見込み生産の生産指示間隔による柔軟性の向上*

高橋 勝彦**・中村 信人**

Improvement of Flexibility by Production Ordering Interval in Make-to-Stock Environment*

Katsuhiko TAKAHASHI** and Nobuto NAKAMURA**

This paper studies the flexibility of production management. Especially, the effects of production ordering intervals upon the flexibility are analyzed for multi-stage production inventory systems in make-to-stock environment. After the introduction, the literature on the flexibility of production management is reviewed. A mathematical model of a production system and its management system is developed. In the model, a multi-stage production inventory system with autocorrelated product demand fluctuation is formulated. Also, the production ordering interval is considered in the production management system. As performance criteria of flexibility, the amplifications of production quantity and inventory level are analyzed, and the relationship between the production ordering interval and the flexibility of production management system is clarified.

1. Introduction

The flexibility of production management is a key factor in responding flexibly to the variety of customers needs and in improving the competitive advantage of a production system. There has been a lot of research on developing production systems with flexibility, such as FMS. Also, there is a lot of research on CIM which aims to respond flexibly to customers needs by integrating the functions surrounding production systems, such as product development, production technology, production, and sales. Not only the production systems considered in the research but also production management systems have a lot of influence upon flexibility.

The flexibility of production management can be considered as a criterion for evaluating the response of production management to changes in environmental factors, such as demand fluctuation and machine breakdown. If the capability to respond the changing factors of production

systems can be improved by selecting an appropriate production management system or setting some control variables of the production management system to an appropriate level, the production management system can be regarded as more flexible than others. There has been a wide variety of research on the flexibility of production management. However, most of the research evaluates not production management systems but production systems primarily from the viewpoint of flexibility.

This paper studies the flexibility of production management. Especially, the effects of production ordering intervals upon the flexibility are analyzed for multi-stage production inventory systems in make-to-stock environment. The production ordering interval is an interval in calculating and releasing a production order for each production process, and it is an important decision variable of production management in make-to-stock environment. Shortening the production ordering interval leads to accurate production ordering quantities because of accurate demand forecasts. And in turn the fluctuations of production quantity and inventory level will be dampened. This means an increase in flexibility. On the other hand, lengthening the production ordering interval results in a smoothing of production ordering

* 原稿受付 1994年10月20日

** 広島大学 工学部 Faculty of Engineering, Hiroshima University; 4-1 Kagamiyama 1-chome, Higashi-Hiroshima city, Hiroshima 739, JAPAN

Key Words: manufacturing flexibility, ordering interval, multi-stage system, ordering system, make-to-stock.

quantities. This also means an increase in flexibility. It can be seen therefore, the production ordering interval and the flexibility have a complicated relationship and it is both important and valuable to study the relationship. This is the motivation of this research.

After the introduction, the paper reviews the literature on the flexibility of production management and the literature on comparing production management systems. Then, the paper develops a mathematical model of a multi-stage production inventory system with autocorrelated product demand fluctuation. In the production management system for the production inventory system, the production ordering interval and two types of production ordering systems are modeled. As performance criteria of flexibility, the amplifications of production quantity and inventory level are analyzed, and the relationship between the production ordering interval and the flexibility of the production management system for each production ordering system is clarified. Finally, the paper summarizes the findings obtained in this paper as conclusions.

2. Literature review

In this section, the literature on the flexibility of production management is briefly reviewed. There is a lot of research on manufacturing flexibility from various view points (some papers review the literature, see, for example, Gupta and Somers¹⁾). Slack⁶⁾ classified the types of manufacturing flexibility. Son and Park⁷⁾ proposed an economic measure of flexibility in advanced manufacturing systems. Also, Muramatsu et al.⁴⁾ proposed physical measures of manufacturing flexibility. Gupta and Somers¹⁾ developed an instrument for measuring and analyzing manufacturing flexibility. Takahashi et al.⁸⁾ classified the changing factors affecting manufacturing flexibility. According to the papers, it can be seen that there are a lot of types of flexibility and performance criteria, and we must thoroughly consider the changing factors of production systems, its management systems, and the performance criteria before discussing the flexibility.

On the other hand, there is also a lot of research on comparing production management systems: however, there is a little research on production ordering intervals. Matsuura and Tsubone³⁾ dealt with the production ordering interval in comparing production ordering systems for two-stage production inventory systems. The effect of production ordering intervals upon flexibility for

more general production systems has not yet been clarified. Also, the production ordering systems in their paper are based on the push concept: that is, they dealt only with Push type production ordering systems, and they did not deal with Pull type production ordering systems.

3. Modeling production management system

In this section, a mathematical model of a production system and its management system is developed. As production management systems, the production ordering interval and two types of production ordering systems are considered in the model.

3.1 Assumptions

The assumptions of the multi-stage production inventory system dealt with in this paper are as follows:

Product: The production system produces a single standard product, that is, a product which can be made to stock. The demand for the product at each planning period, D_t is a steady state (μ_D : mean value, σ_D^2 : variance) time series with an exponential autocorrelation as follows;

$$\frac{\text{Cov}(D_t, D_{t-j})}{\sqrt{V(D_t) \cdot V(D_{t-j})}} = \lambda^j, \quad (1)$$

where, $\text{Cov}(x, y)$ means the covariance between x and y , $V(x)$ does the variance of x . Also, λ is the parameter of autocorrelation. According to the value of λ , a variety of time series from independent to highly autocorrelated time series of product demand can be shown.

Demand forecast: The product demand of the $(t+j)$ th period is forecasted at the end of the t -th period as follows:

$$\hat{D}_{t,t+j} = D_t \quad (2)$$

Eq. (2) is a simplified forecasting rule; however, it does not lose the generality. The variance of forecasting error is calculated as follows;

$$V(e_{t-j,t}) = (1 - \lambda^j) \cdot \sigma_D^2, \quad (3)$$

where, $e_{t-j,t} = \hat{D}_{t-j,t} - D_t$. Eq. (3) shows that the variance of forecasting error decreases as the time lag decreases or the autocorrelation becomes strong.

Production process: The product is produced through the production system that consists of N production processes. The production processes are connected serially. Each production process is

labeled 1st, 2nd, 3rd, ..., or N -th stage according to the upstream stage from the final process. Each process produces a standard item, and has sufficient production capacity.

Production control: The production at each stage is controlled periodically by equal and synchronized periods.

Production lead time: It takes one period at each production process, that is, the production order at the end of t -th period for each stage is produced during the $(t+1)$ st period, and is supplied to the inventory point of the stage at the beginning of $(t+2)$ nd period.

Backlog: The backlog for product demand is allowed.

Safety stock: The sufficient level of buffer stock, $S^{(n)}$ is allocated for each stage as safety stock.

3.2 Production system

The production system dealt with in this paper is formulated as follows:

Production order: The production order for each stage is calculated with an ordering interval by means of the production ordering system as described later.

Production quantity: With the assumptions about the production process and the production lead time, the production quantity at each stage is formulated as follows:

$$P_t^{(n)} = O_t^{(n)}, \quad (4)$$

where, $O_t^{(n)}$ is the production order of the n -th stage for the t -th period.

Inventory level: With the assumptions about the production lead time and the backlog, the inventory level of each stage at the end of t -th period is formulated as follows:

$$I_t^{(1)} = I_{t-1}^{(1)} + P_{t-1}^{(1)} - D_t \quad (5)$$

$$I_t^{(n)} = I_{t-1}^{(n)} + P_{t-1}^{(n)} - P_t^{(n-1)} \quad (n = 2, 3, \dots, N). \quad (6)$$

3.3 Production management system

The production orders of K periods are calculated at the same time, and it is defined that the production ordering interval is equal to K periods. The production order is calculated by means of each of the following two types production ordering systems.

Push type production ordering system: This system is a model of forecast-oriented planning systems, such as MRP (Material Requirements Planning) system⁵⁾. In the system, the production order for each stage is calculated on the basis of product demand forecast and the feedback information. In our model, the production orders

during the $(t+1)$ st through $(t+K)$ th periods are calculated at the same time, and are summed up and divided by the production ordering interval K . Then, the production order of the n -th stage for the $(t+j)$ th period ($j = 1, 2, \dots, K$), $O_{t+j}^{(n)}$ is calculated as follows:

$$O_{t+j}^{(n)} = \frac{1}{K} \sum_{l=1}^{K+1} [\hat{D}_{t,t+n-1+l} - O_t^{(n)} + S^{(n)} - I_t^{(n)}] \\ (n = 1, 2, \dots, N; j = 1, 2, \dots, K). \quad (7)$$

Pull type production ordering system: This system is a model of result-oriented planning system, such as the Kanban system in Just-In-Time(JIT) production²⁾. In the system, the production order of the n -th stage is calculated on the basis of the actual production order for the succeeding stage. In our model of this system, however, the production order of the first stage is defined to be calculated by the same equation to that of the Push type production ordering system. As a result, the production orders during the $(t+1)$ st through $(t+K)$ th periods are calculated at the same time, and they are released for each stage at the end of t -th period.

$$O_{t+j}^{(1)} = \frac{1}{K} \sum_{l=1}^{K+1} [\hat{D}_{t,t+l} - O_t^{(1)} + S^{(1)} - I_t^{(1)}] \quad (8) \\ (j = 1, 2, \dots, K)$$

$$O_{t+j}^{(n)} = O_{t-1+j}^{(n-1)} \\ (n = 2, 3, \dots, N; j = 1, 2, \dots, K) \quad (9)$$

4. Analysis of flexibility

The flexibility of the developed model is analyzed in this section. After formulating performance criteria of flexibility and summarizing the method of analysis, the results of analysis are shown and discussed.

4.1 Performance criteria of flexibility

Flexibility is an ability to respond to changes. Manufacturing flexibility is a performance criteria relating to how a given production management system can respond to the changes of production system. The fluctuation of product demand, a factor of the changes, causes the fluctuations of production quantity and inventory level. One purpose of production management is to dampen the fluctuations, thus leading to an increase in the flexibility of the system^{4),8)}. In this paper, therefore, the variances of production quantity and inventory level of each stage standardized by the variance of product demand are considered. Especially, we define the amplifica-

tions of production quantity and inventory level by averaging the standardized variances during the production ordering interval as follows :

$$Amp(P^{(n)}) = \lim_{t \rightarrow \infty} \frac{1}{K} \sum_{j=1}^K \left[\frac{V(P_{t+j}^{(n)})}{V(D_t)} \right] \quad (10)$$

($n = 1, 2, \dots, N$)

$$Amp(I^{(n)}) = \lim_{t \rightarrow \infty} \frac{1}{K} \sum_{j=1}^K \left[\frac{V(I_{t+j}^{(n)})}{V(D_t)} \right] \quad (11)$$

($n = 1, 2, \dots, N$).

The amplification of production quantity is calculated from the variance of production quantity divided by the variance of product demand. Increasing the square root of the amplification of production quantity at each production process leads to a proportional increase of the production capacity at that stage, and the value can be utilized to design the production capacity. Also, the amplification of inventory level is calculated from the variance of inventory level divided by the variance of product demand. Increasing the square root of the amplification of inventory level at each inventory point leads to a proportional increase of the safety stock level at the inventory point, and the value can be utilized to design the safety stock level. Therefore in this paper, we consider the amplification of production quantity and that of inventory level as the performance criterion for production and that for inventory, respectively.

4.2 Method of analysis

As all of the equations formulated in the preceding section are linear equations, the production quantity and inventory level at each stage modeled by difference equations are calculated, and they can be expressed by an input variable, that is, product demand. Then, the variances of production quantity and inventory level, and finally the amplifications are calculated.

4.3 Results of analysis

The amplifications of Push type production ordering system, $Amp(P^{(n)})_{push}$ and $Amp(I^{(n)})_{push}$ are calculated as follows ;

$$Amp(P^{(n)})_{push} = 2f^2 + \frac{1}{K^2} U_K + \frac{2}{K} f(V_{K-1} + 1) \lambda^{(n-1)K} - 2f^2 \lambda^{nK} - \frac{2}{K} f V_K, \quad (12)$$

($n = 1, 2, \dots, N$)

$$Amp(I^{(n)})_{push} = \frac{1}{K} \sum_{j=1}^K [a_j^2 + b_j^2 U_{K-1} + c_j^2 + d_j^2 U_K + f^2 + 2a_j b_j V_{K-1} + 2a_j c_j \lambda^K - 2a_j d_j \lambda^K V_K - 2a_j f \lambda^{nK} + 2b_j c_j V_{K-1} - 2b_j d_j V_{K-1} V_K - 2b_j f \lambda^{(n-1)K} V_{K-1} - 2c_j d_j V_K - 2c_j f \lambda^{(n-1)K} - 2d_j f (V_{K-1} + 1) \lambda^{n-2 \lfloor K \rfloor}], \quad (13)$$

($n = 1, 2, \dots, N$)

where,

$$a_j = K + 2 - j - \frac{j-1}{K},$$

$$b_j = \frac{j-K-1}{K},$$

$$c_j = \frac{Kj+2j-K-1}{K},$$

$$d_j = \frac{j}{K},$$

$$f = \frac{K+1}{K},$$

$$U_K = 2 \sum_{l=0}^{K-1} (K-l) \lambda^l - K,$$

$$V_K = \sum_{l=1}^K \lambda^l.$$

The amplifications of Pull type production ordering system, $Amp(P^{(n)})_{pull}$ and $Amp(I^{(n)})_{pull}$ are calculated as follows ;

$$Amp(P^{(n)})_{pull} = Amp(P^{(1)})_{push}, \quad (14)$$

($n = 1, 2, \dots, N$)

$$Amp(I^{(1)})_{pull} = Amp(I^{(1)})_{push}, \quad (15)$$

$$Amp(I^{(n)})_{pull} = \frac{1}{K} [4g Amp(P^{(1)})_{push} + h^2 + i^2 + U_{2K-1} - 2h i \lambda^{2K} + 2V_{2K-1}], \quad (16)$$

($n = 2, 3, \dots, N$)

where, $g = K-1$, $h = K+1$, $i = K+2$.

Based on the results of analysis shown above, the effects of production ordering interval upon the amplifications of production quantity and inventory level in the case of no autocorrelation ($\lambda = 0.0$) are shown by Figs. 1 and 2, respectively. Also, those in the case of strong autocorrelation of product demand ($\lambda = 0.8$) are shown by Figs. 3 and 4, respectively.

Fig. 1 The effect of the production ordering interval upon the amplification of production quantity in the case of no autocorrelation of product demand ($\lambda = 0.0$, solid line; Push, broken line; Pull, +; $K = 1$, $\times$; $K = 4$, ∇ ; $K = 7$).

Fig. 2 The effect of the production ordering interval upon the amplification of inventory level in the case of no autocorrelation of product demand ($\lambda = 0.0$, solid line; Push, broken line; Pull, +; $K = 1$, $\times$; $K = 4$, ∇ ; $K = 7$).

Fig. 3 The effect of the production ordering interval upon the amplification of production quantity in the case of strong autocorrelation of product demand ($\lambda = 0.8$, solid line; Push, broken line; Pull, +; $K = 1$, $\times$; $K = 4$, ∇ ; $K = 7$).

From those figures, the following findings are obtained. As the production ordering interval increases, the amplifications of production quantity $Amp(P^{(n)})$ decrease and those of inventory level

Fig. 4 The effect of the production ordering interval upon the amplification of inventory level in the case of strong autocorrelation of product demand ($\lambda = 0.8$, solid line; Push, broken line; Pull, +; $K = 1$, $\times$; $K = 4$, ∇ ; $K = 7$).

$Amp(I^{(n)})$ increase regardless of the parameter of autocorrelation. $Amp(P^{(n)})$ of Pull type production ordering system are smaller than those of Push type only in the case of strong autocorrelation ($\lambda = 0.8$) and short production ordering interval ($K = 1$). Also, $Amp(I^{(n)})$ of Pull type production ordering system is smaller only in the case of no autocorrelation ($\lambda = 0.0$) and long production ordering interval ($K = 7$). That is, the production ordering interval influences the trade-off relationship between $Amp(P^{(n)})$ and $Amp(I^{(n)})$. Also, it has a strong influence upon the flexibility of production ordering systems.

4.4 Total evaluation

As there is the trade-off relationship among the amplifications, this paper introduces total criteria in order to evaluate the production ordering interval and production ordering systems totally as follows ;

$$TP = \sum_{n=1}^N w_p^{(n)} \cdot \sqrt{Amp(P^{(n)})}, \quad (17)$$

$$TI = \sum_{n=1}^N w_i^{(n)} \cdot \sqrt{Amp(I^{(n)})}, \quad (18)$$

where, $w_p^{(n)}$ is the weighting factor for the amplification of production quantity, and $w_i^{(n)}$ is the weighting factor for the amplification of inventory level.

Eq. (17) formulates the total criterion for the amplification of production quantity. Because the production capacity at each stage is proportional to the square root of the amplification of production quantity at the stage as described above, it can be considered that the total criterion is a total cost of production capacity and $w_p^{(n)}$ is the unit cost for the production capacity at the n -th stage.

On the other hand, eq. (18) formulates the total

criterion for the amplification of inventory level. Because the safety stock level at each stage is proportional to the square root of the amplification of the inventory level at the stage as described above, it can be considered that the total criterion is a total cost of safety stock and $w_I^{(n)}$ is the unit cost for the safety stock at the n -th stage.

A single criterion obtained by summing up the total criteria for the amplifications of production quantity and inventory level can be also considered, and it can be utilized to select an optimum production ordering system and to set the production ordering interval to an optimum level, however, we divide it into two criteria as shown in eqs. (17) and (18) in order to analyze the trade-off relationship between the criteria.

Results of evaluation are shown in Figs. 5 and 6. Both Figs. 5 and 6 show the trade-off relationship between the total criterion for the amplification of production quantity and that for the amplification

Fig. 5 The trade-off relationship between the amplification of production quantity and that of inventory level in changing the production ordering interval under the no autocorrelation (+; $\lambda = 0.0$) and strong autocorrelation ($\times$; $\lambda = 0.8$, $w_p^{(n)} = w_I^{(n)} = 1$, solid line; Push, broken line; Pull).

Fig. 6 The trade-off relationship between the amplification of production quantity and that of inventory level in changing the production ordering interval under the increasing weight (+; $w_p^{(n)} = w_I^{(n)} = 0.8^{n-1}$) and the equal weight ($\times$; $w_p^{(n)} = w_I^{(n)} = 1$, $\lambda = 0.0$, solid line; Push, broken line; Pull).

of inventory level. As the production ordering interval is increased from 1 to 12 for each production ordering system, the solid line and the broken line show the values of (TP, TI) of Push type production ordering system and Pull type production ordering system, respectively. In Fig. 5, the parameter of autocorrelation (λ) is fixed at 0.0 and 0.8. Also in Fig. 6, the weighting factors for the amplifications ($w_p^{(n)}$ and $w_I^{(n)}$) are fixed at 0.8^{n-1} (the weighting factors increase as the stage proceeds) and 1^{n-1} ($=1$, the weighting factors are equal to each other).

From the figures, it is seen that the amplification of production quantity can be decreased with the increase in the amplification of inventory level by increasing the production ordering interval. The trade-off relationship between the amplifications suffers from the effect of the parameter of autocorrelation (λ , see Fig. 5). The weighting factors for the amplifications ($w_p^{(n)}$ and $w_I^{(n)}$, see Fig. 6) also affect the values of the amplifications. However, they do not affect the qualitative trade-off relationship.

Setting the production ordering interval to an appropriate level leads to a decrease in the sum of the amplifications of production quantity and inventory level. The level differs from one production ordering system to another, and it suffers from the effect of the parameter of autocorrelation and the weighting factors for the amplifications. By calculating the values of the amplifications based on the results of analysis in this paper (eqs. (13)~(18)), an optimum level of production ordering interval can be determined for each production ordering system under the parameter of autocorrelation and the weighting factors for the amplifications, and an optimum production ordering system with an optimum production ordering interval can be selected. The optimum level of production ordering interval and the optimum production ordering system serve to minimize the amplifications of production quantity and inventory level, and the production management system will be flexible; that is, it can flexibly respond to the demand fluctuation.

5. Conclusions

This paper studied the flexibility of production management. Especially, for multi-stage production inventory systems in make-to-stock environment, this paper studied the effect of production ordering interval upon the flexibility. The findings obtained in this paper are summarized as follows.

- (1) By reviewing the literature on the flexibility of production management, this paper pointed out that the effect of production ordering interval has not yet been comprehensively researched even though it is an important decision variable in make-to-stock environment.
- (2) A mathematical model for investigating the effect of production ordering interval upon flexibility can be developed.
- (3) By analyzing the performance of the developed model, this paper presented the relationship between the production ordering interval and the flexibility. Also, this paper analyzed the trade-off relationship between the performance measures.
- (4) As a result, this paper claims that the production ordering interval has extensive influence upon flexibility as well as the production ordering systems and the condition of product demand autocorrelation. The results of the analysis in this paper can be utilized to design a flexible production management system.

References

- 1) Y. P. Gupta and T. M. Somers : The Measurement of Manufacturing Flexibility ; Euro. J. Opl Res., Vol. 60, No. 2, pp. 166 ~182 (1992)
- 2) O. Kimura and H. Terada : Design and Analysis of Pull System : A Method of Multi-Stage Production Control ; Int. J. Prod. Res., Vol. 19, No. 3, pp. 241~253 (1981)
- 3) H. Matsuura and H. Tsubone : A Comparison of Centralized and Decentralized Control Rules in Push-Type Production Ordering Systems ; Euro. J. Opl Res., Vol. 70, No. 2, pp. 199~211 (1993)
- 4) R. Muramatsu, K. Ishii and K. Takahashi : Some Ways to Increase Flexibility in Manufacturing Systems ; Int. J. Prod. Res., Vol. 23, No. 4, pp. 691~703 (1985)
- 5) J. Orlicky : Material Requirements Planning, McGraw-Hill (1975)
- 6) N. Slack : The Flexibility of Manufacturing Systems ; J. Opns. Prod. Mgt, Vol. 7, No. 4, pp. 35~45 (1987)
- 7) Y. Son and C. S. Park : Economic Measure of Productivity, Quality and Flexibility in Advanced Manufacturing Systems ; J. Manuf. Sys., Vol. 6, No. 3, pp. 193~207 (1987)
- 8) K. Takahashi, S. Hiraki and M. Soshiroda : Flexibility of Production Ordering Systems ; Int. J. Prod. Res., Vol. 32, No. 7, pp. 1739~1752 (1994)