_{2,j,t}$ and $\lambda_{3,j,t}$. The partial Lagrangian function can be written as follows:

$$\begin{aligned} L(\mathbf{X}', \lambda_1, \lambda_2, \lambda_3) = & \sum_{t=1}^T \sum_{(i,j) \in E} r_{ij} I_{ij,t}^{sqr} + \sum_{t \in T} \sum_{i \in N^{pv}} P_{i,t}^{loss} \\ & - \sum_{t=1}^T \sum_{j \in N^{pv}} \lambda_{1,j,t} \left(-P_{j,t}^{pv} + \alpha_t^{pv} S_j^{pv,max} \right) \\ & - \sum_{t=1}^T \sum_{j \in N^{pv}} \lambda_{2,j,t} \left(\sqrt{1 - (\alpha_t^{pv})^2} S_j^{pv,max} + Q_{j,t}^{pv} \right) \\ & - \sum_{t=1}^T \sum_{j \in N^{pv}} \lambda_{3,j,t} \left(\sqrt{1 - (\alpha_t^{pv})^2} S_j^{pv,max} - Q_{j,t}^{pv} \right) \end{aligned} \quad (33)$$

where $\lambda_1 := \{\lambda_{1,j,t} \forall j \in N^{pv}, t = 1, \dots, T\}$, $\lambda_2 := \{\lambda_{2,j,t} \forall j \in N^{pv}, t = 1, \dots, T\}$, $\lambda_3 := \{\lambda_{3,j,t} \forall j \in N^{pv}, t = 1, \dots, T\}$. The partial Lagrangian dual problem (PLDP) of the primal problem (RP) is as follows:

(PLDP)

$$\max_{\lambda_1, \lambda_2, \lambda_3} \phi(\lambda_1, \lambda_2, \lambda_3) \quad (34)$$

$$\text{s.t. } \lambda_2 \geq 0 \quad (35)$$

$$\lambda_3 \geq 0 \quad (36)$$

where

$$\phi(\lambda_1, \lambda_2, \lambda_3) = \min_{\mathbf{X}' \in \Omega} L(\mathbf{X}', \lambda_1, \lambda_2, \lambda_3) \quad (37)$$

Ω represents a set of vector variables $\mathbf{X}'$ that satisfy the constraints of (RP) (i.e., eqs. (9), (10), (12)–(14), (19), (25), (26)). eqs. (35), (36) are also necessary because λ_2, λ_3 are Lagrangian multiples that are related to inequality constraints.

In minimizing the objective function of the primal problem, the solution to the partial dual problem provides a lower bound to the solution of the primal one. Here, eq. (37) can be rewritten as follows:

$$\begin{aligned} \phi(\lambda_1, \lambda_2, \lambda_3) = & \sum_{j \in N^{pv}} \min_{S_j^{pv,max}} L_1(S_j^{pv,max}, \lambda_{1,j}, \lambda_{2,j}, \lambda_{3,j}) \\ & + \sum_{t=1}^T \min_{\mathbf{X}_t''} L_2(\mathbf{X}_t'', \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}) \end{aligned} \quad (38)$$

$$\begin{aligned} L_1(S_j^{pv,max}, \lambda_{1,j}, \lambda_{2,j}, \lambda_{3,j}) = & \left(\sum_{t=1}^T \left\{ -\alpha_t^{pv} \lambda_{1,j,t} - \sqrt{1 - (\alpha_t^{pv})^2} \lambda_{2,j,t} \right. \right. \\ & \left. \left. - \sqrt{1 - (\alpha_t^{pv})^2} \lambda_{3,j,t} \right\} \right) S_j^{pv,max} \end{aligned} \quad (39)$$

$$\begin{aligned} L_2(\mathbf{X}_t'', \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}) = & \sum_{t \in T} \sum_{(i,j) \in E} r_{ij} I_{ij,t}^{sqr} + \sum_{t \in T} \sum_{i \in N^{pv}} P_{i,t}^{loss} \\ & + \sum_{j \in N^{pv}} \left(\lambda_{1,j,t} P_{j,t}^{pv} - \lambda_{2,j,t} Q_{j,t}^{pv} + \lambda_{3,j,t} Q_{j,t}^{pv} \right) \end{aligned} \quad (40)$$

where $\lambda_{1,j} := \{\lambda_{1,j,1}, \dots, \lambda_{1,j,T}\}$, $\lambda_{2,j} := \{\lambda_{2,j,1}, \dots, \lambda_{2,j,T}\}$, $\lambda_{3,j} := \{\lambda_{3,j,1}, \dots, \lambda_{3,j,T}\}$, $\lambda_{1,t} := \{\lambda_{1,1,t}, \dots, \lambda_{1,|N^{pv}|,t}\}$, $\lambda_{2,t} := \{\lambda_{2,1,t}, \dots, \lambda_{2,|N^{pv}|,t}\}$, $\lambda_{3,t} := \{\lambda_{3,1,t}, \dots, \lambda_{3,|N^{pv}|,t}\}$ and $\mathbf{X}'' := \mathbf{X}' \setminus \mathbf{S}^{pv,max}$ (i.e., $\mathbf{X}''$ is generated by removing $\mathbf{S}^{pv,max}$ from $\mathbf{X}'$). The above decomposes the original problem into two kinds of sub-problem that are easier to solve.

The sub-problems associated with the rated power of the PVs are as follows:

(Sub1_j) $j = 1, \dots, |N^{pv}|$

$$\min_{S_j^{pv,max}} L_1(S_j^{pv,max}, \lambda_{1,j}, \lambda_{2,j}, \lambda_{3,j}) \quad (41)$$

s.t. eq. (19).

Here, ψ is defined as

$$\begin{aligned} \psi = & \sum_{t=1}^T \left\{ -\alpha_t^{pv} \lambda_{1,j,t} - \sqrt{1 - (\alpha_t^{pv})^2} \lambda_{2,j,t} \right. \\ & \left. - \sqrt{1 - (\alpha_t^{pv})^2} \lambda_{3,j,t} \right\} \end{aligned} \quad (42)$$

If $\psi > 0$, the optimal solution of (Sub1_j) satisfies $S_j^{pv,max*} = 0$, and if $\psi < 0$, $S_j^{pv,max*} = \overline{S_j^{pv,max}}$. If $\psi = 0$, $S_j^{pv,max*}$ can be an arbitrary value. This is because the objective function, eq. (41), is linear with respect to $S_j^{pv,max}$.

The sub-problems associated with other variables per unit time t are described as

(Sub2_t) $t = 1, \dots, T$

$$\min_{\mathbf{X}_t''} L_2(\mathbf{X}_t'', \lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}) \quad (43)$$

s.t. eqs. (9), (10), (12)–(14), (25), (26)

4.2.2 Lagrange Multiplier Update

The objective function of the Lagrangian dual problem eq. (34) is non-differentiable and concave; thus, we update the Lagrange multipliers by using the sub-gradient method and projected sub-gradient method (see the reference [50] on the convergence and property of the algorithms) described below:

$$\lambda_{1,j,t}^{(k+1)} = \lambda_{1,j,t}^{(k)} + \delta^{(k)} \nabla_{\lambda_{1,j,t}^{(k)}} \phi(\lambda^{(k)}) \quad (44)$$

$$\nabla_{\lambda_{1,j,t}^{(k)}} \phi(\lambda^{(k)}) = P_{j,t}^{pv} - \alpha_t^{pv} S_j^{pv,max} \quad (45)$$

$$\lambda_{2,j,t}^{(k+1)} = \max[\lambda_{2,j,t}^{(k)} + \delta^{(k)} \nabla_{\lambda_{2,j,t}^{(k)}} \phi(\lambda^{(k)}), 0] \quad (46)$$

$$\nabla_{\lambda_{2,j,t}^{(k)}} \phi(\lambda^{(k)}) = -\sqrt{1 - (\alpha_t^{pv})^2 S_j^{pv,max} - Q_{j,t}^{pv}} \quad (47)$$

$$\lambda_{3,j,t}^{(k+1)} = \max[\lambda_{3,j,t}^{(k)} + \delta^{(k)} \nabla_{\lambda_{3,j,t}^{(k)}} \phi(\lambda^{(k)}), 0] \quad (48)$$

$$\nabla_{\lambda_{3,j,t}^{(k)}} \phi(\lambda^{(k)}) = -\sqrt{1 - (\alpha_t^{pv})^2 S_j^{pv,max} + Q_{j,t}^{pv}} \quad (49)$$

where $\lambda^{(k)} := (\lambda_1^{(k)}, \lambda_2^{(k)}, \lambda_3^{(k)})$, and $\delta^{(k)}$ is the step size of the k th iteration. Since there are no constraints on the Lagrange multipliers λ_1 , the sub-gradient method is used to update λ_1 . On the other hand, since Lagrange multipliers λ_2, λ_3 have constraints (i.e., eqs. (35), (36)), the projected sub-gradient method is used for updating them, and it satisfies a diminishing step-size rule as follows:

$$\lim_{k \rightarrow \infty} \delta^{(k)} \rightarrow 0 \quad \text{and} \quad \sum_{k=1}^{\infty} \delta^{(k)} \rightarrow \infty \quad (50)$$

We determine the k th step size as follows:

$$\delta^{(k)} = \frac{a}{2/\sqrt[3]{k}}, \quad a > 0 \quad (51)$$

where the learning rate a is a positive constant. The sub-gradient method and projected sub-gradient method are easy to implement, and they impose a small computational burden. However, they converge slowly to the optimum with oscillating behaviors. This is because the partial Lagrangian dual function is non-differentiable. Furthermore, the oscillating behaviors make it difficult to devise an appropriate stopping criterion. The algorithm of the methods is usually stopped after a certain number of iterations, as follows[51].

Convergence condition:

$$k = k^{th} \quad (52)$$

where k^{th} is the threshold of the iteration number.

4.2.3 Feasible Solution (FS)

For a given diminishing-step-size rule eq. (50), the sub-gradient method and projected sub-gradient method are guaranteed to converge to the optimal value when $k \rightarrow \infty$ [52]. However, we stop the algorithm at a finite step. Therefore, the solution calculated by the

PLR approach is not always a feasible solution of the primal problem (i.e., the solution does not satisfy eq. (15)).

First, we obtain $S_j^{pv,max,(k^{th})}$ from $\lambda_1^{(k^{th})}$ by using complementarity slackness conditions, since the aim of solving the OPSP is obtaining the size of the PVs. The procedure is as follows:

$$S_j^{pv,max,(k^{th})} := \frac{1}{|T_j^f|} \sum_{t \in T_j^f} \frac{1}{\alpha_t^{pv}} P_{j,t}^{pv,(k^{th})} \quad (53)$$

$$T_j^f := \{t | \alpha_t^{pv} > \xi, \lambda_{1,j,t}^{(k^{th})} \neq 0\} \quad (54)$$

where $|T_j^f|$ is the number of telements. Eq. (53) is a transformation of eq. (15) and it takes the average of the $|T_j^f|$ values. We set ξ to a small positive quantity to prevent dividing by 0 in eq. (53).

Second, we obtain the other variables $X''^{(k^{th})}$ in (RP) to determine the size of the PVs calculated in eq. (53). In this case, the (RP) can be divided into optimization problems per unit time t . In so doing, we can easily get the feasible solution of the primal problem.

4.2.4 Solution Procedure

The procedure for solving the Lagrangian dual problem is as follows:

- Step 1)** Initialize Lagrange multipliers $\lambda_1^{(k)}, \lambda_2^{(k)}, \lambda_3^{(k)}$ where $\lambda_2^{(k)} \geq 0$ and $\lambda_3^{(k)} \geq 0$; set $k = 0$.
- Step 2)** Solve the decomposed primal problem by solving one sub-problem per PV generator (Sub1_j) and one sub-problem per unit time t (Sub2_t), corresponding to $\lambda_1^{(k)}, \lambda_2^{(k)}$ and $\lambda_3^{(k)}$ (see section 4.2.1).
- Step 3)** Stop when the convergence condition is reached. Get the feasible solutions from $X^{(k)}$ obtained in Step 2 (see section 4.2.2 and section 4.2.3).
- Step 4)** Update the Lagrange multipliers by using the sub-gradient method and projected sub-gradient method (see section 4.2.2).
- Step 5)** Set $k = k + 1$ and return to Step 2.

5. Numerical Simulations

To evaluate the effectiveness of our method, we implemented it and the previous method[12] in a simulation of a distribution network. To compute the optimization problems, we used Gurobi Optimizer 7.02 on a computer with 3.00 GHz Intel core i7 processors and 16GB of RAM.

5.1 Test Network

The simulations were of a 47-bus distribution feeder model, as shown in Fig. 4. This feeder model is a simplified distribution industrial model integrated with 5 PV buses (Bus 17, 19, 23, 37 and 47). Bus 1 is the substation bus, and the voltage of the substation is set to 1.0 p.u. at all times t (i.e., $V_{1,t}^{sqr} = 1.0$). Detailed network data such as the line impedance are shown

Fig. 4 Schematic diagram of a distribution feeder

in [17]. The peak demand of the loads is 33.04 MW and 24.78 MVar. We assume that the power factor is a constant and the value is 0.8. The base voltage and the base apparent power are set to be 12.35 kV and 1 MVA in (p.u.). We set the parameters $\bar{V}$, $\underline{V}$, and $S_{pv,max}$ to 1.10 p.u., 0.90 p.u., and 6 MVA, respectively.

5.2 Load Profile and PV Output

We selected the actual five-year demand load profiles of Japan (see Fig. 5), which are published by the Tokyo Electric Power Company (TEPCO)[53]. The duration of the data was from January 1, 2010 to December 31, 2014. We also calculated the PV generation rate from eqs. (4), (5) by using actual solar radiation and temperature data collected in Tokyo, as shown in Fig. 6. The data are published by [54]. The duration of each load and PV output for optimization was assumed to be 1 hour.

5.3 Other Parameters

In solving a large-scale problem with our method, the learning rate a greatly affects the accuracy of the solution and the solving speed. However, a trial-and-error approach to determining a is extremely time-consuming and not realistic in a large-scale problem.

Therefore, we performed a performance analysis with a small-scale problem as in [12] and set the learning rate of this simulation to $a = 0.01$. Moreover, we set the initial Lagrange multipliers to $\lambda_{1,t} = \mathbf{0}$, $\lambda_{2,t} = \mathbf{0}$ and $\lambda_{3,t} = \mathbf{0}$. The threshold of the iteration number was $k^{th} = 300$.

5.4 Results and Discussion

Our computers are unable to solve large-scale OP-SPs, because their memory is insufficient for computing the optimal solution. Here, we applied our method to such a large-scale optimization problem and succeeded in solving it in about 1343 hours.

Fig. 7 shows the value of the objective function of the proposed method for each epoch. It can be seen that the objective function converges while finely oscillating. As described in section 4.2.3, this is because the partial Lagrangian dual function is non-differentiable. In addition, since the optimization problem is convex and the problem is solved by using the sub-gradient method or projected sub-gradient method, the solution converges to the optimal one[52].

Fig. 5 5-year active load profile

Fig. 6 5-year PV generation rate α_t^{pv}

Fig. 7 Optimal value of five-year optimization ($a = 0.01$)

5.4.1 Analysis Considering Inverter Loss

In fact, the ratio of inverter losses in the total power loss is not small, and these cannot be ignored when determining the optimal PV capacity to minimize the total power loss. Since the previous method[12] was unable to minimize the total power loss including inverter losses, in this section, we analyze the effect of our proposed method on minimizing these losses, and calculate the total PV capacity. We maintained the same conditions with the previous method other than

Table 1 Comparison of optimization results

	Previous[12] (MW)	Proposed (MW)
Network loss	2615.13	2668.49
Inverter loss	1165.98	1059.94
Total loss	3781.11	3728.43

Fig. 8 Apparent PV power

inverters, such as parameters, in the simulations.

Table 1 shows the results of the total power loss between the proposed method and previous method. It can be seen that the previous method has a smaller network loss compared to the proposed one. This is due to the fact that it only focuses on minimizing the network loss, which resulted in a greater inverter loss. On the contrary, the proposed method was able to minimize the total power loss by considering inverter losses. As a result, the total power loss of the proposed method is about 98.6% of the previous one.

Fig.8 compares the PV allocations. The total amount of PV capacity of the proposed method was 7.52 MW, which is about 78% of the previous one of 9.66 MW. In other words, considering the effect of inverter losses, it becomes possible to reduce the total power loss with a smaller PV installation.

5.4.2 Performance Analysis

In the above simulation, we solved $|N^{pv}| + T$ sub-problems in series by using one computer. However, since the sub-problems can be solved in parallel, our algorithm is easy to parallelize. Thus, the computational time can be made much shorter. In the optimization problem, the sub-problems are completely independent, so the theoretical calculation time per epoch can be described as

$$\frac{[(|N^{pv}| + T)/n]}{|N^{pv}| + T} \cdot \tau \quad (55)$$

where n is the number of processors, $\lceil \cdot \rceil$ represents the ceiling function, and τ is the CPU time using one processor.

In order to verify that there is a computational performance improvement by parallelizing the proposed method, we actually implemented parallel processing for the large-scale optimization problem and

Fig. 9 Calculation time per epoch for three bus models

compared the theoretical computational time and the experimental value. In this verification, the parallel processing was carried out using the CPUs of an Intel Xeon (R) (2.60 GHz $\times$ 30 processors).

Moreover, in order to show that the proposed method is effective even if the network becomes large-scale, we also implemented parallel processing on the 85 bus model[55] and the 118 bus model[56]. We randomly selected 5 PV buses for each bus model, and the other conditions in the simulation are the same as in the 47 bus case.

Fig.9 shows the theoretical and experimental computational time per epoch of three bus models (i.e., 47-bus, 85-bus and 118-bus). From this figure, it can be seen that the CPU time decreases as the number of processors increases for both theoretical and experimental values of all models, and these values converge to almost the same value. This is because even if the network becomes large, the CPU time to solve the sub-problem hardly changes. Thus, the proposed algorithm can significantly reduce the CPU time by parallel processing and verified that it does not depend on the scale of the network.

6. Conclusion

We proposed a new approach to determine the optimal size of PVs for minimizing network losses and inverter losses. The OPSP was formulated as a radial optimal power flow problem that minimizes the network losses and inverter losses. However, the problem is generally non-convex and large-scale due to many variations in demand loads and weather conditions. As a result, it is hard to solve.

In order to resolve this issue, our approach consists of an exact convexification and decomposition of the target problem. By using exact convexification, the non-convex model can be transformed into a convex SOCP model, and we proved the exactness of the relaxation, provided that over-satisfaction of the load is allowed. In addition, the decomposition divides up the problem into two kinds of sub-problem by using the PLR approach for solving a large-scale OPSP. The decomposed sub-problems can be computed in

parallel.

In the numerical simulations implementing our approach, we succeeded in solving a large-scale OPSP that is hard to solve. Furthermore, we obtained new knowledge about the effect of inverter losses on the PV allocation and verified that parallel processing improved the computational performance.

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Appendix

The solution $\tilde{X}'$ satisfies constraints (eqs. (9), (10), (12)–(17), (19), (25), (26), (28), (29)) at all nodes, but here, in order to show whether it holds specifically, examples will be given focusing on node i and node j in eq. (9).

First, we consider the case focusing on node i .

Assuming that a current flows from node h to node i , the following equation holds.

$$P_{hi,t}^* = \sum_{k:(i,k) \in E} P_{ik,t}^* + r_{hi} I_{hi,t}^{sqr*} - P_{i,t}^{pv*} + P_{i,t}^{d*} \quad (A1)$$

The above equation can be rewritten as follows when node j is separated from node k .

$$P_{hi,t}^* = \sum_{k \setminus j:(i,k) \in E} P_{ik,t}^* + P_{ij,t}^* + r_{hi} I_{hi,t}^{sqr*} - P_{i,t}^{pv*} + P_{i,t}^{d*} \quad (A2)$$

From the definition of $\tilde{X}'$, the following relationship holds.

$$P_{hi,t}^* = \tilde{P}_{hi,t}, \quad I_{hi,t}^{sqr*} = \tilde{I}_{hi,t}^{sqr*}, \quad (A3)$$

$$P_{i,t}^{pv*} = \tilde{P}_{i,t}^{pv}, \quad P_{i,t}^{d*} = \tilde{P}_{i,t}^d - \frac{r_{ij}\epsilon_1}{2} \quad (A4)$$

$$P_{ik,t}^* = \begin{cases} \tilde{P}_{ik,t} + \frac{r_{ij}\epsilon_1}{2} & (k=j) \\ \tilde{P}_{ik,t} & (k \neq j) \end{cases} \quad (A5)$$

By substituting eqs.(A3)-(A5) for eq.(A2), it can be seen that $\tilde{X}'$ satisfies eq. (9) at node i as follows:

$$\tilde{P}_{hi,t} = \sum_{k \setminus j:(i,k) \in E} \tilde{P}_{ik,t} + \tilde{P}_{ij,t} + \frac{r_{ij}\epsilon_1}{2} + r_{hi} \tilde{I}_{hi,t}^{sqr*} - \tilde{P}_{i,t}^{pv} + \tilde{P}_{i,t}^d - \frac{r_{ij}\epsilon_1}{2} \quad (A6)$$

$$\Leftrightarrow \tilde{P}_{hi,t} = \sum_{k:(i,k) \in E} \tilde{P}_{ik,t} + r_{hi} \tilde{I}_{hi,t} - \tilde{P}_{i,t}^{pv} + \tilde{P}_{i,t}^d \quad (A7)$$

Second, we focus on node j and the following equation holds.

$$P_{ij,t}^* = \sum_{k:(j,k) \in E} P_{jk,t}^* + r_{ij} I_{ij,t}^{sqr*} - P_{j,t}^{pv*} + P_{j,t}^{d*} \quad (A8)$$

Similarly, from the definition of $\tilde{X}'$, the following equation holds.

$$P_{ij,t}^* = \tilde{P}_{ij,t} + \frac{r_{ij}\epsilon_1}{2}, \quad P_{jk,t}^* = \tilde{P}_{jk,t} \quad (A9)$$

$$I_{ij,t}^* = \tilde{I}_{ij,t} + \epsilon_1, \quad P_{j,t}^{pv*} = \tilde{P}_{j,t}^{pv}, \quad P_{j,t}^{d*} = \tilde{P}_{j,t}^d - \frac{r_{ij}\epsilon_1}{2} \quad (A10)$$

By substituting eqs.(A9), (A10) for eq.(A8), it is shown that $\tilde{X}'$ satisfies eq. (9) at node j as follows:

$$\tilde{P}_{ij,t} + \frac{r_{ij}\epsilon_1}{2} = \sum_{k:(j,k) \in E} \tilde{P}_{jk,t} + r_{ij}(\tilde{I}_{ij,t} + \epsilon_1) - \tilde{P}_{j,t}^{pv} + \tilde{P}_{j,t}^d - \frac{r_{ij}\epsilon_1}{2} \quad (A11)$$

$$\Leftrightarrow \tilde{P}_{ij,t} = \sum_{k:(j,k) \in E} \tilde{P}_{jk,t} + r_{ij} \tilde{I}_{ij,t} - \tilde{P}_{j,t}^{pv} + \tilde{P}_{j,t}^d \quad (A12)$$

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