Three Degree-of-freedom Modeling of the Random Fluctuation Arising in Human-bicycle Balance*

Katsutoshi YOSHIDA[†], Mikinori ITO[†], Hayate KAIDO[†] and Yoshikazu YAMANAKA[†]

A novel three degree-of-freedom (DOF) fluctuation model that accurately reproduces the probability density functions (PDFs) of human-bicycle balance motions is proposed. We experimentally obtain the time series of the roll angular displacement, wheel's lateral displacement, steering angular displacement, and each velocity of them. Constructing the PDFs of these time series as training data, we identify the model parameters through the use of particle swarm optimization (PSO) that minimizes the squared residuals between the experimental participants' PDFs and those simulated by our model. Over 97% PDF fitnesses were obtained for all participants, indicating that our proposed model can successfully simulate the measured human PDFs.

1. Introduction

Bicycles provide a useful means of short-distance transportation and their utilization is expected to contribute to building a healthy and environmentally friendly society[1]. However, at least on Japanese roads, bicycle transportation is not always safe due to collisions with automobiles. The Japan National Police Agency reported that over 83% of bicycle accidents in Japan in the last ten years have involved automobiles[2]. To avoid such accidents, autonomous vehicle technology will play an important role; if it can predict bicycle motions, the resulting self-driving cars may reduce such accidents. For this purpose, accurate simulation models of bicycle motions are required and they should be provided as simply as possible for the potential use of electronic control units in self-driving cars.

Bicycle motions with human riders in traffic seem to be classified into two types: voluntary and involuntary. The former comprises purposeful motions such as right and left turning at a street intersection. Google[3] reported that their sensors can detect a cyclists' hand signals as an indication of an intention to make a turn or shift over. On

the other hand, the latter comprises unconscious motions such as human fluctuated balance motions, which have already been found universally in human quiet standing[4–6], human stick balancing[7,8], visuomotor tracking[9–11], and so on.

In this study, we propose a new three DOF stochastic model that accurately reproduces the PDFs of human-bicycle motions with respect to the roll angular displacement, wheel's lateral displacement, steering angular displacement, and respective velocities. The proposed model in this study provides a larger DOF extension to our previous single DOF model[12] with respect to the roll angle only and is expected to provide a key technology of simulating the bicycle running paths. To this end, we have conducted an experiment in which each human participant rides a bicycle on bicycle-trainer rollers, allowing it to move without rolling or yawing constraints. During this experiment, we measure the time-series of the abovementioned displacements and velocities and obtain their PDFs. Next, we design our proposed human-bicycle model as a carted inverted pendulum mechanism controlled by our human controller model, which was successfully used in our previous studies to simulate random human fluctuations during visuomotor tracking[11] and our previous single DOF human-bicycle experiment[12]. We then identify the model parameters based on the measured PDFs as training data, using PSO to minimize the squared residuals (SRs) between the measured PDFs and those simulated by our proposed human-bicycle model. It will be shown that our proposed model successfully reproduces the measured PDFs with SR fitnesses of over 97%.

Our approach mentioned above sharply contrasts with other studies in the fields of autonomous

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[†] Department of Mechanical and Intelligent Engineering, Utsunomiya University; 7-1-2 Yoto, Utsunomiya, Tochigi 321-8585, JAPAN

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Fig. 1 Photograph of our experimental device and a human participant

or unmanned bicycle-control systems[13–21] because their models have been deterministic and not designed to have randomly fluctuating terms. Our approach also differs from Google’s study on voluntary bicycle motions, as mentioned above[3]. Although there has been another study on stochastic modeling of bicycle fluctuated motion[22], it addressed the large-scale bicycle running paths, unlike our study, which deals with small fluctuations.

The rest of the paper is structured as follows: Section 2 describes the human-bicycle experiment. In Section 3, our proposed stochastic model is presented. In Sections 4 and 5, we describe our method of parameter identification and present the results. Section 6 concludes our study.

This paper is an extended version of our conference paper published in[23]. We extend our previous work by showing further experimental data and describing detailed explanations of our experimental method, conditions, and results, as well as those for numerical calculations.

2. Human-bicycle Balance Experiment

2.1 Experimental setup and procedure

Fig.1 shows a photograph of our experimental device and a human participant. The experimental device consists of five units: a bicycle (BE-ELL03, Panasonic, Japan), a set of bicycle-trainer rollers (E-MOTION, Elite, Italy), a set of motion capture cameras (GV500, Library, Japan), a three-dimensional measurement software (Move-tr/3D, Library, Japan), and a computer.

The experimental participants were three healthy males in their early twenties. They were first instructed on the operation of the experimental device, the number of trials, and the duration of each trial. The experiment was performed according to the principles of the Declaration of Helsinki and informed consent was obtained.

In each trial, the participant rode the bicycle on

Fig. 2 Schematic front view of the bicycle during the experiment

Fig. 3 Schematic top view of the bicycle front wheel during the experiment

the trainer rollers for significantly more than 257 s at a speed about 12 km/h. Several practice trials were performed prior to measurement. The air pressure of the tires was set to 680 kPa.

The experiment was performed within a quiet room and the running process on the rollers was sufficiently smooth; we suppose that environmental disturbances in this study are negligible.

2.2 Experimental Data

During each trial, the time series of the measurement vector,

$$\mathbf{x}(t) = (x_1(t), x_2(t), \dots, x_6(t))^T \\ := (\theta(t), \dot{\theta}(t), y(t), \dot{y}(t), \phi(t), \dot{\phi}(t))^T \quad (1)$$

(hereafter, $(\cdot)^T$ denotes a transpose), was obtained by the motion capture cameras. Here, θ [rad] is the roll angle from the vertical line to the bicycle’s vertical axis, $\dot{\theta} := d\theta/dt$ [rad/s] is the corresponding angular velocity, y [m] is the wheel’s lateral displacement, $\dot{y} := dy/dt$ is the corresponding wheel’s lateral velocity, ϕ is the steering angle from the reference longitudinal axis to the bicycle’s longitudinal axis, and $\dot{\phi} := d\phi/dt$ is the corresponding angular velocity.

Fig. 4 The measured time series of the human-bicycle balance for $(s,n)=(3,1)$

Fig.2 schematically shows the definition of the roll angle θ in the front view of the bicycle and Fig.3 shows those of the wheel's lateral displacement y and the steering angle ϕ in the top view of the bicycle front wheel. The bicycle's vertical and longitudinal axes were nominally determined based on the direction of the bicycle frame and the bicycle handle stem, respectively. The axis of the lateral wheel's displacement were nominally determined based on the direction of the rotation axis of the bicycle-trainer's front roller.

Note that, in our experiment, the rotational origins $\theta=0$ and $\phi=0$ were calibrated to fit the bicycle's vertical parking position; therefore, these do not indicate, respectively, the physical upright equilibrium and the physical straight running position of the unmanned bicycle. In addition, the wheel's lateral origin $y=0$ does not indicate vibrational center of y , which center depends on the human riding motions.

Throughout the experiment, $\mathbf{x}(t)$ were stored in the computer in the following form:

$$\{\mathbf{x}_{\text{hum}}^{(s,n)}(t_0), \dots, \mathbf{x}_{\text{hum}}^{(s,n)}(t_i), \dots, \mathbf{x}_{\text{hum}}^{(s,n)}(t_{I-1})\},$$

$$i=0, \dots, I-1, s=1, \dots, S, n=1, \dots, N, \quad (2)$$

where $t_i := i\Delta t$ [s] is a discrete time with a sampling period of Δt , I is the length of the time series, s and S are an index and the number of participants, respectively, and n and N are an index and the number of trials, respectively.

For this study, we chose $\Delta t = 1/70$ s and $I = 18001$ to obtain the physical data length $(I-1)\Delta t = 257$ s. The number of participants was $S=3$ and the number of trials undertaken by each participant was $N=5$.

Fig.4 shows the measured time series for $(s,n)=(3,1)$, i.e., for the third participant's first trial. Thus, we obtained the time series for all $s=1, \dots, S$ and $n=1, \dots, N$. For reference, the result seems to exhibit human specific fluctuations in which large-amplitude spikes intermittently arise among the moderate-

amplitude fluctuation process, which have already been recognized as temporal intermittency in the field of nonlinear physics[6–8,24].

As shown in the left column graphs of Fig.4, the vibrational centers of $x_1 := \theta$, $x_3 := \phi$, and $x_5 := y$ are shifted from their nominal origins ($\theta=0$, $\phi=0$, and $y=0$) as mentioned in 2.2. We therefore statistically evaluate these shifts by identifying them as the mean values of these displacements given by

$$E^{(s)}[x_k] := \frac{1}{N \times I} \sum_{n=1}^N \sum_{i=0}^{I-1} x_k^{(s,n)}(t_i),$$

$$s=1, \dots, S, \quad k=1, 3, 5, \quad (3)$$

which is the temporal average of the s th participant's $\theta(t)$, $y(t)$, and $\phi(t)$ further averaged over all of his trials. These mean values are hereafter denoted by $\hat{\theta}^{(s)}$, $\hat{y}^{(s)}$, and $\hat{\phi}^{(s)}$ (or simply $\hat{\theta}$, $\hat{y}$, and $\hat{\phi}$), respectively. The resulting values are listed in Table 1 with standard deviations given by

$$SD^{(s)}[x_k] = \sqrt{V^{(s)}[x_k]},$$

$$V^{(s)}[x_k] := \sum_{n=1}^N \sum_{i=0}^{I-1} \frac{(x_k^{(s,n)}(t_i) - E^{(s)}[x_k])^2}{N \times I - 1},$$

$$s=1, \dots, S, \quad k=1, 3, 5. \quad (4)$$

It appears in Table 1 that the $E^{(s)}[x_k]$ ($k=1, 3, 5$) values are slightly different; these mainly depend on the respective riding forms of the participants.

2.3 Construction of Measured PDFs

We obtain $P_{\text{hum}}^{(s,n)}(x_k)$, the PDF with respect to the k th component of the time series in eq.(2) for the s th participant's n th trial. This was achieved by normalizing the histogram of the k th component $\{x_k^{(s,n)}(t_i)\}_{i=0}^{I-1}$ with bin width $(\bar{x}_k - x_k)/N_{\text{bin}}$. Here, N_{bin} is the number of histogram bins and $\bar{x}_k$ and x_k are the upper and lower limits of x_k , respectively.

The resulting $P_{\text{hum}}^{(s,n)}(x_k)$ is averaged over all trials $n=1, \dots, N$ to obtain the s th participant's PDF as

Fig. 5 The measured PDFs $P_{\text{hum}}^{(s)}(x_k)$, $k=1, \dots, 6$ from the third participant ($s=3$)

Table 1 Time series statistics of the roll angle x_1 , wheel's lateral displacement x_3 , and steering angle x_5 for the s th participant

s		x_1	x_3	x_5
1	$E^{(s)}[\cdot]$	-1.07×10^{-2}	-5.72×10^{-1}	-1.01×10^{-1}
	$SD^{(s)}[\cdot]$	1.88×10^{-2}	3.68×10^{-2}	3.30×10^{-2}
2	$E^{(s)}[\cdot]$	-1.86×10^{-3}	-6.51×10^{-1}	-1.60×10^{-1}
	$SD^{(s)}[\cdot]$	2.39×10^{-2}	3.30×10^{-2}	3.99×10^{-2}
3	$E^{(s)}[\cdot]$	-1.56×10^{-2}	-5.64×10^{-1}	-7.59×10^{-2}
	$SD^{(s)}[\cdot]$	1.61×10^{-2}	3.70×10^{-2}	3.12×10^{-2}

$$P_{\text{hum}}^{(s)}(x_k) = \frac{1}{N} \sum_{n=1}^N P_{\text{hum}}^{(s,n)}(x_k), \quad k=1, \dots, 6. \quad (5)$$

We call eq.(5) the measured PDF of the s th participant.

Fig.5 shows the measured PDF of the third participant ($s=3$). In this way, we obtained the measured PDFs for all $s=1,2,3$. In this study, we commonly set $N_{\text{bin}}=40$, while $\bar{x}_k$ and $\underline{x}_k$ are chosen for each participant to make them cover the respective distribution of experimental data.

In our previous study[12], we performed a human-bicycle experiment quite similar to that in this study and statistically guaranteed that the human-bicycle roll angular motion $x_1(t)$ and $x_2(t)$ yielded typical probability distributions which significantly differ from Gaussian ones in terms of Kolmogorov–Smirnov (KS) statistical-hypothesis-testing[12]. Since the state variables added in this study, $x_3(t), \dots, x_6(t)$, are mechanically coupled with the non-Gaussian $x_1(t)$ and $x_2(t)$, they are supposed to be non-Gaussian as well. Unfortunately, as rigorous formulation of multidimensional KS testing seems to be still an open problem, statistical testing of our six-dimensional data will be a future problem.

Additionally, note that the double-indexing of $P_{\text{hum}}^{(s,n)}(x_k)$ to store the measured PDF is intended for potential use in our future work on the effects of the number of trials n although the numerical calculation of $P_{\text{hum}}^{(s)}(x_k)$ by eq. (5) can equivalently be replaced with calculating the histogram from the union of s th participant's data.

3. Fluctuation Model of the Human-bicycle Balance

In this section, we propose a new three DOF stochastic model that allows us to reproduce the measured PDFs obtained above as simply as possible.

3.1 A Human-bicycle Fluctuation Model

We model the human-bicycle mechanics by a carted inverted pendulum. Based on the time series statistics in Table 1, we also assume that the roll angle $x_1 = \theta$ about the equilibrium and steering angle $x_5 = \phi$ about the straight running is sufficiently small; the maximal three standard deviation indicates $3 \times \max_s SD^{(s)}[x_1] \approx 7.17 \times 10^{-2}$ rad (or 4.10° deg) and $3 \times \max_s SD^{(s)}[x_5] \approx 1.19 \times 10^{-1}$ rad (or 6.81° deg). Therefore, we model the bicycle's motion by

a linearized carted inverted pendulum of the form:

$$\begin{cases} \ddot{y} + \ddot{\theta} - g(\theta - \hat{\theta}) + c_\theta \dot{\theta} = 0, \\ \ddot{y} + c_y \dot{y} + c_\phi \dot{\phi} = V \dot{\phi}. \end{cases} \quad (6)$$

This can be transformed, using the state vector $\mathbf{x} = (x_1, \dots, x_6)^T := (\theta, \dot{\theta}, y, \dot{y}, \phi, \dot{\phi})^T$, into the state-space form:

$$\begin{cases} \dot{x}_1 = x_2, \dot{x}_2 = -Vx_6 + c_y x_4 + g(x_1 - \hat{\theta}) - c_\theta x_2, \\ \dot{x}_3 = V(x_5 - \hat{\phi}), \dot{x}_4 = Vx_6 - c_y x_4 - c_\phi x_6, \\ \dot{x}_5 = x_6, \dot{x}_6 = u(t), \end{cases} \quad (7)$$

where $u(t)$ is a human input, $g := 9.81 \text{ m/s}^2$ is gravitational acceleration, c_θ , c_y , and c_ϕ are viscous dampings, and V is a bicycle speed.

The human input $u(t)$ is specified to simulate the human fluctuation during human-bicycle motion. As successfully demonstrated in our previous studies[11,12], some human fluctuations can be accurately simulated by the following state-feedback mechanism:

$$\begin{aligned} u(t) = & F_1(x_1 - \hat{\theta}) + F_2x_2 + F_3(x_3 - \hat{y}) + F_4x_4 \\ & + \mu\{1 + \sigma_1\xi_1(t)\}(x_5 - \hat{\phi}) + F_6x_6 + \sigma_2\xi_2(t). \end{aligned} \quad (8)$$

Here, $F_1, \dots, F_4, F_6$ are deterministic gains and $\xi_1(t)$ and $\xi_2(t)$ are independent white Gaussian noises with zero mean and unit variance. Thus, $\mu\{1 + \sigma_1\xi_1\}$ represents a random proportional gain with mean μ and variance $(\mu\sigma_1)^2$ while $\sigma_2\xi_2(t)$ represents an additive random perturbation.

Finally, we substitute eq.(8) into eq.(7) and propose a *human-bicycle fluctuation model* with the eleven-dimensional parameter vector:

$$\begin{aligned} \mathbf{p} = & (p_1, \dots, p_{11}) \\ := & (F_1, F_2, F_3, F_4, F_6, c_\theta, c_y, c_\phi, \mu, \sigma_1, \sigma_2) \in \mathbb{R}^{11}, \end{aligned} \quad (9)$$

where $\mathbb{R}^n$ denotes n -dimensional real space. The parameter vector $\mathbf{p}$ parameterizes the random dynamical properties of the human-bicycle motion of each participant in terms of classical mechanics as well as control engineering.

The remarkable feature of our proposed model (eq.(7) and eq.(8)) is that it is driven by the random noises ξ_1 and ξ_2 only, which simulate human endogenous fluctuations mainly due to hemodynamics and cardiorespiratory dynamics[25]. This means that, in our model identification problem, the input data (ξ_1 and ξ_2) is experimentally unavailable as time-series data. Instead, in this study, we mathematically assume probability distributions of the input data and experimentally measure those of the output data $P_{\text{hum}}^{(s)}(x_k)$, which is compared with the simulated output data $P_{\text{sim}}(x_k; \mathbf{q}, \mathbf{p})$ as described in the next section.

3.2 Calculation of Simulated PDFs

Using given $\mathbf{q} := (\hat{\theta}, \hat{y}, \hat{\phi})$ and $\mathbf{p}$, we obtain N' samples of the stationary numerical solution of eq. (7) and eq. (8) as

$$\{\mathbf{x}_A^{(n')}(t_i; \mathbf{q}, \mathbf{p})\}_{i=0}^{I-1}, \quad n' = 1, \dots, N', \quad (10)$$

by means of a fourth-order Runge–Kutta–Gill method with time step $\Delta t = 10^{-2}$ s. To generate these samples, N' different sequences of normal pseudo-random numbers[26]:

$$\{\nu_i^{(n')}\}_{i=0}^{I-1}, \quad n' = 1, \dots, N' \quad (11)$$

are used to simulate the independent white Gaussian noises $\xi_1(t)$ and $\xi_2(t)$ by

$$\xi_l(t_i) \approx \nu_i^{(n')}(\Delta t)^{-1/2}, \quad l = 1, 2, \quad (12)$$

where $(\Delta t)^{-1/2}$ is the numerical factor required for integrating stochastic differential equations[27].

From the k th component of the simulated time series in eq.(10), we construct the n' th sample's PDF $P_{\text{sim}}^{(n')}(x_k; \mathbf{q}, \mathbf{p})$ using the same procedure and conditions as applied for deriving the measured PDF in 2.3. We also take the average $P_{\text{sim}}^{(n')}(x_k; \mathbf{q}, \mathbf{p})$ over all samples by

$$\begin{aligned} P_{\text{sim}}(x_k; \mathbf{q}, \mathbf{p}) = & \frac{1}{N'} \sum_{n'=1}^{N'} P_{\text{sim}}^{(n')}(x_k; \mathbf{q}, \mathbf{p}), \\ & k = 1, 2, \dots, 6. \end{aligned} \quad (13)$$

We call eq.(13) the simulated PDFs for $\mathbf{q}$ and $\mathbf{p}$, which are to be compared with the measured $P_{\text{hum}}^{(s)}(x_k)$.

4. Method of Parameter Identification

In this section, we formulate the identification problem of an unknown parameter vector $\mathbf{p}$ of our proposed model. However, as pointed out in 3.1, our identification problem suffers from the lack of input time-series and thus time-series-based approaches are not applicable to our problem. Instead, we here formulate a PDF-based identification approach.

4.1 Parameter Identification Problem

We solve the optimization problem

$$\underset{\mathbf{p}}{\text{Minimize}} C(\mathbf{p}) \quad (14)$$

with the cost function

$$\begin{aligned} C(\mathbf{p}) := & \sum_{k=1}^6 a_k C_k(\mathbf{p}), \quad a_k > 0, \\ C_k(\mathbf{p}) := & \frac{\int_{\underline{x}_k}^{\overline{x}_k} \{P_{\text{sim}}(x_k; \mathbf{q}, \mathbf{p}) - P_{\text{hum}}^{(s)}(x_k)\}^2 dx_k}{\int_{\underline{x}_k}^{\overline{x}_k} \{P_{\text{hum}}^{(s)}(x_k)\}^2 dx_k}. \end{aligned} \quad (15)$$

Here, a_k are weight coefficients and $C_k(\mathbf{p})$ is the integral of the SR between the simulated $P_{\text{sim}}(x_k; \mathbf{q}, \mathbf{p})$ and the measured $P_{\text{hum}}^{(s)}(x_k)$, as normalized by the integral of the squared $P_{\text{hum}}^{(s)}(x_k)$. Therefore, this cost function evaluates the PDF's reproduction error and satisfies $C(\mathbf{p}) = 0$ if $P_{\text{sim}}(x_k; \mathbf{q}, \mathbf{p}) = P_{\text{hum}}^{(s)}(x_k)$, $k = 1, \dots, 6$.

4.2 Procedure of PSO

We employ PSO[28] to solve eq. (14). Consider a swarm of M candidate solutions,

$$\{\mathbf{p}^1, \mathbf{p}^2, \dots, \mathbf{p}^i, \dots, \mathbf{p}^M\}, \quad \mathbf{p}^i \in \mathbb{R}^{11}, \quad (16)$$

which are called particles. Each component of $\mathbf{p}^i$ is recursively updated by

$$\begin{cases} v_j^i(k+1) = \omega v_j^i(k) + c_1 \eta_{1j}^i(k) (pb_j^i(k) - p_j^i(k)) \\ \quad + c_2 \eta_{2j}^i(k) (gb_j(k) - p_j^i(k)), \\ p_j^i(k+1) = p_j^i(k) + v_j^i(k+1), \quad k = 0, 1, \dots, K \end{cases} \quad (17)$$

where $p_j^i(k)$ denotes the j -th component of $\mathbf{p}_i$ at iteration k ; $v_j^i(k)$ is the corresponding velocity; ω , c_1 , and c_2 are system parameters of PSO; $\eta_{1j}^i(k)$ and $\eta_{2j}^i(k)$ are random numbers independently generated by [26] for each i , j , and k with a uniform distribution in $[0, 1]$; and $pb_j^i(k)$ and $gb_j(k)$ are the j -th components of the vectors $\mathbf{pb}_i(k)$ and $\mathbf{gb}(k) \in \mathbb{R}^{11}$, respectively. $\mathbf{pb}_i(k)$ is the position of the particle taking the lowest cost among those at $\mathbf{p}_i(0), \dots, \mathbf{p}_i(k)$; this is called the personal best. $\mathbf{gb}(k)$ is the position of the particle with the lowest cost among all particles for all iterations up to k ; this is called the global best. For sufficiently large K , $\mathbf{gb}(K)$ is getting close to the optimal solution $\mathbf{p}^*$.

5. Identification Results

5.1 Identification Condition

The PSO was employed with $M = 2048$ particles and $K = 500$ iterations. The PSO system parameters were set to the optimized values $\omega = 0.729$ and $c_1 = c_2 = 1.49445$ as described in [29]. The initial particles $\mathbf{p}^i(0)$, $i = 1, \dots, M$ were given by random points uniformly distributed within the eleven-dimensional hyperrectangle empirically given by

$$\begin{aligned} D_0 := \{ & (p_1, \dots, p_{11}) \mid -30 \leq p_1 \leq 0, 0 \leq p_2 \leq 10, \\ & -30 \leq p_3 \leq 0, 0 \leq p_4 \leq 30, -30 \leq p_5 \leq 0, \\ & 0 \leq p_6 \leq 15, 0 \leq p_7 \leq 10, 0 \leq p_8 \leq 3, \\ & 0 \leq p_9 \leq 10, 0 \leq p_{10} \leq 20, 0 \leq p_{11} \leq 5 \}. \end{aligned} \quad (18)$$

The weights in eq. (15) were empirically set to $a_k = 1$ for all k by which the cost values were converged within the $K = 500$ iterations and the proper fitness was obtained as described in the next section.

The number of the simulated samples in eq. (13) was $N' = 5$, the same as the number of experimental trials per participant. Hereafter, $\mathbf{p}^{(s)}$ denotes the optimized solution $\mathbf{gb}(K)$ obtained from the s th participant's data: $P_{\text{hum}}^{(s)}(x_k)$ and $\mathbf{q}^{(s)} =$

$(\hat{\theta}^{(s)}, \hat{\gamma}^{(s)}, \hat{\phi}^{(s)})$. We also use the notation $P_{\text{sim}}^{(s)}(x_k) := P_{\text{sim}}(x_k; \mathbf{q}^{(s)}, \mathbf{p}^{(s)})$ for the simulated PDFs derived from the s th participant's data.

5.2 Identification Results

Table 2 lists the identified vector components of $\mathbf{p}^{(s)}$ by PSO for all participants $s = 1, 2, 3$ and the corresponding SR cost value $C(\mathbf{p}^{(s)})$. The last column indicates the cost value as a PDF fitness value of the form:

$$\{1 - C(\mathbf{p}^{(s)})\} \times 100\%, \quad (19)$$

which indicates the accuracy of our human-bicycle fluctuation model in eq. (7) and eq. (8) in terms of the reproducibility of PDFs. The resulting PDF fitness values indicate that our proposed model in eq. (7) and eq. (8) successfully achieved over 97% PDF fitness, even in the worst case ($s = 3$).

In Fig. 6, the solid curves indicate our simulated $P_{\text{sim}}^{(s)}(x_k)$ ($k = 1, \dots, 6$) for (a) the best case ($s = 2$) and (b) the worst case ($s = 3$). The small circles indicate the corresponding $P_{\text{hum}}^{(s)}(x_k)$ measurements. As already indicated by the PDF fitness values in Table 2, our simulated PDFs mostly agree with the measured PDFs even in the worst case ($s = 3$). Furthermore, the best case ($s = 2$) resulted in much better agreement with the measured PDFs.

6. Conclusion

In this study, we have constructed a three DOF human-bicycle fluctuation model that reproduces the PDFs of the experimentally measured human-bicycle balance motions.

First, we measured the time series of the roll angular displacement, wheel's lateral displacement, steering angular displacement, and each velocity of the human-bicycle balance motions and constructed their PDFs. Using these PDFs as the training data, we identified the model parameters by PSO, minimizing the SR between the measured PDF from the participant and the simulated PDF by our model. The resulting PDF fitnesses were over 97%, indicating that the simulated PDFs reproduced the measured ones.

The above result leads to the conclusion that our proposed model can provide a three degree-of-freedom fluctuation model of human-bicycle fluctuations.

In future work, we plan to simulate the bicycle's running paths based on our proposed model to reduce the bicycle accidents; for this purpose, nonlinear analysis of temporal intermittency [6–8, 24] specific to the human-bicycle fluctuations will be a critical issue. Furthermore, we plan to apply a semi-analytical method of calculating PDF residuals [30] towards faster calculation of our method.

Table 2 Identified vector components of $\mathbf{p}^{(s)} = (p_1, \dots, p_{11})$ and its cost value $C(\mathbf{p}^{(s)})$ for the s th participant

s	p_1	p_2	p_3	p_4	p_5	p_6	
1	54.28647	14.61676	4.156761	18.44603	5.104233	9.158946	
2	57.28823	7.932240	2.008461	15.50420	1.911434	6.228664	
3	26.99948	10.64740	1.428514	6.469611	3.570648	10.08310	
s	p_7	p_8	p_9	p_{10}	p_{11}	$C(\mathbf{p}^{(s)})$	PDF fitness
1	12.61928	10.45926	36.59455	0.8424895	1.766464	1.510×10^{-2}	98.49%
2	9.352436	8.206091	12.56037	0.1545174	0.9200015	8.749×10^{-3}	99.12% (best)
3	10.95418	10.43443	10.18957	1.241405	0.5660814	2.924×10^{-2}	97.07% (worst)

(a) The best case ($s = 2$) (b) The worst case ($s = 3$) Fig. 6 Comparison between the measured $P_{\text{hum}}^{(s)}(x_k)$ and the simulated $P_{\text{sim}}^{(s)}(x_k)$, $k = 1, \dots, 6$, for (a) the best case ($s = 2$) and (b) the worst case ($s = 3$). The small circles indicate the measured PDFs and the solid curves indicate the simulated PDFs.

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Authors

Katsutoshi YOSHIDA (Member)

Katsutoshi Yoshida received the Ph.D. degree in Engineering from Utsunomiya University, Utsunomiya, Japan, in 1996. Since 2012, he is Professor at the Graduate School of Engineering, Utsunomiya University.

His research interests are in nonlinear dynamics, stochastic mechanics, complex systems, and human dynamics. He is a member of ISCIE and JSME.

Mikinori ITO

Mikinori Ito received the B.S.(2019) and M.S.(2021) degrees in Engineering from Utsunomiya University, Utsunomiya, Japan. His research interests include human dynamics.

Hayate KAIDO

Hayate Kaido received the B.S.(2020) degree in Engineering from Utsunomiya University, Utsunomiya, Japan. His research interests include human dynamics.

Yoshikazu YAMANAKA

Yoshikazu Yamanaka received the Ph.D. degree in Engineering from Nagasaki University of Technology, Niigata, Japan, in 2017. Since 2017, he is Assistant Professor at the Graduate School of Engineering, Utsunomiya University.

His research interests are in evolutionary computation, swarm intelligence, and nonlinear dynamics. He is a member of IEICE and IEEE.