A Case Study on Optimal Allocation of Call Center Staff – With Un-used Phones and Repeated Calls Situation Considered –

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Abstract: In this paper, we consider the customer call center operation which is one of the personal computer end-user services. We deal with the problem as a case study where the optimal allocation of the call center staff which is called agents is determined. This previous proposed model was to provide the optimal agents' allocation for the call center service based on queuing (Erlang loss formula) [18, 19] and linear programming theory [20]. Here, besides it, we're going to address or examine the mode, considering the queuing space of not-using phones or the phones that no agents are using and the repeated calls that the customers would call when they couldn't reach any agents directly or immediately, and they couldn't even get into the queuing space of not-using phones. Then we show the effectiveness or the extent of the effectiveness of the model.

Key Words: Call center, Poisson distribution, Erlang distribution, Erlang loss formula, Linear programming

1. Introduction

In one of the Outsourcing Businesses of IT industry, there is the call center service for the end-users of the customers. It provides the customers with a single point of contact for the solution support of various IT problems, especially of the most familiar product of PCs'. You could say that the call center is also the first contact point for the customers; it is, for that reason, one of the most important services to establish or enhance the product brand and to improve the customer satisfaction.

In this study, we're considering the model with the un-used phones and the repeated calls based on the previous model, where the agent numbers are fixed according to the arrival rate each hour.

There have been many other studies on the agents' scheduling problem of the call center so far. As some of the latest papers on it, they addressed the enhanced model itself [1, 2, 4, 6, 7-9, 13], the new solution approaches [3, 5, 11, 12, 14, 15], and then tried practically to make the call center more effective [10, 17], so on. In those papers, they considered the agents' cyclic work shift and their detailed skills levels, or they basically touched on the agents' work-force allocation from the points of a broad scale and long time range or long term. It is often difficult to estimate the arrival calls for a long time span.

While we take it into account that the arrival calls are fluctuating and depend on the types of the PCs that the customers use or the timing of the software up-grades and so on. So we focus on doing the arrival calls' forecast for quite a short time period or the unit time of one hour at each service day. Then, we con-

Table 1 Call Arrival Rate

Time	8	9	10	11	12	13	14	15	16	17	18	19
Arrival Rate	13.37	14.40	11.48	6.51	7.73	8.91	9.27	9.14	7.47	4.63	2.37	0.34

sidered making a model of the agents' scheduling dynamically and flexibly based on it [21]. We're now going over or exploring the above situation, plus the un-used phones available (as the waiting space) and repeated calls considered. And we're going to get the result that we'd find the previous model suited or applicable enough, in the actual call center situation except such special or typical cases as system programs' changes or updates, etc.

2. Arrival Rate

The table 1 shows the average call arrival rate each hour over actual 84 months' period. We're going to focus on addressing the most frequent call arrival time of 10 a.m. -11 a.m. to explain the core of this study. And we show that the actual call arrival number's frequency and its theoretical Poisson arrival in the figure 1.

The expected values in the figure 1 are the theoretical values of Poisson arrival based on the actual arrival calls. We used the Chi-squared test to show how well the Poisson arrival fits for the actual arrival rate in the table 2. We got the result, good enough within the rejection region of 5% and its average of 14.40, standard deviation of 3.07. And the actual maximum call arrival number is 25.38 and its arrival possibility is 0.33%, theoretically. And 25.38 is also the highest call arrival number over 84 months' data-tracking period of all the service time of 8 a.m. to 20 p.m.

3. Service Rate

In the same manner as arrival rate, we got the service time distribution as we measured the actual service time over the past

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Fig. 1 Call Arrival Distribution

Table 2 Fitness of Poisson distribution for Call Arrival

Class	Actual value	Expected value	Chi-Squared Test	
0~10	2	11.665	8.008	Chi-Squared Theoretical Value Freedom degree 8 Significance Level 0.05
10~12	18	18.404	0.009	
12~14	22	21.060	0.042	
14~16	23	18.275	1.221	
16~18	9	12.438	0.950	
18~20	5	6.817	0.484	
20~22	3	3.073	0.002	
22~24	1	1.159	0.022	
24~26	1	0.371	1.063	
			11.802	15.057

Fig. 2 Service time Distribution

84 months' period, and below was depicted the figure 2 with the vertical line of Frequency to the service time range of every 50 second. The actual average service time is 428.57 seconds. While the theoretical expected service time is expressed as Erlang distribution, and its fitness is shown in the table 3 with Chi-squared test.

You would find the fitness quite good from the table 3. Here, we assume that the service time doesn't depend on the time itself or each time. Now we're going to address or look into the

call center model with the repeated calls and the waiting space involved, based on these arrival rate and service time.

Table 3 Fitness of Erlang distribution for Service Time

Class	Actual value	Expected value	Chi-Squared Test	
0~200	0	0.48	0.481	Chi-Squared Theoretical Value Freedom degree 8 Significance Level 0.05
200~250	0	3.07	3.070	
250~300	11	9.21	0.348	
300~350	17	16.40	0.022	
350~400	20	19.94	0.000	
400~450	12	18.12	2.067	
450~500	11	13.08	0.331	
500~550	3	7.84	2.987	
550~600	6	4.03	0.964	
			10.269	15.057

4. State Transition Model

We're going to set the probability variable of i, j , and l as the number of agents working each hour, the number of users in the waiting space, and the number of users trying to re-call for the services, respectively, and to define the notation or the probability as (i, j, l) and $\Pi_{i,j,l}$. Also, we set the number of the physical existing phones available as M . Here, we're going to address or think about the probability of the steady-state system where one situation is moving to another during a very tiny time period of Δt or the state transition probability. Whenever the changes occur, we have such situations as a call arrival, a service completion, a call dropping-away from the waiting space, or a repeated call occurring (including a giving-up repeated call).

Here, the probability of one call's arrival in tiny bits of time Δt is always, regardless of the system situation, $\lambda \Delta t + o(\Delta t)$, according to the Poisson arrival assumption. We're going to discuss, using the actual service time as a service time parameter μ , which we found as Erlang distribution. The probability that the number of i agents ($i \leq 1$) are servicing in tiny bits of time is $i\mu \Delta t + o(\Delta t)$. And we also assume that the abandon rate (or the dropping-away rate) is $j\beta \Delta t + o(\Delta t)$, where β is the dropping-away rate in the waiting space, and the probability of the repeated calls in the tiny bits of time similarly is $l\gamma \Delta t + o(\Delta t)$, where l ($l \leq 1$) is the number of users who would be going to re-call for the services and γ is the interval time of recalling, that is the time until when the users could wait and try to re-call for the services. And the users have the options that they re-call or not, with the probability of α for re-calling and $(1 - \alpha)$ for NOT re-calling. And $o(\Delta t)$ means that $\lim_{\Delta t \rightarrow 0} o(\Delta t)/\Delta t = 0$.

5. State Transition Probability

5.1 State Transition

In the call center operation, we have been taking the approach that a minimum number of agents ($i = c$) is assigned or fixed each hour according to each hour's call arrival rate. From here, we're going to take more realistic situation where the minimal number of agents ($i = c$) is already set, there is the number of unused phones available ($M - c$) as the waiting space and repeated calls are considered. Here, M is the total number of phones existed. The table 4 shows each state transition status according to the state transition diagram (the figure 4). And potential re-calling users ($l \leq 1$) are trying to re-call with the

probability of $l\gamma$, but there are no agents who can take the calls and provide the services. Therefore, they can not receive the services, or they are back to the current state of $(c, M - c, l)$. $l\gamma$ does not give any changes to the state of $(c, M - c, l)$ so that we exclude it in the table 4. The figure 3 shows this state transition. We're thinking about the system or the situation where the number of agents ($i = c$) is already set or fixed each hour, and up to the number of calls of c , that is, the agents assigned ($i = c$) can take the calls of up to calls of c and give the services immediately. This means that the row of $(i - 3)$ in the figure 4, for example, has no waiting queues or no users in the waiting space (the horizontal cells of j). Once agents ($i = c$) can not take care of calls ($i > c$), the state transition moves to the horizontal columns of the waiting space. Therefore, we'll address or solve the state transition equation, only focusing on the horizontal cells of (i, j, l) in the figure 4. In the notation of (i, j, l) of the figure 4, if $l = 0$, then we simply set (i, j) and the figure 4 specifically shows the case of $i = 5$.

Fig. 3 Impact of Probability of $l\gamma$

Table 4 State Transition Probability

State Transition	Transition Probability
$(i+1, 0, 0)$ $\uparrow (0 \leq i \leq c-1, j=0)$ $(i, 0, 0)$	$\lambda\Delta t + o(\Delta t)$
$(i, 0)$ $\downarrow (1 \leq i \leq c-1, j=0)$ $(i-1, 0, 0)$	$i\mu\Delta t + o(\Delta t)$
$(i, j, 0) \rightarrow (i, j+1, 0)$ $(0 \leq j \leq M-i)$	$\lambda\Delta t + o(\Delta t)$
$(i, j-1, 0) \leftarrow (i, j, 0)$ $(1 \leq j)$	$i\mu\Delta t + o(\Delta t)$
$(c, j-1, 0) \leftarrow (c, j, 0)$ $(1 \leq j \leq M-i)$	$j\beta\Delta t + o(\Delta t)$
$(c, M-c, l) \leftarrow (c, M-c, l+1)$ $(1 \leq l \leq N)$	$\lambda\alpha\Delta t + o(\Delta t)$
$(c, k, l-1) \leftarrow (c, k, l)$ $(1 \leq l \leq N)$	$k\gamma(1-\alpha)\Delta t + o(\Delta t)$

5.2 State Equation

We could generally notify the state transition as the following equation.

When

$$1 \leq i \leq c$$

$$0 \leq j \leq (M-i)$$

$$0 \leq l \leq k,$$

$$\square \lambda\Pi_{0,0} = \mu\Pi_{1,0}$$

$$\square (\lambda + \mu)\Pi_{1,0} = \lambda\Pi_{0,0} + 2\mu\Pi_{2,0}$$

Fig. 4 State Transition

$$\square (\lambda + 2\mu)\Pi_{2,0} = \lambda\Pi_{1,0} + 3\mu\Pi_{3,0}$$

$$\square (\lambda + 3\mu)\Pi_{3,0} = \lambda\Pi_{2,0} + 4\mu\Pi_{4,0}$$

$$\square (\lambda + 4\mu)\Pi_{4,0} = \lambda\Pi_{3,0} + 5\mu\Pi_{5,0}$$

$$\square (\lambda + 5\mu)\Pi_{5,0} = \lambda\Pi_{4,0} + (5\mu + \beta)\Pi_{5,1}$$

$$\square (\lambda + 5\mu + \beta)\Pi_{5,1} = \lambda\Pi_{5,0} + (5\mu + 2\beta)\Pi_{5,2}$$

$$\square (\lambda + 5\mu + 2\beta)\Pi_{5,2} = \lambda\Pi_{5,1} + (5\mu + 3\beta)\Pi_{5,3}$$

$$\square (\lambda + 5\mu + 3\beta)\Pi_{5,3} = \lambda\Pi_{5,2} + (5\mu + 4\beta)\Pi_{5,4}$$

$$\square (\lambda + 5\mu + 4\beta)\Pi_{5,4} = \lambda\Pi_{5,3} + (5\mu + 5\beta)\Pi_{5,5}$$

$$\square (\lambda + 5\mu + 5\beta)\Pi_{5,5} = \lambda\Pi_{5,4} + \gamma(1-\alpha)\Pi_{5,5,1}$$

$$\square (\lambda\alpha + \gamma(1-\alpha))\Pi_{5,5,1} = \lambda\alpha\Pi_{5,5} + 2\gamma(1-\alpha)\Pi_{5,5,2}$$

$$\square (\lambda\alpha + 2\gamma(1-\alpha))\Pi_{5,5,2} = \lambda\alpha\Pi_{5,5,1} + 3\gamma(1-\alpha)\Pi_{5,5,3}$$

$$\square (\lambda\alpha + 3\gamma(1-\alpha))\Pi_{5,5,3} = \lambda\alpha\Pi_{5,5,2} + 4\gamma(1-\alpha)\Pi_{5,5,4}$$

$$\square (\lambda\alpha + 4\gamma(1-\alpha))\Pi_{5,5,4} = \lambda\alpha\Pi_{5,5,3} + 5\gamma(1-\alpha)\Pi_{5,5,5}$$

...

$(k-1)$ re-calling users,

$$(\lambda\alpha + (k-1)\gamma(1-\alpha))\Pi_{5,5,k-1} = \lambda\alpha\Pi_{5,5,k-2} + (k\gamma(1-\alpha))\Pi_{5,5,k}$$

k re-calling users,

$$(k\gamma(1-\alpha))\Pi_{5,5,k} = \lambda\alpha\Pi_{5,5,k-1}$$

5.3 Case of State Transition of $(c = 5, M = 10, l = 5)$

To understand the state equation of 5.2 more clearly, we specifically set (i, j, l) as $(5, 10, 5)$, then we're going to have the following state equation with 16 unknown variables of $\Pi_{i,j,l}$ and 16 equations. Therefore, we can find out each state transition probabilities of $\Pi_{i,j,l}$.

$$\square \lambda\Pi_{0,0} = \mu\Pi_{1,0}$$

$$\square (\lambda + \mu)\Pi_{1,0} = \lambda\Pi_{0,0} + 2\mu\Pi_{2,0}$$

$$\square (\lambda + 2\mu)\Pi_{2,0} = \lambda\Pi_{1,0} + 3\mu\Pi_{3,0}$$

$$\square (\lambda + 3\mu)\Pi_{3,0} = \lambda\Pi_{2,0} + 4\mu\Pi_{4,0}$$

$$\square (\lambda + 4\mu)\Pi_{4,0} = \lambda\Pi_{3,0} + 5\mu\Pi_{5,0}$$

$$\square (\lambda + 5\mu)\Pi_{5,0} = \lambda\Pi_{4,0} + (5\mu + \beta)\Pi_{5,1}$$

$$\begin{aligned}
\Box (\lambda + 5\mu + \beta)\Pi_{5,1} &= \lambda\Pi_{5,0} + (5\mu + 2\beta)\Pi_{5,2} \\
\Box (\lambda + 5\mu + 2\beta)\Pi_{5,2} &= \lambda\Pi_{5,1} + (5\mu + 3\beta)\Pi_{5,3} \\
\Box (\lambda + 5\mu + 3\beta)\Pi_{5,3} &= \lambda\Pi_{5,2} + (5\mu + 4\beta)\Pi_{5,4} \\
\Box (\lambda + 5\mu + 4\beta)\Pi_{5,4} &= \lambda\Pi_{5,3} + (5\mu + 5\beta)\Pi_{5,5} \\
\Box (\lambda + 5\mu + 5\beta)\Pi_{5,5} &= \lambda\Pi_{5,4} + \gamma(1 - \alpha)\Pi_{5,5,1} \\
\Box (\lambda\alpha + \gamma(1 - \alpha))\Pi_{5,5,1} &= \lambda\Pi_{5,5} + 2\gamma(1 - \alpha)\Pi_{5,5,2} \\
\Box (\lambda\alpha + 2\gamma(1 - \alpha))\Pi_{5,5,2} &= \lambda\alpha\Pi_{5,5,1} + 3\gamma(1 - \alpha)\Pi_{5,5,3} \\
\Box (\lambda\alpha + 3\gamma(1 - \alpha))\Pi_{5,5,3} &= \lambda\alpha\Pi_{5,5,2} + 4\gamma(1 - \alpha)\Pi_{5,5,4} \\
\Box (\lambda\alpha + 4\gamma(1 - \alpha))\Pi_{5,5,4} &= \lambda\alpha\Pi_{5,5,3} + 5\gamma(1 - \alpha)\Pi_{5,5,5} \\
\Box 5\gamma(1 - \alpha)\Pi_{5,5,5} &= \lambda\alpha\Pi_{5,5,4}
\end{aligned}$$

Focusing on the equations of $\Box$ and $\Box$ underlined, we're starting to solve sequentially from the equation of $\Box$ in the descending order. Then, we got the following simpler forms of equations.

$$\begin{aligned}
\Box' \mu\Pi_{1,0} &= \lambda\Pi_{0,0} \\
\Box' 2\mu\Pi_{2,0} &= \lambda\Pi_{1,0} \\
\Box' 3\mu\Pi_{3,0} &= \lambda\Pi_{2,0} \\
\Box' 4\mu\Pi_{4,0} &= \lambda\Pi_{3,0} \\
\Box' 5\mu\Pi_{5,0} &= \lambda\Pi_{4,0} \\
\Box' (5\mu + \beta)\Pi_{5,1} &= \lambda\Pi_{5,0} \\
\Box' (5\mu + 2\beta)\Pi_{5,2} &= \lambda\Pi_{5,1} \\
\Box' (5\mu + 3\beta)\Pi_{5,3} &= \lambda\Pi_{5,2} \\
\Box' (5\mu + 4\beta)\Pi_{5,4} &= \lambda\Pi_{5,3} \\
\Box' (5\mu + 5\beta)\Pi_{5,5} &= \lambda\Pi_{5,4} \\
\Box' \gamma(1 - \alpha)\Pi_{5,5,1} &= \lambda\alpha\Pi_{5,5} \\
\Box' 2\gamma(1 - \alpha)\Pi_{5,5,2} &= \lambda\alpha\Pi_{5,5,1} \\
\Box' 3\gamma(1 - \alpha)\Pi_{5,5,3} &= \lambda\alpha\Pi_{5,5,2} \\
\Box' 4\gamma(1 - \alpha)\Pi_{5,5,4} &= \lambda\alpha\Pi_{5,5,3} \\
\Box' 5\gamma(1 - \alpha)\Pi_{5,5,5} &= \lambda\alpha\Pi_{5,5,4}
\end{aligned}$$

These 15 equations are showing the relationships of a neighboring variable of $\Pi_{i,j,l}$. Hence, with these equations and the total sum of each state transition probability to 1, we can notify each state transition probability as the form of the probability of $\Pi_{0,0}$.

Table 5 Solution of Steady-state transition equation ($\lambda = 14.40$)

λ	14.40	calls/hour
μ	8.40	calls/hour
γ	6	calls/hour
α	0.5	
β	0.6	

	State Transition	Probability	User in Queue
$\Box$	$\Pi_{0,0}$	17.93%	0.000
$\Box$	$\Pi_{1,0}$	30.73%	0.000
$\Box$	$\Pi_{2,0}$	26.34%	0.000
$\Box$	$\Pi_{3,0}$	15.05%	0.000
$\Box$	$\Pi_{4,0}$	6.45%	0.000
$\Box$	$\Pi_{5,0}$	2.21%	0.000
$\Box$	$\Pi_{5,1}$	0.75%	0.007
$\Box$	$\Pi_{5,2}$	0.25%	0.005
$\Box$	$\Pi_{5,3}$	0.08%	0.002
$\Box$	$\Pi_{5,4}$	0.03%	0.001
$\Box$	$\Pi_{5,5}$	0.01%	0.000
$\Box$	$\Pi_{5,5,1}$	0.04%	0.002
$\Box$	$\Pi_{5,5,2}$	0.05%	0.003
$\Box$	$\Pi_{5,5,3}$	0.04%	0.003
$\Box$	$\Pi_{5,5,4}$	0.02%	0.002
$\Box$	$\Pi_{5,5,5}$	0.01%	0.001
	Total	1	0.029

Fig. 5 State Transition Probability ($\lambda = 14.40$)

6. Steady-state equation and its solution

The table 5 shows the result that we've solved the steady-state equation of the specific case of 5.3 to get each state transition probability. Similarly, we could get the probability of the general steady-state equation of 5.2, given the parameters fixed or set. The parameters of the case of 5.3 are shown in the table 5. The figure 5 shows the probability of each state transition. The agents of 5 could take care of 98.72% calls. Then, they could take care of 99.84% calls with the waiting space provided. In the total system, there are 0.029 users waiting, with the waiting space and re-calling users included.

In the same way, at λ of 20.57 which is the point of the right hand of $+2\sigma$ from the center of arrival rate of 14.40, we get the solution of the steady-state equation in the table 6 and their probabilities' status in the figure 6. The agents of 5 could take care of 87.99% calls. Furthermore, they could take care of 93.17% calls with the waiting space. And there are 0.0637 users waiting totally. We would say that the agents of 5 could do the services with almost no impact to the services or users.

From the figure 6, we guess that we would have more impacts to the system or the services if the arrival rate is increas-

Table 6 Solution of Steady-state transition equation
($\lambda = 20.57$)

λ	20.57	calls/hour
μ	8.40	calls/hour
γ	6	calls/hour
α	0.5	
β	0.6	

	State Transition	Probability	User in Queue
☐	$\Pi_{0,0}$	7.91%	0.000
☐	$\Pi_{1,0}$	19.36%	0.000
☐	$\Pi_{2,0}$	23.71%	0.000
☐	$\Pi_{3,0}$	19.35%	0.000
☐	$\Pi_{4,0}$	11.85%	0.000
☐	$\Pi_{5,0}$	5.80%	0.000
☐	$\Pi_{5,1}$	2.80%	0.028
☐	$\Pi_{5,2}$	1.33%	0.027
☐	$\Pi_{5,3}$	0.63%	0.019
☐	$\Pi_{5,4}$	0.29%	0.012
☐	$\Pi_{5,5}$	0.13%	0.007
☐	$\Pi_{5,5,1}$	0.91%	0.055
☐	$\Pi_{5,5,2}$	1.56%	0.109
☐	$\Pi_{5,5,3}$	1.78%	0.143
☐	$\Pi_{5,5,4}$	1.53%	0.137
☐	$\Pi_{5,5,5}$	1.05%	0.105
Total		1	0.640

Fig. 6 State Transition Probability ($\lambda = 20.57$)

ing with the current service situation of 5 agents and service rate of 8.40 calls/hour. The theoretical probability of the arrival rate of 25.38 is 0.33%. But, once the agents of 5 would get this arrival rate of 25.38, the re-calling status would suddenly increase, as shown in the table 7 or the figure 7. In the month of having the arrival rate of 25.38, we had actually known the fact or the event that we had the system program's update. And we made ready for it and assigned 7 agents to do the services well enough, as shown in the table 8 or the figure 8.

Here, the table 9 shows the number of waiting users according to the arrival rate λ with the agents of 5. Supposed the abandon rate is less than or equal to 5%, we would do the services well enough with the arrival rate of 17.5 (waiting users; 0.145).

From the standpoint of the daily abandon rate in the previous study, we could do the services with the service rate of 20.56 (waiting users; 0.637).

We have been doing the agent scheduling, based on the past call arrival rate and any IT changes, such as software changes or updates. We would say from this study that we could provide the services with our previous proposed model if we have the

Table 7 Solution of Steady-state transition equation
($\lambda = 25.38$)

λ	25.38	calls/hour
μ	8.40	calls/hour
γ	6	calls/hour
α	0.5	
β	0.6	

	State Transition	Probability	User in Queue
☐	$\Pi_{0,0}$	2.83%	0.000
☐	$\Pi_{1,0}$	8.54%	0.000
☐	$\Pi_{2,0}$	12.90%	0.000
☐	$\Pi_{3,0}$	12.99%	0.000
☐	$\Pi_{4,0}$	9.81%	0.000
☐	$\Pi_{5,0}$	5.93%	0.000
☐	$\Pi_{5,1}$	3.53%	0.035
☐	$\Pi_{5,2}$	2.08%	0.042
☐	$\Pi_{5,3}$	1.20%	0.036
☐	$\Pi_{5,4}$	0.69%	0.028
☐	$\Pi_{5,5}$	0.39%	0.019
☐	$\Pi_{5,5,1}$	3.28%	0.197
☐	$\Pi_{5,5,2}$	6.94%	0.486
☐	$\Pi_{5,5,3}$	9.78%	0.783
☐	$\Pi_{5,5,4}$	10.35%	0.931
☐	$\Pi_{5,5,5}$	8.75%	0.875
Total		1	3.432

Fig. 7 State Transition Probability ($\lambda = 25.38$) Fig. 8 State Transition Probability
($\lambda = 25.38$, Agents of 7)

call arrival rate range of up to $+2\sigma$ from the average call arrival rate of 10 to 11 a.m. in the table 1.

If we have an arrival rate increasing by the impact of re-calling in a unit hour, we would have the higher arrival rate in the next hour. But, take, for example, the arrival rate of 11 a.m. — 12 noon in the table 1, we have lower arrival rate of

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