低次 IIR デジタルフィルタによる直線位相 FIR デジタルフィルタの振幅と位相の近似*

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Magnitude and Phase Approximations of Linear Phase FIR Digital Filters by Reduced-Order IIR Digital Filters*

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This paper compares the time and memory requirements of an approximately linear phase IIR digital filter with those of a linear phase FIR digital filter, and shows that, only if a substantial difference between the orders of the filters exists, the former can be considered as an attractive substitute for the latter. Furthermore, it presents a method which makes feasible the design of reduced-order IIR digital filters that resemble the characteristics of a linear phase FIR digital filter. Some design examples are shown, illustrating how the proposed method can be used to design approximately linear phase IIR digital filters that are faster and require less memory than their equivalent linear phase FIR counterparts.

1. Introduction

In digital signal processing there are several applications for which the phase response of digital filters is very important and cannot be ignored. Although some applications may require a phase response with some special characteristic, the cases most commonly found in practice are those requiring linear phase response. In such cases finite impulse response digital filters (FIR DFs) are normally used, due to their optional linear phase characteristic. However, FIR DFs are not the only possible choice, as several methods have been proposed so far for the design of approximately linear phase infinite impulse response digital filters (IIR DFs). The advantage of using an IIR DF instead of an FIR DF is that, for a given frequency response, the order of the former can be set much lower than that of the latter, simplifying the filtering system. Here, the definition of filter order is the same as that presented in reference 1).

The conventional approach is to design a standard

IIR DF with nonlinear phase followed by an all-pass filter which equalizes the group delay of it, i.e., makes the phase response approximately linear¹⁾. Although this method works well in the design of low-order IIR DFs, it is impractical when higher-order filters are necessary. Other methods are based on the approximation of the IIR filter frequency responses to previously specified ones (model approximation in the frequency domain)^{1)~4)}. There are also methods that take place in the time domain^{1), 2)}. However, the tradeoffs involved in the choice of the IIR DF's order are not discussed in detail in such works. A comparison between the characteristics and orders of linear phase FIR DFs and approximately linear phase IIR DFs without group delay equalizers can be found in reference 5). Although good approximations are obtained from the method proposed in that reference, the large storage and computational requirements make it difficult to implement and limit its use in real time applications. Another design method for approximately linear phase IIR DFs is proposed in reference 6), where a statistical approach is taken. However, that method is not suited to the design of high-order filters, as very large matrices are used. Furthermore, the orders of the numerator and denominator of the filter transfer function cannot be known before the design finishes, as filters that are unstable are discarded by

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the algorithm, without any attempt to make them stable.

If an IIR DF is compared with an FIR DF with the same order, it is found that the number of coefficients in the IIR DF's transfer function can be up to two times larger than those in an FIR DF without linear phase, and up to four times larger, in case of a linear phase FIR DF. Therefore, the question on whether the use of such an approximated IIR DF has in practice any advantage over the FIR DF must be answered, before the former replaces the latter.

In this paper the time and space complexities of the FIR DF are compared with those of an IIR DF. It is shown that to make the IIR DF advantageous in practical terms over the FIR DF, a large reduction in order is necessary. The paper also extends the design method proposed in reference 7), presenting two techniques that make feasible a reduction in the order of the filters designed by that method. These techniques are combined into a new design method which can derive approximately linear phase IIR DFs that are faster and smaller (in terms of memory space) than equivalent linear phase FIR DFs.

2. Relation between FIR DF and IIR DF's Orders

FIR DFs and IIR DFs have been compared in several textbooks on digital signal processing. In reference 1), for example, it is found that, under certain conditions, IIR and FIR DFs with similar magnitude responses can have orders up to 10 times different. In such cases the phase of the IIR DF is not of concern, being normally highly nonlinear. However, if an approximately linear phase IIR DF is desired, the difference of order cannot be very large. In the comparison between FIR DFs and delay equalized IIR DFs found in 1), it is observed that to achieve equalization to within a 3% error, the order of the IIR DF plus the order of the all-pass equalizer has to be larger than half the order of equivalent FIR DF. Moreover, for higher-order IIR DFs, the order of the necessary equalizer has to be so large that it is not practical to implement such delay equalized filters.

IIR DFs without equalizers, but still having an approximately linear phase response, can also be designed by one of the design methods proposed so far. However, as is shown in reference 7), satisfactory approximations of magnitude and phase responses can be obtained only for IIR DFs with order values that are at least equal to half the order of the equivalent FIR DFs.

2.1 Memory Requirements

In typical implementations, an N th-order IIR DF (assuming that both numerator and denominator of

the filter's transfer function are N th-order polynomials) needs up to $2N+1$ memory words to store filter coefficients and N words to store previous samples, making a total of $3N+1$ words. An M th-order linear phase FIR DF requires up to $3M/2+1$ words (even M) or $(3M-1)/2+1$ (odd M) for the same purpose. Therefore, for high-order filters, only if the condition $N < M/2$ holds, an economy in memory space is obtained by using an IIR DF instead of a linear phase FIR DF. In this statement it is considered that the filter's order is large, and that the difference of memory space occupied by the FIR DF and IIR DF's programs is negligible when compared with the data memory.

2.2 Computation Time Requirements

Now, if the processing speed is taken into account, it is found that up to $2N+1$ multiplications per sample are required for the IIR DF, and $M/2+1$ (even M) or $(M-1)/2+1$ (odd M) multiplications per sample for the linear phase FIR DF. Therefore, for high-order filters, if the condition $N < M/4$ is true the IIR DF has faster processing speed than the linear phase FIR DF.

Then, only if $N < M/4$, IIR DFs are faster and smaller (in terms of storage) than linear phase FIR DFs.

3. Design of Approximately Linear Phase IIR DFs

This section describes the design of digital filters in the time and frequency domains, stressing their weak and strong points. It gives also the background necessary to understand the design method proposed later in this paper.

3.1 Design in the Time and Frequency Domains

Most of the design methods for approximately linear phase IIR DFs proposed in the literature take place only in the time or in the frequency domain. The design in the frequency domain presents the advantage of approximating directly the IIR DF frequency response to that used as model. However, when both magnitude and phase have to be approximated, the process becomes more complicated, as two nonlinear optimizations are necessary. Moreover, before starting the design process, it is necessary to obtain somehow good initial values for the filter coefficients, because the objective function to be minimized is normally multimodal and convergence to an unsatisfactory local minimum representing an unstable filter is very likely to occur.

On the other hand, time domain design basically approximates the IIR DF impulse response to that of a system possessing the desired magnitude and phase responses. Such impulse response based

design allows simultaneous approximation of filter magnitude and phase responses, simplifying the process. Another strong point of time domain design is that, if the model is a stable system (which is true for an FIR DF) and if a good approximation to its impulse response is obtained, the designed IIR filter is likely to be also a stable system⁴⁾. The problems with this type of design are the necessity of knowing the impulse response of the model, and the fact that small difference between the two impulse responses can cause rather big difference between the magnitude responses. Therefore, a check on the filter characteristics in the frequency domain is necessary, to see if it attends or not the design specifications (normally expressed in terms of the magnitude and phase responses).

The design method proposed in this paper is partially based on the method presented in reference 7). It is a two-step process which takes place in both the time and frequency domains, exploring the advantages of both. First, the impulse response of the model (a linear phase FIR DF) is approximated by an IIR DF, using Kalman filter techniques. The resulting IIR DF has an impulse response and, consequently, magnitude and phase responses, similar to those of the FIR DF. However, due to the fact that small errors in the time domain can produce large errors in the frequency domain, a further refinement of the IIR DF frequency responses is recommended to achieve a higher degree of similarity. Such refinement can be done using an optimization technique as, for example, the Fletcher-Powell method, one of the most powerful optimization methods for multi-variable problems. Notice that choosing an adequate starting point is not a problem here, as the IIR DF coefficients derived by the Kalman filter algorithm are already close to a satisfactory minimum of the objective function to be minimized. Because of that, the poles of the IIR DF transfer function do not change very much during the optimization, and the filter is likely to remain stable. However, during this process, a stability check is made and in case that a pole moves outside the stable region (unit circle), it is replaced by its mirror image, and the optimization procedure continues⁷⁾.

This two-step procedure makes possible the design of a stable approximately linear phase IIR DF with better frequency responses than those obtained for a filter designed only in the time or in the frequency domain.

3.2 The Basis of the Design

As the design method proposed in this paper is partially based on that presented in reference 7), a brief description of it is given. Further details can be

found in that reference.

Approximately linear phase IIR DFs can be designed through parameter estimation techniques in the time domain, followed by an optimization procedure in the frequency domain, having linear phase FIR DFs as models. The design in the time domain can be made through the use of Kalman filter techniques. It is assumed that the FIR DF and IIR DF's outputs are given by

$$y_f(t) = \sum_{m=0}^M h_{f,m} u(t-m) \quad (1)$$

and

$$y(t) = \mathbf{x}^T(t) \boldsymbol{\theta} \quad (2)$$

respectively, where T denotes the transpose, and

$$\mathbf{x}^T(t) = [-y(t-1), \dots, -y(t-N), u(t), u(t-1), \dots, u(t-N)] \quad (3)$$

$$\boldsymbol{\theta} = [a_1, a_2, \dots, a_N, b_0, b_1, b_2, \dots, b_N]^T. \quad (4)$$

In the above equations $u(t)$ represents the input samples to the filter, $h_{f,m}$ the FIR filter coefficients, and (a_n, b_n) the IIR filter coefficients. The transfer functions of the above FIR and IIR DFs are given by

$$H_f(z) = \sum_{m=0}^M h_{f,m} z^{-m} \quad (5)$$

and

$$H(z) = \frac{\sum_{n=0}^N b_n z^{-n}}{1 + \sum_{n=1}^N a_n z^{-n}} \quad (6)$$

respectively.

The vector $\boldsymbol{\theta}$ of filter coefficients can be estimated from the following algorithm, which is an adaptation of the general Kalman filter method to this filter design problem.

$$\hat{\boldsymbol{\theta}}(t+1) = \hat{\boldsymbol{\theta}}(t) + \mathbf{k}(t+1)v(t+1) \quad (7)$$

$$v(t+1) = y_f(t+1) - y(t+1) \quad (8)$$

$$\mathbf{k}(t+1) = \frac{\mathbf{R}(t)\mathbf{x}(t+1)}{\sigma_v^2(t) + \mathbf{x}^T(t+1)\mathbf{R}(t)\mathbf{x}(t+1)} \quad (9)$$

$$\mathbf{R}(t+1) = [\mathbf{I} - \mathbf{k}(t+1)\mathbf{x}^T(t+1)] \mathbf{R}(t) \quad (10)$$

$$\hat{\boldsymbol{\theta}}(0) = \mathbf{0} \quad (11)$$

$$\mathbf{R}(0) = \gamma \mathbf{I}, \gamma > 0 \quad (12)$$

The matrix $\mathbf{R}(t)$ denotes the parameter error covariance, which can be represented by

$$\mathbf{R}(t) = E[\tilde{\boldsymbol{\theta}}(t)\tilde{\boldsymbol{\theta}}^T(t)] \quad (13)$$

where

$$\tilde{\boldsymbol{\theta}}(t) = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}(t). \quad (14)$$

The value of $\sigma_v^2(t)$ represents the variance of the output error, which can be calculated approximately by

$$\begin{aligned}\sigma_v^2(t) &= E \left[\{v(t) - \bar{v}(t)\}^2 \right] \\ &\cong \frac{1}{P} \sum_{i=t-P+1}^t [v(i) - \bar{v}_P(t)]^2\end{aligned}\quad (15)$$

where

$$\bar{v}(t) = E[v(t)] \quad (16)$$

and

$$\bar{v}_P(t) = \frac{1}{P} \sum_{i=t-P+1}^t v(i). \quad (17)$$

A block diagram of the system is shown in Fig. 1.

Fig. 1 Block diagram of the estimation process

The parameters obtained from the above equations are further refined through an optimization procedure in the frequency domain. Now, an objective function that measures the difference between the IIR DF and FIR DF's frequency responses is defined and minimized using the Fletcher-Powell method. The objective function is

$$E(Q, J) = Q(\theta) + \alpha J(\theta) \quad (18)$$

where

$$Q(\theta) = \sum_{i=1}^{K_1} [|H(e^{j\omega_i})| - |H_f(e^{j\omega_i})|]^2 \quad (19)$$

and

$$J(\theta) = \sum_{i=K_2}^{K_3} [\beta(e^{j\omega_i}) - \beta_f(e^{j\omega_i})]^2. \quad (20)$$

The functions $Q(\theta)$ and $J(\theta)$ represent the squared errors of magnitude and phase, respectively. The filter frequency responses are obtained from their transfer functions $H(z)$ and $H_f(z)$, evaluated at $z = e^{j\omega}$. The magnitude and phase responses are obtained from the following expressions.

IIR DF :

$$|H(e^{j\omega_i})| = \left[\frac{B_R^2(e^{j\omega_i}) + B_I^2(e^{j\omega_i})}{A_R^2(e^{j\omega_i}) + A_I^2(e^{j\omega_i})} \right]^{\frac{1}{2}} \quad (21)$$

$$\beta(e^{j\omega_i}) = \arctan$$

$$\frac{A_R(e^{j\omega_i})B_I(e^{j\omega_i}) - A_I(e^{j\omega_i})B_R(e^{j\omega_i})}{A_R(e^{j\omega_i})B_R(e^{j\omega_i}) + A_I(e^{j\omega_i})B_I(e^{j\omega_i})} \quad (22)$$

where

$$A_R(e^{j\omega_i}) = \text{real} \left[1 + \sum_{n=1}^N a_n e^{-jn\omega_i} \right] \quad (23)$$

$$A_I(e^{j\omega_i}) = \text{imag} \left[1 + \sum_{n=1}^N a_n e^{-jn\omega_i} \right] \quad (24)$$

$$B_R(e^{j\omega_i}) = \text{real} \left[\sum_{n=0}^N b_n e^{-jn\omega_i} \right] \quad (25)$$

and

$$B_I(e^{j\omega_i}) = \text{imag} \left[\sum_{n=0}^N b_n e^{-jn\omega_i} \right]. \quad (26)$$

FIR DF :

$$|H_f(e^{j\omega_i})| = \left| \sum_{m=0}^M h_{f,m} e^{-jm\omega_i} \right| \quad (27)$$

$$\beta_f(e^{j\omega_i}) = -\frac{M\omega_i}{2}. \quad (28)$$

The weight α in the objective function corrects the initial difference of scale between the two errors, and gives more flexibility to the process. It is calculated by

$$\alpha = \delta \frac{Q(\theta_0)}{J(\theta_0)} \quad (29)$$

where θ_0 represents the vector of parameters obtained from the design in the time domain, and δ is the weighting coefficient. The fraction in Eq. (29) makes the scale adjustment.

The index K_1 indicates the number of samples taken from the magnitude responses, and the indexes K_2 and K_3 stand for the first and last samples of the phase responses which are inside the pass band frequency range. The use of the above expressions simplifies the calculation of the gradient of the objective function $E(Q, J)$, necessary in the Fletcher-Powell method. The algorithm runs until the relative difference between two successive errors $E(Q, J)$ or its gradient falls below a certain tolerance value ϵ . This method is well suited to the design of high-order filters, as it is a recursive procedure that does not use large matrices. For a detailed explanation about the above algorithm refer to reference 7).

4. Reduction in the Order of the IIR DF

It is worth noticing that in almost all the methods based on impulse response approximation, like that described above, not only the linearity of the phase response is approximated, but also its inclination, i. e., its group delay. If only a linear phase is desired and the value of the group delay can be chosen arbitrarily (the normal case found in practice), a reduction in the order of the IIR DF can be expected. In what follows, two techniques that extends the method proposed in reference 7) are presented.

These techniques allow the design of reduced-order approximately linear phase IIR DFs, which are more attractive than equivalent FIR DFs, from a practical point of view.

4.1 Technique I : The Use of Phase Model

To obtain an IIR DF with phase response that is linear, but different from that of the model, time domain design cannot be used directly, as the impulse responses differ from each other. However, an auxiliary model for the IIR filter, called "phase model", can be used during the time domain design. This phase model is a linear phase FIR DF with order given by

$$M_p = 2\tau_p \quad (30)$$

where τ_p is the desired constant group delay. If the condition

$$M_p \leq 2N \quad (31)$$

is satisfied, good approximations can be expected⁷⁾. In order to have the magnitude response for the M_p th-order FIR DF close to that of the main FIR DF model, it is recommended to adopt an M_p as large as possible, i. e.,

$$M_p = 2N. \quad (32)$$

Although an IIR DF with the desired characteristic for phase is obtained in this stage, its magnitude response has yet to be approximated to that of the main FIR DF model. After the time domain design is finished, the frequency domain design takes place, using now, as model, the original M th-order FIR DF that attends the design specifications for magnitude response. The phase response used in this stage is the same as before, i. e.,

$$\beta_{f,p}(e^{j\omega_i}) = -\tau_p \omega_i. \quad (33)$$

At the end of this optimization, an approximately linear phase IIR DF with a group delay smaller than that of the main FIR DF model is obtained. More important is the fact that such IIR DF has an order lower than half the order of the main FIR DF model, making it smaller than this model in terms of memory space.

In order to illustrate the above technique, a 20th-order IIR DF is designed using as models a 40th-order (phase model for time domain design) and a 100th-order (frequency domain design) low-pass linear phase FIR DFs. These FIR DFs are designed by the Remez exchange algorithm¹⁾, which is well known by producing optimal filters for a given set of specifications. These specifications are :

- pass band magnitude : 1(0 dB)
- stop band magnitude : 0($-\infty$ dB)
- transition band width : 0.09π

- cutoff frequency : 0.2π

The tolerance ϵ for the relative change of $E(Q, J)$ in the Fletcher-Powell method is set to 10^{-6} , and the value of δ to 2. The magnitude response in linear and dB scales, and the phase response of the filters are shown in Figs. 2 to 4. Although the IIR DF's magnitude response in the stop band is larger than that of the FIR DF, it is below the -40 dB level, which is sufficient for most practical applications. Notice that the ripple in the pass band is small and that the phase response is practically linear within this region (Fig. 5). As the frequency components that are outside the pass band are attenuated by the filter, an eventual nonlinearity in the phase response outside such region is not important. Because the algorithm does not attempt to make the phase linear outside the pass band, it is expected to achieve better results

Fig. 2 Magnitude response using Technique I : (---) FIR DF, (—) IIR DF, (···) design constraints

Fig. 3 Magnitude response in dB using Technique I : (---) FIR DF, (—) IIR DF, (···) design constraints

Fig. 4 Phase response using Technique I : (---) FIR DF (adjusted), (—) IIR DF, (···) cutoff frequency

Fig. 5 Ripple in the pass band using Technique I :
(---) FIR DF, (—) IIR DF

than other design procedures that spend efforts in making all the phase response linear. The final values of $Q(\theta)$ and $J(\theta)$ are 0.0128 and 3.5364, respectively, for $K_1 = 512$, $K_2 = 1$ and $K_3 = 102$. The maximum errors of the magnitude and phase responses, defined as

$$E_H(\theta) = \max_i \left| |H(e^{j\omega_i})| - |H_f(e^{j\omega_i})| \right| \quad (34)$$

$$E_\beta(\theta) = \max_{K_2 \leq i \leq K_3} |\beta(e^{j\omega_i}) - \beta_f(e^{j\omega_i})| \quad (35)$$

are equal to 0.0236 and 0.5329, respectively.

4.2 Technique II : Implementation of Time Delays

Another way of achieving a change in the filter's group delay is through the use of time delay units. Mathematically this procedure is equivalent to multiply the IIR DF transfer function by z^{-d} , where d is a positive integer representing the desired delay. Such artifact was also used in reference 6), and it has the advantage of causing no effect in the filter magnitude response, as the magnitude of z^{-d} is equal to 1. There is only a change in the inclination of the phase response. The idea here is to use the delay z^{-d} during the design of the IIR DF and to remove it after the design is finished. Then, a new filter results, with the same magnitude response as before, but with a different (smaller) group delay. An important characteristic of Technique II is that, by using z^{-d} , the transfer function of the system composed by the IIR DF and the time delay unit looks like a higher-order function ($N + d$), facilitating the impulse response approximation. Fig. 6 shows how the time delay is inserted in the parameter estimation scheme.

An important detail is that in practice the use of time delay is equivalent to ignore the first d samples of the FIR DF impulse response, i.e., equivalent to consider these samples as being zero. Therefore, this method is well suited to the approximation of high-order linear phase FIR filters, as they are characterized by having impulse responses with very small

Fig. 6 Block diagram of the design method using time delay

initial values. Based on this, some useful hints to find an appropriate value of d can be obtained. First of all, the condition

$$d + N \geq \frac{M}{2} \quad (36)$$

has to be fulfilled if a satisfactory approximation is desired in the time domain design. Another one is that a very large d is not allowed, as it implies disregarding many initial samples of the FIR DF impulse response. If among these samples there are one or more that are higher than a certain tolerance value (found experimentally to be around 5% of the impulse response's highest peak), satisfactory results may not be obtained for the phase response. This is readily understood if we remember that the symmetry of the FIR DF impulse response is responsible for the linearity of the phase response. Therefore, the value of d must be as low as possible, in order to reduce the asymmetry of the model impulse response, caused by the inclusion of it. In the frequency domain optimization, d is used to adjust the inclination of the phase response to be used as model. This phase reference is obtained by

$$\beta_{f,aux}(e^{j\omega_i}) = -\frac{M-2d}{2} \omega_i. \quad (37)$$

To exemplify Technique II, a 20th-order IIR DF is designed and its frequency responses are shown in Figs. 7 and 8, together with those of the model, which is the same 100th-order linear phase FIR DF used in the previous example. The value of the delay is set to 30, and the other design specifications are the same as before. Notice that the phase response obtained by this method is also practically linear within the pass band (Fig. 9). Although an attenuation smaller than that of the FIR DF is obtained here again for the IIR DF, it is still below the level of -40 dB. It is worth noticing that within the pass band the ripple is very small (Fig. 10) and that the phase response is approximately linear, making this IIR DF a suitable candidate to replace the FIR DF. If this replacement takes place,

Fig. 7 Magnitude response using Technique II :
(---) FIR DF, (—) IIR DF, (···) design constraints

Fig. 8 Magnitude response in dB using Technique II :
(---) FIR DF, (—) IIR DF, (···) design constraints

Fig. 9 Phase response using Technique II :
(---) FIR DF (adjusted), (—) IIR DF,
(···) cutoff frequency

Fig. 10 Ripple in the pass band using Technique II :
(---) FIR DF, (—) IIR DF

an output sample will be obtained with 19% fewer multiplications and the filter will use 59% less memory space. For this example, the values of $Q(\theta)$, $J(\theta)$, $E_H(\theta)$ and $E_\beta(\theta)$ are equal to 0.0028, 3.3277,

0.0085 and 0.2988, respectively, where K_1 , K_2 and K_3 are the same as before.

4.3 A New Design Method

The previous two techniques can be combined into a new design method that takes advantage of both. As a result, the order of the IIR DF can be further reduced, making it even more attractive in terms of size and speed over equivalent FIR DFs. The process is still divided into two steps. First, the IIR DF approximates an M_{aux} th-order FIR DF used as phase model (auxiliary model), using the time domain algorithm. Then, the resulting filter parameters are optimized in the frequency domain, using the main M th-order FIR DF as model. The difference from Technique I is that now the time delay d is also used (Technique II), making possible the design of a filter with order lower than the IIR DF derived by that technique.

The design method is exemplified by the results presented in Figs. 11 to 14. The 15th-order IIR DF shown in these figures is based on an initial approximation to a 60th-order FIR DF, using a delay $d = 15$. Following this initial design, comes the frequency domain optimization, which has the previous 100th-order FIR DF as model. In Fig. 15 the poles of the IIR DF transfer function are plotted before and after the optimization. This figure shows that, as expected, the position of the poles does not change substantially, as the initial conditions for the filter parameters are already close to satisfactory minimum of the objective function $E(Q, J)$. Because of that, the stable IIR DF obtained from the time domain design is likely to remain stable after the frequency domain optimization. However, in the case that a pole goes outside the unit circle, it will be still very close to this circle, and replacing it by its mirror image will not have a strong influence on the phase response. The 15th-order IIR DF above represents an economy of 69% in memory and 39% in computation time over the 100th-order linear phase FIR DF. The squared and maximum errors $Q(\theta)$, $J(\theta)$, $E_H(\theta)$ and $E_\beta(\theta)$ are, in this case, 0.0177, 3.8860, 0.0306 and 0.3447, respectively. Also here the values of K_1 , K_2 and K_3 are as in the first example.

In order to obtain satisfactory results, the tradeoffs involved in the choice of the auxiliary FIR DF's order M_{aux} and delay d must be taken into account. Although a closed formula to choose these parameters is difficult to derive, it is possible to obtain an idea of the relations among them from the experiments. If the order M_{aux} is close to M , the difference $M_{\text{aux}} - N$ will be large, and the design in the time domain becomes difficult. On the other hand, if M_{aux} is close to $2N$, the magnitude response of the IIR DF ob-

Fig. 11 Magnitude response using the new design method : (---) FIR DF, (—) IIR DF, (···) design constraints

Fig. 12 Magnitude response in dB using the new design method : (---) FIR DF, (—) IIR DF, (···) design constraints

Fig. 13 Phase response using the new design method : (---) FIR DF (adjusted), (—) IIR DF, (···) cutoff frequency

Fig. 14 Ripple in the pass band using the new design method : (---) FIR DF, (—) IIR DF

tained in the time domain will be more different from the desired magnitude response (M -th order FIR DF), and then the optimization in the frequency

Fig. 15 Poles of IIR DF using the new design method : (*) before optimization (o) after optimization

domain may be difficult. In any case, the relation in Eq.(38) must be observed, if good approximations during the time domain design are desired.

$$d + N \geq \frac{M_{\text{aux}}}{2} \quad (38)$$

5. Some Remarks

The use of the new design method makes feasible the design of approximately linear phase IIR DFs with orders that are lower than those of the filters obtained by one of the techniques used alone, and much lower than those of the filters presented in reference 7).

In the examples shown here the stop band attenuation of the FIR DF is not always achieved by the IIR DF, due to the reduction in its order. However, the latter has an attenuation beyond the -40dB level, which is sufficient for most of the practical applications. Furthermore, reduced ripple in the pass band and sharp cutoff characteristic are strong points of the filters designed by the proposed method. Similar results to those presented here were also obtained for band-pass and high-pass filters, which are not presented due to space limitations.

To what extent can the order of the IIR DF be reduced? The answer is not unique, and it depends on how much the IIR DF's magnitude response must be close to the FIR DF's one. If the order difference is large, the difference between the two magnitude responses increases, and for certain critical applications the approximations will not be satisfactory anymore.

6. Conclusions

In this paper, it has been shown that the implementation of an approximately linear phase IIR DF is advantageous over an equivalent linear phase FIR DF with respect to memory space only if the former's order is lower than 50% of the latter's one. Moreover, if the order of the IIR DF is under 25% of

the linear phase FIR DF's order, not only memory space is saved, but also the time spent to calculate an output sample is reduced. To make feasible the design of approximately linear phase IIR DFs that attend the above conditions, a new design method has been proposed, and some design examples have been shown. The use of this method makes possible the design of approximately linear phase IIR DFs that are faster and require less memory than the conventional linear phase FIR DFs.

References

- 1) L. Rabiner and B. Gold : Theory and Application of Digital Signal Processing, Prentice-Hall Inc. , New Jersey (1975)
- 2) T. Yahagi : Theory of Digital Signal Processing, Vols. 2 and 3, Corona Publ. Co., Ltd., Tokyo(1985 and 1986)
- 3) T. Inukai : A Unified Approach to Optimal Recursive Digital Filter Design ; IEEE Trans. Circuits and Systems, Vol. CAS-27, No. 7, pp. 646~649 (1980)
- 4) G. Gu and B. A. Shenoi : A Novel Approach to the Synthesis of Recursive Digital Filters with Linear Phase ; IEEE Trans. Circuits and Systems, Vol. CAS-38, No. 6, pp. 602~612 (1991)
- 5) S. R. Powell and P. M. Chau : A Technique for Realizing Linear Phase IIR Filters ; IEEE Trans. Acoustics, Speech, and Signal Processing, Vol. ASSP-39, No. 11, pp. 2425~2435 (1991)
- 6) T. Komatsu, Y. Monden and S. Arimoto : Statistical Design of Linear-Phase ARMA Filters—Concentrating on Their Frequency-Domain Characteristics ; Trans. Society of Instrument and Control Engineers, Vol. 18, No. 12, pp. 1202~1206 (1982)
- 7) M. A. A. Henriques and T. Yahagi : Time and Frequency Domain Design of Approximately Linear Phase IIR Digital Filters ; IEICE Trans. Fundamentals of Electronics, Communications and Computer Sciences, Vol. E75-A, No. 10, pp. 1429~1437(1992)