コロケイト入出力の場合の放物型分布定数系に 対する適応レギュレータの設計*

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Adaptive Regulator Design for Collocated Parabolic Distributed Parameter Systems*

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In this paper an adaptive regulator is designed for collocated parabolic distributed parameter systems with bounded deterministic disturbances in the case of the input and output operators being unbounded. The adaptive regulator is constructed by the concept of high-gain output feedback and the estimation mechanism of the unknown parameters for the disturbance. In the control system, the convergence of the system state to zero will be guaranteed.

1. Introduction

One of the most important applications of feedback is to achieve regulation and servoaction, that is, to obtain a stable closed-loop system that rejects a given class of external disturbances and tracks a given class of reference signals with zero asymptotic error. This problem has been well investigated provided that the plant is linear and time-invariant and the plant uncertainty is sufficiently small (see Ref. 2) for finite-dimensional systems, and Ref. 12) for infinite-dimensional systems).

If the plant uncertainty is large, which is the case if only certain structural information on the plant is available, it is desirable to construct an adaptive regulator or an adaptive servomechanism, that is, a fixed controller that achieves regulation and servoaction for a whole prescribed class of linear time-invariant systems without explicit identification of the system parameters. Non-identifier-based high-gain adaptive stabilization and adaptive servomechanism design have been also investigated for a class of distributed param-

eter systems^{4),5),9),8),7),14)}. In the design of adaptive control, however, rather strong conditions for distributed parameter systems have been required. These conditions exclude not only the systems with unbounded input and output operators but also the systems with some bounded input and output operators. In Ref. 14) adaptive stabilization has been studied for detectable and stabilizable distributed parameter systems of Prithard-Salamon type which are externally stabilized by some scalar output feedback. This class does not allow for collocated actuators and sensors, an interesting case often used in applications. In this paper we shall show that an adaptive regulator can be constructed for a class of collocated parabolic distributed parameter systems with unbounded input and output operators in the case of bounded deterministic disturbances. The adaptive regulator is constructed by the concept of high-gain output feedback and the estimation mechanism of the unknown parameters for the disturbance. In the control system, the convergence of the system state to zero will be guaranteed.

2. System description and preliminary results

Consider the distributed parameter system described by the following evolutionary equation on a Hilbert space H

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$$\begin{cases} \frac{dx(t)}{dt} = Ax(t) + B[u(t) + \theta^T w(t)], \\ 0 < t < t_f, x(0) = x_0 \in H \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where the operator A is the generator of a strongly continuous semigroup on H and B, C are appropriate linear operators. In this paper we only treat the collocated case such that $C=B^*$ (B^* denotes the adjoint of B). The function $x(t)$ is the system state, $u(t) \in \mathbf{R}$ is the input and $y(t) \in \mathbf{R}$ is the output. The disturbance vector function $w(t)$ is bounded and known, but θ is the l -dimensional unknown constant vector. For example, we shall consider a disturbance $w(t)$ such that

$$w(t) = \begin{bmatrix} 1 \\ \sin(t) \\ \cos(2t) \end{bmatrix}$$

The object of adaptive regulator design is to construct the control input u such that the closed-loop system will be stable and reject a given class of disturbances without explicit knowledge of A, B, C and θ .

The main conditions required in the design of adaptive stabilizers for the system (1) are the following^{(4), (5), (9)}

- (C1) The system(1) is minimum-phase, that is, the system has no unstable zeros.
- (C2) $CB \neq 0$.
- (C3) $\text{Range}(B) \subset \text{Domain}(A)$ and $\text{Range}(C^*) \subset \text{Domain}(A^*)$.

These conditions come from that the inverse system of system (1) is governed by the operator $A_\gamma = A - B(CB)^{-1}C(A - \gamma I)$ whose eigenvalues are γ and the zeros of system (1)^{(4), (5)}. The condition (C1) is essential in the design of adaptive control systems even for finite-dimensional systems. The condition (C2), in the case of high-gain adaptive stabilization, is the rather major limitation even for finite-dimensional systems. The condition (C3) is proper to distributed parameter systems. This condition excludes some bounded input and output operators such that $B \in L(\mathbf{R}, H)$ (the space of bounded linear operators from $\mathbf{R}$ to H), $C \in L(H, \mathbf{R})$ and any unbounded input and output operators B, C . It is a severe restriction from a practical point of view.

In this paper we shall construct a high-gain adaptive regulator for the system (1) in a special case of unbounded input and output operators B and $C = B^*$, even if the conditions (C1), (C2) and (C3) do not hold.

Hereafter system description is following Lions⁽⁶⁾ in order to treat some unbounded input and output operators. Let H and V be two Hilbert spaces with V, H and V' (the dual space of V) satisfying the inclusion relation

$$V \subset H = H' \subset V',$$

where $\subset$ means continuous dense injection. We denotes by $\langle \cdot, \cdot \rangle_X$ and $\|\cdot\|_X$ the scalar product in X and the norm on X , respectively. If $f \in V'$ and $v \in V$, $\langle f, v \rangle$ denotes their scalar product. If $f \in H$, it coincides with the scalar product in H . Let the operator A be in $L(V, V')$ and satisfy the coercivity condition

$$\text{Re} \langle -Ax, x \rangle + c \|x\|_H^2 \geq \delta \|x\|_V^2, \quad \delta > 0, x \in V. \quad (2)$$

Then the operator A generates a strongly continuous semigroup $S(t)$ on H .

For the operators B and C we assume that $B \in L(\mathbf{R}, V')$ and $C = B^* \in L(V, \mathbf{R})$. These B and C can express boundary or pointwise controls and observation. They are called unbounded input and output operators, because $\text{Range}(B)$ and $\text{Domain}(C)$ do not coincide with the state space H . Then CB is not well-defined and the condition (C3) dose not hold.

For $u \in L^2(0, t_f)$ the system (1) has unique solution x such that $x \in L^2(0, t_f; V)$ and $\dot{x} \in L^2(0, t_f; V')$ ⁽⁶⁾. Here $L^2(0, t_f; X)$ denotes the space (equivalent class) of square integrable functions on $[0, t_f]$ with values in X .

In this paper we shall show that a high-gain adaptive regulator can be designed for the system (1) with collocated actuators and sensors (that is, $C = B^*$) under the following assumptions.

Assumption 1 The operator $-A$ is self-adjoint and coercive with compact resolvent, and

$$C = B^* \in L(V, \mathbf{R}).$$

Assumption 2 There exists a positive constant $\hat{k}$ such that the operator

$$A_{\hat{k}} = A - \hat{k}BB^* \quad (3)$$

generates an exponentially stable semigroup on H .

Remark 1 Assumption 2 does not require that the system is minimum-phase.

3. Adaptive regulators

In this section, under Assumption 1 and Assumption 2, we shall design an adaptive regulator for the system (1) by the concept of high-gain

output feedback and the estimation mechanism of the unknown parameters for the disturbance. If the parameters $\hat{k}$ and θ are known, using the feedback control law

$$u(t) = -\hat{k}y(t) - \theta^T w(t) \quad (4)$$

for the system (1), we have the closed-loop system

$$\frac{dx(t)}{dt} = (A - \hat{k}BB^*)x(t). \quad (5)$$

Thus a regulator is constructed under **Assumption 1** and **Assumption 2**. We shall show that, even if the parameters $\hat{k}$ and θ are unknown, an adaptive controller

$$\begin{cases} u(t) = -k(t)y(t) - \hat{\theta}(t)^T w(t), \\ \dot{k}(t) = ry^2(t), k(0) = 0, r > 0, \\ \dot{\hat{\theta}}(t) = \Gamma w(t)y(t), \hat{\theta}(0) = \hat{\theta}_0 \end{cases} \quad (6)$$

(where Γ is an l -dimensional positive definite matrix) will regulate the system (1) under **Assumption 1** and **Assumption 2**.

If the-adaptive controller (6) is applied to the system (1), the resulting closed-loop system becomes

$$\begin{aligned} \frac{dx(t)}{dt} &= [A - k(t)BB^*]x(t) \\ &\quad - B[\hat{\theta}(t)^T - \theta^T]w(t) \end{aligned} \quad (7)$$

$$\begin{cases} \dot{k} = ry^2(t), k(0) = 0 \\ \dot{\hat{\theta}}(t) = \Gamma w(t)y(t), \hat{\theta}(0) = \hat{\theta}_0 \\ y(t) = B^*x(t) \end{cases} \quad (8)$$

In this time-variant system (7), the operator $-A + k(t)BB^* \in L(V, V')$ satisfies the coercivity condition for $k(t) < \infty$. When $k(t) < \infty$ and $\|\hat{\theta}(t)\| < \infty$ for $t > 0$, the system (7) also has a unique solution x such that $x \in L^2(0, t_f; V)$ and $\dot{x} \in L^2(0, t_f; V')$ for any $t_f < \infty$ ⁶⁾. Under **Assumption 1** and **Assumption 2**, the following main theorem holds.

[Theorem 1] Suppose that **Assumption 1** and **Assumption 2** hold. Then the resulting closed-loop system (7) and (8) has following properties. For all $x_0 \in H$ and $k(0) = 0$, the solution of eqs.(7) and (8) satisfies

$$\lim_{t \rightarrow \infty} \|x(t)\|_H = 0 \quad (9)$$

and

$$k(t) < \infty \text{ and } \|\hat{\theta}(t)\| < \infty \text{ for any } t > 0. \quad (10)$$

(Proof) The system (7) and (8) can be rewritten as

$$\begin{cases} \frac{dx(t)}{dt} = (A - \hat{k}BB^*)x(t) - [k(t) - \hat{k}]BB^*x(t) \\ \quad - B[\hat{\theta}(t)^T - \theta^T]w(t) \\ \dot{k} = ry^2(t), k(0) = 0 \\ \dot{\hat{\theta}}(t) = \Gamma w(t)y(t), \hat{\theta}(0) = \hat{\theta}_0 \\ y(t) = B^*x(t) \end{cases} \quad (11)$$

From **Assumption 1**, $A - kBB^* \in L(V, V')$ is self-adjoint with compact resolvent¹¹⁾, so the spectrum of $A - kBB^*$ is discrete and has orthonormal eigenfunctions ϕ_{kn} with corresponding eigenvalues λ_{kn} . The eigenvalues depend continuously on parameter k ¹¹⁾. Moreover from **Assumption 2** it holds that

$$\begin{aligned} \langle (A - \hat{k}BB^*)x, x \rangle &= \sum_{n=1}^{\infty} \lambda_{kn} \langle x, \phi_{kn} \rangle_H^2 \\ &\leq \max_n \lambda_{kn} \|x\|_H^2 \end{aligned} \quad (12)$$

for $x \in V$ and $-\omega_{\hat{k}} = \max_n \lambda_{kn} < 0$.

Now consider a Lyapunov-like functional on $V \times \mathbf{R} \times \mathbf{R}^l$

$$\begin{aligned} V(t) &= \|x(t)\|_H^2 + \frac{1}{r} [k(t) - \hat{k}]^2 \\ &\quad + [\hat{\theta}(t) - \theta]^T \Gamma^{-1} [\hat{\theta}(t) - \theta]. \end{aligned} \quad (13)$$

The time derivative of $V(t)$ along the solution of system (11) can be estimated as follows.

$$\begin{aligned} \dot{V} &= 2 \langle (A - \hat{k}BB^*)x(t), x(t) \rangle \\ &\quad - 2[k(t) - \hat{k}] \langle BB^*x(t), x(t) \rangle \\ &\quad + 2[k(t) - \hat{k}]y^2(t) \\ &\quad - 2 \langle B[\hat{\theta}(t) - \theta]^T w(t), x(t) \rangle \\ &\quad + 2[\hat{\theta}(t) - \theta]^T w(t)y(t) \\ &= 2 \langle (A - \hat{k}BB^*)x(t), x(t) \rangle \\ &\leq -2\omega_{\hat{k}} \|x(t)\|_H^2 \end{aligned} \quad (14)$$

From this $V(t) \leq V(0)$, and $k(t) < \infty$ and $\|\hat{\theta}(t)\| < \infty$ for any $t > 0$ which implies that $y \in L^2(0, \infty)$. Since the operator $A_{\hat{k}} = A - \hat{k}BB^*$ is self-adjoint and negative-definite, it generates an exponentially stable analytic semigroup $S_{\hat{k}}(t)$ on H such that

$$\|S_{\hat{k}}(t)x\|_H \leq Me^{-\gamma t} \|x\|_H \text{ for any } t > 0 \quad (15)$$

where M is a positive constant and $0 < \gamma \leq \omega_{\hat{k}}$ ¹⁰⁾

We can define $\sqrt{-A_{\hat{k}}}$ and $(\sqrt{-A_{\hat{k}}})^{-1}$ in the usual way. Moreover it holds that the domain of $\sqrt{-A_{\hat{k}}}$ is V (Ref. 13), p. 29). We shall estimate the solution $x(t)$ of the system (9). At first, since from eq.(14)

$$\|x(t)\|_H^2 \leq -\frac{1}{2\omega_{\hat{k}}} \frac{dV(t)}{dt},$$

$$\int_0^\infty \|x(t)\|_H^2 dt \leq \frac{1}{2\omega_{\hat{k}}} [V(0) - V(\infty)] < \infty. \quad (16)$$

In order to investigate the convergence of $x(t)$ in H we use the eigenvalues $\lambda_{\hat{k}n}$ and the eigenfunctions $\phi_{\hat{k}n}$ of the operator $A - \hat{k}BB^*$. In this case we can estimate $x(t)$ as follows. The solution $x(t)$ of the system (7) is given by

$$x(t) = S_{\hat{k}}(t)x_0 - \int_0^t S_{\hat{k}}(t-\tau)[k(\tau) - \hat{k}]By(\tau)d\tau$$

$$- \int_0^t S_{\hat{k}}(t-\tau)B[\hat{\theta}(\tau) - \theta]^T w(\tau)d\tau. \quad (17)$$

Here we decompose $x(t)$ such that

$$\begin{cases} x(t) = x_1(t) + x_2(t), \\ x_1(t) = S_{\hat{k}}(t)x_0 + \int_0^t S_{\hat{k}}(t-\tau)Bg_1(\tau)d\tau \\ x_2(t) = \int_0^t S_{\hat{k}}(t-\tau)Bg_2(\tau)d\tau, \end{cases} \quad (18)$$

where $g_1(t) = -[k(t) - \hat{k}]y(t)$ and $g_2(t) = -[\hat{\theta}(t) - \theta]^T w(t)$. It should be noted that $x_1(t)$ and $x_2(t)$ are the solutions of the evolution equations

$$\dot{x}_1(t) = (A - \hat{k}BB^*)x_1(t) + Bg_1(t), \quad x_1(0) = x_0,$$

and

$$\dot{x}_2(t) = (A - \hat{k}BB^*)x_2(t) + Bg_2(t), \quad x_2(0) = 0,$$

respectively.

First we shall estimate $x_1(t)$. From eq.(15)

$$\|S_{\hat{k}}(t)x_0\|_H \leq Me^{-\gamma t} \|x_0\|_H. \quad (19)$$

For the second term we can estimate the function $a(t)$

$$a(t) = \int_0^t S_{\hat{k}}(t-\tau)Bg_1(\tau)d\tau$$

in H as follows. We can have

$$\begin{aligned} & |\langle a(t), \phi_{\hat{k}n} \rangle_H| \\ &= \left| \langle \int_0^t S_{\hat{k}}(t-\tau)[k(\tau) - \hat{k}]By(\tau)d\tau, \phi_{\hat{k}n} \rangle_H \right| \\ &\leq \sup_{t \geq 0} \|k(t) - \hat{k}\| |B^* \phi_{\hat{k}n}| \int_0^t e^{\lambda_{\hat{k}n}(t-\tau)} |y(\tau)| d\tau \\ &= K |B^* \phi_{\hat{k}n}| \left(\int_0^{\frac{t}{2}} e^{\lambda_{\hat{k}n}(t-\tau)} |y(\tau)| d\tau \right. \\ &\quad \left. + \int_{\frac{t}{2}}^t e^{\lambda_{\hat{k}n}(t-\tau)} |y(\tau)| d\tau \right) \\ &= K |B^* \phi_{\hat{k}n}| \left(\int_{\frac{t}{2}}^t e^{\lambda_{\hat{k}n}\tau} |y(t-\tau)| d\tau \right. \\ &\quad \left. + \int_{\frac{t}{2}}^t e^{\lambda_{\hat{k}n}(t-\tau)} |y(\tau)| d\tau \right) \\ &\leq K |B^* \phi_{\hat{k}n}| \\ &\quad \left\{ \left(\int_{\frac{t}{2}}^t e^{2\lambda_{\hat{k}n}\tau} d\tau \right)^{\frac{1}{2}} \left(\int_{\frac{t}{2}}^t y^2(t-\tau) d\tau \right)^{\frac{1}{2}} \right. \\ &\quad \left. + \left(\int_{\frac{t}{2}}^t e^{2\lambda_{\hat{k}n}(t-\tau)} d\tau \right)^{\frac{1}{2}} \left(\int_{\frac{t}{2}}^t y^2(\tau) d\tau \right)^{\frac{1}{2}} \right\} \\ &\leq K |B^* \phi_{\hat{k}n}| \\ &\quad \left\{ \left[\frac{\exp(2\lambda_{\hat{k}n}t) - \exp(\lambda_{\hat{k}n}t)}{2\lambda_{\hat{k}n}} \right]^{\frac{1}{2}} \right. \\ &\quad \times \left(\int_0^\infty y^2(\tau) d\tau \right)^{\frac{1}{2}} \\ &\quad \left. + \left[\frac{\exp(\lambda_{\hat{k}n}t) - 1}{2\lambda_{\hat{k}n}} \right]^{\frac{1}{2}} \left(\int_{\frac{t}{2}}^\infty y^2(\tau) d\tau \right)^{\frac{1}{2}} \right\} \\ &\leq K |B^* \phi_{\hat{k}n}| \\ &\quad \left\{ \left[\frac{-\exp(\lambda_{\hat{k}n}t)}{2\lambda_{\hat{k}n}} \right]^{\frac{1}{2}} \left(\int_0^\infty y^2(\tau) d\tau \right)^{\frac{1}{2}} \right. \\ &\quad \left. + \left[\frac{-1}{2\lambda_{\hat{k}n}} \right]^{\frac{1}{2}} \left(\int_{\frac{t}{2}}^\infty y^2(\tau) d\tau \right)^{\frac{1}{2}} \right\} \end{aligned}$$

From this estimate we can obtain

$$\begin{aligned} \|a(t)\|_H^2 &= \sum_{n=1}^\infty |\langle a(t), \phi_{\hat{k}n} \rangle_H|^2 \\ &\leq 2K^2 \sum_{n=1}^\infty \left\{ \frac{-\exp(\lambda_{\hat{k}n}t)}{2\lambda_{\hat{k}n}} \int_0^\infty y^2(\tau) d\tau \right. \\ &\quad \left. + \left[\frac{-1}{2\lambda_{\hat{k}n}} \right] \left(\int_{\frac{t}{2}}^\infty y^2(\tau) d\tau \right) \right\} |B^* \phi_{\hat{k}n}|^2 \\ &\leq K^2 \left\{ e^{\max \lambda_{\hat{k}n} t} \int_0^\infty y^2(\tau) d\tau \right. \\ &\quad \left. + \int_{\frac{t}{2}}^\infty y^2(\tau) d\tau \right\} \\ &\quad \sum_{n=1}^\infty \frac{1}{|\lambda_{\hat{k}n}|} |B^* \phi_{\hat{k}n}|^2 < \infty, \quad (20) \end{aligned}$$

since the domain of $\sqrt{-A_{\hat{k}}}$ is V and $B \in L(\mathbf{R}, V')$. Eq.(20) implies that $a(t)$ in H for all $t > 0$ and $\|a(t)\|_H \rightarrow 0$ as $t \rightarrow \infty$, since $\int_t^\infty y^2(\tau) d\tau \rightarrow 0$ as $t \rightarrow \infty$, if $y \in L^2(0, \infty)$. From eqs.(19) and

(20) it holds that $\|x_1(t)\|_H < \infty$ for any $t > 0$ and $\lim_{t \rightarrow \infty} \|x_1(t)\|_H = 0$.

Moreover estimating directly $S_{\hat{k}}(t)x_0$ and $a(t)$, we can obtain

$$\int_0^\infty \|x_1(t)\|_H^2 dt < \infty. \quad (21)$$

Next we shall estimate $x_2(t)$. Consider the functional

$$f(t) = \frac{1}{2} \|x_2(t)\|_H^2.$$

From eqs.(16) and (21)

$$\begin{aligned} \int_0^\infty f(t) dt &\leq \int_0^\infty \|x(t)\|_H^2 dt \\ &\quad + \int_0^\infty \|x_1(t)\|_H^2 dt < \infty, \end{aligned}$$

that is, $f \in L^1(0, \infty)$. On the other hand it holds that

$$\begin{aligned} \frac{df(t)}{dt} &= \langle \dot{x}_2(t), x_2(t) \rangle \\ &= \left\langle \left(\sqrt{-A_{\hat{k}}} \right)^{-1} \dot{x}_2(t), \sqrt{-A_{\hat{k}}} x_2(t) \right\rangle_H \\ &\leq \left\| \sqrt{-A_{\hat{k}}} x_2(t) \right\|_H \cdot \left\| \left(\sqrt{-A_{\hat{k}}} \right)^{-1} \dot{x}_2(t) \right\|_H \\ &< \infty \quad \text{for any } t > 0 \end{aligned}$$

from **Lemma 1** (Lemma 1 will be given below). The function $f(t)$ is uniformly continuous for $t > 0$ and $f \in L^1(0, \infty)$. Hence $\lim_{t \rightarrow \infty} f(t) = 0$, that is, $\lim_{t \rightarrow \infty} \|x_2(t)\|_H = 0$. Consequently we have that $\lim_{t \rightarrow \infty} \|x(t)\|_H = 0$. We have proved the theorem.

It should be noted that the exponential convergence of $x(t)$ has not been shown³⁾

Lastly we must prove the following Lemma.

【Lemma 1】 Suppose that the assumptions of Theorem 1 hold. Then

$$\left\| \sqrt{-A_{\hat{k}}} x_2(t) \right\|_H < \infty \quad \text{for any } t > 0$$

and

$$\left\| \left(\sqrt{-A_{\hat{k}}} \right)^{-1} \dot{x}_2(t) \right\|_H < \infty \quad \text{for any } t > 0.$$

(Proof) First we can estimate the function $b(t) = \sqrt{-A_{\hat{k}}} x_2(t)$.

$$\begin{aligned} b(t) &= \sqrt{-A_{\hat{k}}} x_2(t) \\ &= \sqrt{-A_{\hat{k}}} \int_0^t S_{\hat{k}}(t-\tau) B g_2(\tau) d\tau \end{aligned}$$

in H as follows.

$$\begin{aligned} &\left| \langle b(t), \phi_{\hat{k}n} \rangle_H \right| \\ &= \left| \left\langle \int_0^t \sqrt{-A_{\hat{k}}} S_{\hat{k}}(t-\tau) B g_2(\tau) d\tau, \phi_{\hat{k}n} \right\rangle_H \right| \\ &\leq \sup_{t \geq 0} \left| \left[\hat{\theta}(t) - \theta \right]^T w(t) \right| |B^* \phi_{\hat{k}n}| \\ &\quad \sqrt{-\lambda_{\hat{k}n}} \int_0^t e^{\lambda_{\hat{k}n}(t-\tau)} d\tau \\ &\leq M_\theta |B^* \phi_{\hat{k}n}| \frac{1}{\sqrt{-\lambda_{\hat{k}n}}}, \end{aligned}$$

where $M_\theta = \sup_{t \geq 0} \left| \left[\hat{\theta}(t) - \theta \right]^T w(t) \right| < \infty$. From this we obtain

$$\begin{aligned} \|b(t)\|_H^2 &= \sum_{n=1}^\infty \left| \langle b(t), \phi_{\hat{k}n} \rangle_H \right|^2 \\ &\leq M_\theta^2 \sum_{n=1}^\infty \frac{1}{|\lambda_{\hat{k}n}|} |B^* \phi_{\hat{k}n}|^2 \\ &< \infty \quad \text{for any } t > 0, \end{aligned} \quad (22)$$

since the domain of $\sqrt{-A_{\hat{k}}}$ is V and $B \in L(\mathbf{R}, V')$. Next we have

$$\begin{aligned} \left(\sqrt{-A_{\hat{k}}} \right)^{-1} \dot{x}_2(t) &= -\sqrt{-A_{\hat{k}}} x_2(t) \\ &\quad + \left(\sqrt{-A_{\hat{k}}} \right)^{-1} B g_2(t). \end{aligned}$$

For the second term it holds that

$$\begin{aligned} &\left\| \left(\sqrt{-A_{\hat{k}}} \right)^{-1} B g_2(t) \right\|_H \\ &\leq \left\| \left(\sqrt{-A_{\hat{k}}} \right)^{-1} B \right\|_{H \sup_{t \geq 0}} |g_2(t)| \\ &< \infty \quad \text{for any } t > 0. \end{aligned} \quad (23)$$

From eqs.(22) and (23) we have

$$\left\| \left(\sqrt{-A_{\hat{k}}} \right)^{-1} \dot{x}_2(t) \right\|_H < \infty \quad \text{for any } t > 0. \quad (24)$$

We have proved the lemma.

4. Example

In this section we give an example to illustrate our theory. Let us consider the following unstable heat system.

$$\begin{cases} x_t(t, z) = x_{zz}(t, z) + a x(t, z), \quad a > 0 \\ x(0, z) = x_0(z) \\ x_z(t, 0) = -u(t) - \theta_1 - \theta_2 \sin(t), \quad x_z(t, 1) = 0 \\ y(t) = \int_0^1 c(z) x(t, z) dz = x(t, 0) \end{cases} \quad (25)$$

where a is an unknown positive constant parameter, and θ_1, θ_2 are unknown constants. It can be shown that this system is equivalent to a Cauchy problem of eq.(1)¹⁾. For the system the operator

A is $d^2/dz^2 + aI$ with obvious domain and A has the eigenset

$$\begin{cases} \lambda_n = -(n-1)^2\pi^2 + a, \\ \quad \quad \quad n = 1, 2, \dots \\ \phi_1(z) = 1, \quad \phi_n(z) = \sqrt{2}\cos(n-1)\pi z, \\ \quad \quad \quad n = 2, 3, \dots \end{cases}$$

The operator A is self-adjoint and generates an analytic semigroup $S(t)$ on $H = L^2(0,1)$. The operator B can be identified with the delta function $\delta(z)^1$, and thus the actuator and sensor are collocated, that is, $C = B^*$. The system has unbounded B and C operators. One way of obtaining V and V' spaces is to introduce the following weighted Hilbert space.

$$V = \left\{ x = \sum_{n=1}^{\infty} x_n \phi_n \left| \sum_{n=1}^{\infty} \beta_n |x_n|^2 < \infty \right. \right\}$$

with the inner product $\langle x, h \rangle_V = \sum_{n=1}^{\infty} \beta_n x_n h_n$, where $x_n = \langle x, \phi_n \rangle_H$ and

$$\beta_n = \begin{cases} 1 & \text{if } n \in \Delta = \{n \in \{1, 2, \dots\} | \lambda_n > -1\} \\ |\lambda_n| & \text{if } n \in \Gamma = \{n \in \{1, 2, \dots\} | \lambda_n \leq -1\}. \end{cases}$$

Then the dual space V' is defined by

$$V' = \left\{ x = \sum_{n=1}^{\infty} x_n \phi_n \left| \sum_{n=1}^{\infty} \frac{1}{\beta_n} |x_n|^2 < \infty \right. \right\}$$

The coercivity condition holds as follows. For $x \in V$,

$$\begin{aligned} \langle -Ax, x \rangle &= \sum_{n=1}^{\infty} (-\lambda_n) x_n^2 \\ &= - \sum_{n \in \Delta} \lambda_n x_n^2 + \sum_{n \in \Gamma} \beta_n x_n^2 \\ &= \sum_{n=1}^{\infty} \beta_n x_n^2 - \sum_{n \in \Delta} (\lambda_n + 1) x_n^2 \\ &\geq \|x\|_V^2 - \max_{n \in \Delta} (\lambda_n + 1) \sum_{n \in \Delta} x_n^2 \\ &\geq \|x\|_V^2 - \max_{n \in \Delta} (\lambda_n + 1) \|x\|_H^2. \end{aligned}$$

Moreover it can be easily shown that $B \in L(\mathbf{R}, V')$. Thus **Assumption 1** holds. We investigate the eigenvalues of the operator $A - kBB^*$. This leads the following eigenvalue problem.

$$\begin{cases} x_{zz}(z) + ax(z) = \lambda x(z) \\ x_z(0) = kx(0), \quad x_z(1) = 0 \end{cases}$$

In this case the equation providing the eigenvalues is

$$\sqrt{a - \lambda} \tan \sqrt{a - \lambda} = k.$$

This equation has infinitely many zeros $\{\lambda_{kn}\}$ satisfying

$$-\left(n - \frac{1}{2}\right)^2 \pi^2 + a < \lambda_{kn} < -(n-1)^2 \pi^2 + a, \quad n = 1, 2, \dots$$

Moreover it holds that

$$\max_n \lambda_{kn} = \lambda_{k1} \rightarrow a - \frac{\pi^2}{4}, \quad k \rightarrow \infty.$$

Corresponding to the eigenvalues λ_{kn} we have the complete orthonormal system of eigenfunctions

$$\begin{aligned} \phi_{kn}(z) &= \kappa_n \cos(\mu_n(1-z)), \quad n = 1, 2, \dots, \\ \mu_n^2 &= -\lambda_n \\ \kappa_n &= \sqrt{\frac{2(k^2 + \mu_n^2)}{k^2 + \mu_n^2 + k}} \end{aligned}$$

in $H = L^2(0,1)$. The eigenvalues λ_{kn} are simple. Since the eigenvalues depend continuously on k , there exists a positive constant $\hat{k}$ such that $\lambda_{\hat{k}} = \max_n \lambda_{\hat{k}n} < 0$, if $a < \pi^2/4$. Thus **Assumption 2** holds. It should be noted that the transfer function of the system (25) is

$$G(s) = \frac{\cosh \sqrt{s-a}}{\sqrt{s-a} \sinh \sqrt{s-a}}$$

and the largest zero of $G(s)$ is a $-\pi^2/4$.

We can construct an adaptive output-feedback stabilizer for the system of (25)

$$\begin{cases} u(t) = -k(t)y(t) - \hat{\theta}_1(t) - \hat{\theta}_2(t)\sin(t), \\ \dot{k}(t) = ry^2(t), \quad k(0) = 0, \\ \dot{\hat{\theta}}_1(t) = \gamma_1 y(t), \quad \hat{\theta}_1(0) = \hat{\theta}_{10}, \quad \gamma_1 > 0, \\ \dot{\hat{\theta}}_2(t) = \gamma_2 \sin(t)y(t), \quad \hat{\theta}_2(0) = \hat{\theta}_{20}, \quad \gamma_2 > 0, \end{cases}$$

if the unknown parameter $a < \pi^2/4$.

We give a simulation result by eigenfunctions expansion in the case where $a = 2$, $\theta_1 = 2$, $\theta_2 = 1$, $x_0(z) = 1$. Fig. 1 shows plots of the state $x(t, z)$. Fig. 2 shows plots of the input $u(t)$, the gain $k(t)$, the parameters $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$, respectively. In simulation we choose $\hat{\theta}_{10} = \hat{\theta}_{20} = 0$, $r = 200$, $\gamma_1 = 15$, $\gamma_2 = 100$. These numerical results demonstrate the efficiency of the theory.

5. Conclusion

We have designed an adaptive regulator for some collocated distributed parameter systems with unbounded input and output operators in the case of bounded deterministic disturbances. The adaptive regulator has been constructed by the

Fig. 1 Trajectory of $x(t, z)$ Fig. 2 Plots of $u(t)$, $k(t)$, $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$

concept of high-gain output feedback and the estimation mechanism of the unknown parameters for the disturbance. In the control system, the convergence of the system state to zero is guaranteed. Lastly we should note that the self-adjointness condition of the operator A is not essential. We can directly extend our theory to the systems with non self-adjoint discrete spectral operator A .

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