A Generalization of Direct Strain Feedback Control for a Flexible Structure with Spatially Varying Parameters and a Tip Body — A New Example of the A-dependent Operator*

Motofumi HATTORI**, Satoshi TADOKORO** and Toshi TAKAMORI**

1. Introduction

Recently, great attention is paid to control flexible structures. Many control laws are proposed for stabilizing flexible structures. As a simple and robust controller, H. G. Lee et al. proposed the strain feedback\(^2\(^3\)\). If the strain (the bending moment) signal at the root end of the flexible robot arm is measured and directly fed back, then the closed loop system becomes asymptotically stable and the vibration can be suppressed (damping enhanced). In their research, the asymptotic stability was proved only for the finite dimensional approximation model of the flexible robot arm.

But, since these flexible structures are continua physically, the rigorous dynamics model of flexible structures are infinite dimensional model. For uniform one-link flexible robot arms, Z. H. Luo proved the asymptotic stability of the original infinite dimensional closed loop system\(^4\). Most large flexible space structures are more complicated than the above flexible structures(uniform one-link flexible robot arms\(^1\(^5\))\). It is important to extend the availability of direct strain feedback to more complicated flexible structures\(^6\). In this paper, the asymptotic stability is proved for infinite dimensional models of flexible structures with spatially varying parameters (of stiffness and damping) and a tip body (tip mass). For this direct strain feedback strategy, both the system structure and measurement structure are extended.

2. The dynamics model

To describe the dynamics of a cantilevered beam with spatially varying parameters and a flexible robot arm which carries a rigid payload (a tip body or a tip mass), the following dynamics model is considered.

Here, \(L\) is the length of the structure, and \(M\) is the mass of the payload (a tip mass or a tip body). \(x=0\) corresponds to the root end and \(x=L\) corresponds to the free end of the structure. \(\rho(x)\) is the mass density at \(x\) per unit length of the structure, and \(EI(x)\) is the stiffness coefficient at \(x\). \(u_1(t,x)\) is the bending displacement at time \(t\) at position \(x\) (\(0 < x < L\)) and \(u_2(t,x) = \frac{\partial u_1}{\partial t}\). \(f_2(t,x)\) is a control input. \(\cdot\) represents \(\partial/\partial x\).

As a dynamics model, it is assumed that the state variable \(u(t,x) = (u_1(t,x), u_2(t,x))^T\) satisfies the following evolution equation\(^1\(^5\))\):

\[
\frac{\partial u}{\partial t} = A_x u(t,x) + f(t,x) \quad (1)
\]

where \(f(t,x) = (0, f_2(t,x))^T\), and

\[
A_x u(x) = (u_2(x), -A_x u_1(x) - D_x u_2(x))^T \quad (2)
\]

for \(u(x) = (u_1(x), u_2(x))^T\).

The operator \(A\) represents stiffness and defined by

\[
A_x = \frac{1}{\rho(x)} \frac{\partial^2}{\partial x^2} \left\{ EI(x) \frac{\partial^2(\cdot)}{\partial x^2} \right\} \quad (3)
\]

The domain of the operator \(A\) is as follows:
Let $H$ be a function space consists of all square integrable functions about the following measure, i.e.

$$H = L^2([0, L], \mu).$$

(5)

Here $\mu$ is a measure on the closed interval $[0, L]$ defined by

$$\mu(dx) = \rho(x)dx + M \delta_L(dx).$$

(6)

where $\delta_L$ is the Dirac measure at $x = L$.

$H$ becomes a Hilbert space with the inner product

$$(v, w)_H = \int_0^L v(x)w(x)\mu(dx).$$

(7)

The Hilbert space $H$ and its inner product $(\cdot, \cdot)_H$ are mathematically rigorous expressions of ones which were introduced by Sakawa, Matsuno and Fukushima. The norm in $H$ is denoted by $\|\cdot\|_H$.

$D$ is assumed to be a Rayleigh damping, i.e.

$$D = \alpha I + \beta A$$

(8)

for some non-negative constants $\alpha$ and $\beta$ ($\alpha > 0$ or $\beta > 0$), where $I$ is an identity operator.

Since

$$(Av, w)_H = \int_0^L EI(x) \frac{d^2v}{dx^2} \frac{dw}{dx^2} dx$$

for any $v, w \in D(A)$

the operator $A$ becomes self-adjoint and positive definite on $H$. By the eq.(8), $DA^{-1}$ becomes positive definite.

Suppose we can measure the time derivative of the bending moment (strain) of the flexible structure by means of several strain gauges installed at each preassigned location $x = a_1, a_2, \ldots, a_N$. That is, the observation mechanism is described as follows:

$$u_{obs}(t) = C u(t, \cdot)$$

(10)

where the operator $C$ is defined as follows:

$$C u(\cdot) = (EI(a_k)u_2(a_k); k \downarrow 1, 2, \ldots, N)$$

(11)

3. Direct strain feedback

Let the observation signals eq.(10) be fed back into the input signals $f(t, x)$ of the system eq.(1) directly, i.e.

$$f(t, x) = G(x) u_{obs}(t)$$

(12)

where $G(x)$ is a feedback gain. By this static output feedback, we obtain the following closed loop system.

$$\frac{\partial u}{\partial t} = (\mathcal{A} + G(x)C) u(t, x)$$

(13)

We propose a feedback gain

$$G(x) = \begin{pmatrix} 0, \ldots, 0 \\ -c_1(x), \ldots, -c_N(x) \end{pmatrix}$$

(14)

for the closed loop system eq.(13) to be asymptotically stable, where

$$c_k(x) = c_k \max \{0, x - a_k\}$$

(15)

and $c_k$ is a positive constant.

Then the closed loop system eq.(13) is rewritten as

$$\frac{\partial u_1(t, \cdot)}{\partial t} = \begin{pmatrix} u_2(t, \cdot) \\ -A u_1(t, \cdot) - (D + II) u_2(t, \cdot) \end{pmatrix}$$

(16)

where the operator $\Pi$ is defined as

$$\Pi u(\cdot) = \sum_{k=1}^N g_k(\cdot) EI(a_k) v''(a_k).$$

(17)

Since we can show that the operator $\Pi$ becomes $A$-bounded, $A$-symmetric and $A$-positive semidefinite (these will be proved in the next section), $\Pi$ is the one of $A$-dependent operators which were introduced by Z. H. Luo. Since $\Pi$ becomes an $A$-dependent operator, it follows that the closed loop system eq.(16) becomes asymptotically stable in the Hilbert space $H = D(A^{1/2}) \times H$ in the same way as shown in the paper.

4. $A$-dependent Operator

In this section, it will be shown that the operator $\Pi$ is $A$-bounded, $A$-symmetric and $A$-positive semidefinite.

For any $v \in D(A)$, since

$$\left( EI(x) v''(x) \right)' = \left( EI(\xi) v''(\xi) \right)'_{\xi=L}$$

$$- \int_x^L \left( EI(\xi) v''(\xi) \right)' d\xi$$

(18)
we have
\[
\left\{ (EI(x)v''(x)) \right\}^2 \\
\leq 2 \left\{ \int_{x=L}^{x=L} (EI(x)v''(x)) \right\}^2 \\
+ 2 \left\{ \int_{x=L}^{x=L} (EI(x)v''(x))'' \right\}^2 \\
= 2 \left\{ \int_{x=L}^{x=L} (EI(x)v''(x))'' \right\}^2 \\
\leq 2M^2 (A_L v(x))_x^2 + 2 \int_{x=L}^{x=L} (EI(x)v''(x))^2 dx \\
\leq 2M^2 (A_L v(x))_x^2 + 2 (L-x) \rho(x) \int_{x=L}^{x=L} (EI(x)v''(x))'' dx \\
\leq 2 \left\{ M + (L-x) ||\rho||_\infty \right\} ||Av||_H^2 \tag{22}
\]
where || \cdot ||_\infty means
\[
||\rho||_\infty = \text{ess.sup} \{ \rho(\xi) : \xi \in [0,L] \}. \tag{23}
\]
Since
\[
0 - EI(a_k) v''(a_k) = \int_{a_k}^{L} \left( EI(x)v''(x) \right) dx \tag{24}
\]
we have
\[
\left( EI(a_k) v''(a_k) \right)^2 \\
\leq \left( \int_{a_k}^{L} 1^2 dx \right) \left( \int_{a_k}^{L} (EI(x)v''(x))'' dx \right) \tag{25}
\]
\[
\leq (L-a_k) \int_{a_k}^{L} 2 \left( M + (L-x) ||\rho||_\infty \right) ||Av||_H^2 dx \\
= C_{1,k} ||Av||_H^2 \tag{26}
\]
for some positive constant C_{1,k}. It is seen that
\[
||IIv||_H^2 = M \left\{ \sum_{k=1}^{N} g_k(L) EI(a_k)v''(a_k) \right\}^2 \tag{27}
\]
\[
+ \int_{0}^{L} \rho(x) \left( \sum_{k=1}^{N} g_k(x) EI(a_k)v''(a_k) \right)^2 dx \\
\leq M \sum_{k=1}^{N} g_k^2(L) \sum_{k=1}^{N} (EI(a_k)v''(a_k))^2 \tag{28}
\]
\[
+ \int_{0}^{L} \rho(x) \left( \sum_{k=1}^{N} g_k^2(x) \right)^2 dx \\
\leq C_{2} ||Av||_H^2 \tag{29}
\]
for some positive constant C_{2}. Thus, the operator II is A-bounded.

For v,w \in D(A), since
\[
\left( \frac{1}{\rho(\cdot)} (EI(\cdot)v''(\cdot))'' g_k(\cdot) EI(a_k) w''(a_k) \right)_H \\
= EI(a_k) w''(a_k) \int_{0}^{L} (EI(x) v''(x))'' g_k(x) dx \\
- \left[ (EI(x) v''(x))'' g_k(L) EI(a_k) w''(a_k) \right]_{x=L}^{x=L} \\
= EI(a_k) w''(a_k) c_k EI(a_k) v''(a_k) \tag{30}
\]
we have
\[
(Av, IIw)_H = \sum_{k=1}^{N} c_k (EI(a_k))^2 v''(a_k) w''(a_k).
\]
Thus, the operator II is A-symmetric and A-positive semidefinite.

5. Conclusions

A new example of the A-dependent operator is obtained. This shows that the direct strain feedback control is effective (the closed loop system becomes asymptotically stable) for a flexible structure with a tip body whose stiffness and damping coefficients are spatially varying.

Further, a mathematically rigorous model of a flexible robot arm with tip body(tip mass) is obtained by introducing a new Hilbert space $H$ (cf. eq.(5)) as a state space.

The first author would like to express his thanks to Prof. Makiko Nisio (Osaka Electro-Communication Univ.) for her kind advices.

References

5) H. T. Banks and K. Kunisch: Estimation Techniques for Distributed Parameter Systems, Chapter 1, Section 3, Birkhauser (1989)