ABSTRACT

This paper presents a mixed integer bi-level dynamic hub location problem for freight transportation. The problem is formulated as a mixed integer bi-level optimization model. The upper-level problem maximizes the profit of a freight company and the lower level problem minimizes the cost paid by customers. The bi-level problem is reformulated into a single level model using the KKT reformulation and a numerical example is provided to show the validity of the proposed model.

INTRODUCTION

Transporting goods to customers in today’s competitive world has become one of the main supply chain interfaces that companies can transmute this important interface to be one of their strengths. They can do this function themselves or outsource that to transportation companies. Customers also have become very impatient and seek to get as quick as possible services. As online shopping is evolving and people trust it more than before, and also by the changes happen in business models in which more small businesses and entrepreneurs have been flourished, the demand for transporting goods and products even in international level is increasing.

According to International Energy Agency report with no dedicated policies, road travel likely to double by 2050 [1] and according to Boeing 2016-1017 report on world air cargo forecast [2], World air cargo traffic will more than double over the next 20 years. This creates high potentials for freight companies to increase their capacities and define better strategies to optimize their performances in future competitive environment. This paper focuses on two main purpose. First, with regard to increasing demand for freight transportation in all sectors, air, road and rail freight, companies should have some extension strategies for their future to gain more market share and to be able to survive in the competitive globalized environment. The possible extension scenarios for companies performing in this field has been considered in this paper. These scenarios are as follow:

First extension scenario: The hub-and-spoke system became the norm for most major transportation systems, because of the numerous advantages that it has such as: Needs less links, needs less carriers, can sum up the flow in hubs, can reduce the half loaded carriers, in summary increases productivity and profit and reduces cost. These are from companies’ perspective but it is also better for customers and markets to use such a system. They can get faster services and catch their destination by changing in hubs instead of waiting for a long time to reach the direct service. Also it would be cheaper than direct service. To deal with future increasing demands, companies may decide to increase number of hubs and cover more customers. In this scenario they will most probably compete with their rivals in using same hubs to get more market share. Second scenario: another scenario is to increase number of vehicles to transport demands. Companies can decide to buy or lease more vehicles in some time periods in future so that they can carry more flow.

The second purpose of the paper is to suppose that the customers (thereafter markets) are involved in pricing decisions, in other words markets have rational behavior and have the possibility to choose their providers according to their interest. There would be a price for carrying goods which is paid by the markets and the markets seek to minimize this amount, on the other hand this price would be the company’s income, which the company desires to maximize this amount. To formulate the markets behavior, its objective function should be included in the constraints of problem. This formulation yields to a bi-level optimization problem in which in the upper level problem the leader (Freight Company) maximizes its profit and the lower level problem is the response of markets that select amount of
flow to be carried by companies with the unique interest of satisfying their demands at lowest cost.

This paper is organized as follows. In the next sections after having a description about problem, we introduce the parameters and decision variables and then the mathematical model will be presented. Then we have discussion about solving strategy and present an example to see how the model works. Finally the conclusion is presented.

**PROBLEM DESCRIPTION**

The dynamic maximal hub location problem for freight transportation planning is described as follows:

Suppose there is a freight company even operating in air freight transportation or road or other kind of carriers. This company uses the hub and spoke system to traverse the flow among nodes and has predefined decisions on the number of established hubs. The company has some rivals that are operating in the same business environment and even may use the same hubs that our company is using. Also the company has information about the increasing demand in the future and is eager to gain benefit through establishing some extending plans. The company has some dominant customers that have great power and can control the prices by sourcing the demand to the rival. The nodes of the network are shown by \( N = \{1, \ldots , N\} \) and \( i,j,k,m \) are used to refer to the nodes of this graph. The markets place some order to be traversed from origin node \( i \) to destination node \( j \). The company has freedom in choosing the hubs to carry this order and also the order can be divided and carried using different hubs and seeks to maximize the flow carried by carriers to maximize the income. If the company does not have enough capacity or possibility to transport the order and the order has been already booked, then the company can outsource it to a sister company. So up to here our problem is a multi-allocating maximal hub location problem (\( p \) is the number of hubs to be utilizes at the first period of time and the \( q \) is the number of hubs that the company is going to add to its current hubs as extension plan) in which markets are rational and have the possibility to select their providers according to their own interest i.e. price.

**MATHEMATICAL FORMULATION**

The model parameters and decision variables are listed below.

**INDECIES**

- \( i,k,m,j \): Index of nodes
- \( l \): Index of transportation vehicle type. Index \( l\) in exclusively used to represent the outsourcing carrier no matter what specific type it is.
- \( t \): Index of planning horizon, \( t=0 \) is the first time period that initial hubs are allowed. \( t=1,\ldots , T \) is used to refer to time periods that expansion is allowed in them.

**PARAMETERS**

- \( c_{ij} \): Travel time (distance) of each pair of nodes from node \( i \) to node \( j \).
- \( rev_{ijt} \): Price paid by market for carrying one Kg weight from node \( i \) to node \( j \) in period \( t \).
- \( d_{ijt} \): Markets demand to be carried from origin \( i \) to destination \( j \) at period \( t \).
- \( cap_l \): The capacity of transportation vehicle type \( l \).
- \( cap_{ck} \): The capacity of node \( k \) as chosen to be hub.
- \( E_k \): Fixed cost of locating a hub at node \( k \) at the first period of the planning horizon.
- \( F_{kt} \): Fixed cost of locating a hub at node \( k \) at period \( t \).
- \( A_{kmlt} \): Hourly transportation cost of using vehicle type \( l \), from node \( k \) to node \( m \) at period \( t \).
- \( MSh_r \): Market share for the rival company \( r \).
- \( x_{rkm} \): The number of carrier type \( l \) in the link \( k-m \) for rivals.
- \( a \): Discount factor to pass by hubs on the route of \( i-j \), where \( 0 \leq a \leq 1 \).
- \( p \): The number of hubs at the first period of planning horizon.
- \( q \): The number of hubs decided as expansion.
- \( \beta \): The deadline traveling time (distance) from node \( i \) to node \( j \), set to assess the coverage of logistics.
- \( a_{km} \): 1 if there are no outsourcing services between nodes \( i \) and \( j \), 0 otherwise.
- \( cost_i \): The cost of buying or leasing the carrier type \( l \)
- \( budg_t \): Available budget at period \( t \).
- \( g_{ikmj} \): Binary parameter, which indicates whether the origin-destination pair \( (i-j) \) distance using the hubs \( k-m \) can be covered by \( \beta \) as:

\[
g_{ikmj} = \begin{cases} 
1 & \text{if } C_{ik} + \alpha C_{km} + C_{jm} \leq \beta \\
0 & \text{otherwise}
\end{cases}
\]

**DECISION VARIABLES**

Upper level decision variables

- \( H_k \): Binary variable, 1 if node \( k \) is selected as an initial hub at the first period.
- \( w_{kt} \): Binary variable, 1 if node \( k \) is selected to be hub at period \( t \).
- \( v_{kt} \): Binary variable, 1 if node \( k \) is selected as an expansion hub at period \( t \).
- \( h_{kt} \): Binary variable, 1 if node \( k \) is operating as a hub at period \( t \).
- \( x_{kmlt} \): The number of carrier type \( l \) in the link \( k-m \) at period \( t \). \( x_{kmlt} \) can be interpreted as outsourcing amount on link \( k-m \).

Lower level decision variables

- \( y_{ikmj} \): The amount of flow carried from node \( i \) to node \( j \) through hubs \( k-m \) at period \( t \)

The problem is formulated as a bi-level optimization problem in (1)-(23). The first term in the objective function in (1) maximizes the income gained from carrying flow through the hubs that are controlled by the leader, while the second and third terms are for the fixed cost of establishing initial hubs and expansion hubs, and the forth term is for the transportation cost of the self-owned carriers and outsourcing. Constraint (2) determines the primary number of established initial hubs to be \( p \). Constraints (3) restrict
that each hub is chosen as either an initial hub or an expansion hub in each period. Constraints (4) imply that the total number of opened hubs in and after the periods, which the expansions is allowed, is equal to \( p + q \). Constraints (5) enforce that once a hub is selected as an initial hub, it can be used for whole time periods. Constraints (6) state that an expansion hub becomes available when \( v_k = 1 \). Constraints (7) once a hub is opened in each period it should continue operating in the forthcoming periods. Constraints (8) state that no carrier can be assigned to link \( k - m \) unless one of these nodes are operating as hub. Constraints (9) mean that unless there is not outsourcing service for the link \( k - m \), no out sourcing is allowed for that link. Constraints (10) are related to this assumption that usually the number of the carriers operating in backward direction is the same as that in the forward direction. Constraints (11) are for the budget limitation of buying or leasing the carriers. Constraints (12) and (13) define the variables in the upper level problem as binary and positive integer variables.

The objective function (14) as the lower level objective, optimizes the cost paid by the markets. Constraints (15) and (16) imply that no flow can be carried through nodes unless they use hub/hubs. Constraints (17) imply that the total flow from node \( i \) to node \( j \) at each period \( t \) should be equal to the demand of that origin and destination. Constraints (18) make sure that the total capacity of employed carriers should not be violated. In this constraint the left hand side is the total flow occurring on link \( k - m \) utilizing by our company, including the amount \( \sum_i \sum_j y_{ikmjt} \) that transfers from other \( i - j \) pairs, and the amount \( \sum_o \sum_d (y_{kmodt} + y_{odkmt}) \) that is transferred to or from other node pairs, these pairs are shown by \( o \) and \( d \). The right hand side is the total capacity of the carriers employed to link \( k - m \) utilizing by our company. Constraints (19) state that the total flow for the rivals’ hubs could at most be equal to the situation it had since past. In addition, constraints (20) state that the amount of flow carried through the rivals’ hub should be less than its market share of the demand. Constraints (21) and (22) make sure that the total capacity of established hubs should not be violated. It worth to note that all variables in upper level problem take discrete values while the variables in lower level problem has continuous variables. Constraints (23) enforce the decision variables to be non-negative. The nature of the model variables plays an important role in reformulation techniques of bi-level problems. In the bi-level reformulation literature, there are many papers studying models having different kind of variables in upper level and lower level problem theoretically and practically.

\[
\sum_{k \in E_U} \sum_{t} w_{kt} = p + q \tag{4}
\]

\[
h_{kt} \geq H_k \quad \forall k \in U, t \tag{5}
\]

\[
h_{kt} \geq v_{kt} \quad \forall k \in U, t \tag{6}
\]

\[
h_{kt} \geq h_{kt-1} \quad \forall k \in U, t \tag{7}
\]

\[
x_{kmi} \leq M \left(h_{kt} + h_{m} \right) \quad \forall k \in U, m \in U, t \tag{8}
\]

\[
x_{kmi,t} = 0 \quad \forall k \in U, m \in U, t \tag{9}
\]

\[
x_{kmi} = x_{kmit} \quad \forall k \in U, m \in U, l \neq 1, t \tag{10}
\]

\[
\sum_{k \in E_U} \sum_{t} \text{cost}_{kmi} \leq \text{budget}_t \quad \forall t \tag{11}
\]

\[
H_k, w_{kt}, v_{kt}, h_{kt} \in \{0,1\} \tag{12}
\]

\[
x_{kmi} \in \mathbb{Z}^+ \tag{13}
\]

\[
\min \sum_{i \in E_K} \sum_{\ell \in E_D} \sum_{m \in D_{\ell}} \sum_{t} \left( r_{ijt}y_{ikmjt}g_{ikmjt} \right) \tag{14}
\]

\[
y_{ikmjt} \leq Mw_{kt} \quad \forall i,j,k \in U, m \in U, t \tag{15}
\]

\[
y_{ikmjt} \leq Mw_{mt} \quad \forall i,j,k \in U, m \in U, t \tag{16}
\]

\[
\sum_{k \in E_U} \sum_{t} \text{y}_{ikmjt} = d_{ijt} \tag{17}
\]

\[
\sum_{i} \sum_{j} \sum_{\ell \in E_K} \sum_{m \in D_{\ell}} \left( y_{kmodt} + y_{odkmt} \right) \leq \sum_{t} \text{cap}_{i}x_{kmit} \tag{18}
\]

\[
\sum_{k \in E_K} \sum_{t} \sum_{j} \left( y_{ikmjt} + \sum_{o \in E_D} \sum_{d \in D_{o}} \left( y_{kmodt} + y_{odkmt} \right) \right) \leq MSh_t \tag{19}
\]

\[
\sum_{i} \sum_{j} \sum_{\ell \in E_K} \sum_{m \in D_{\ell}} \left( y_{kmodt} + y_{odkmt} \right) \leq MSh_t \tag{20}
\]

\[
\sum_{k \in E_K} \sum_{t} \sum_{j} \sum_{\ell \in E_K} \sum_{m \in D_{\ell}} \left( y_{ikmjt} \right) \leq \text{cap}_{m}w_{mt} \quad \forall m \in U, t \tag{21}
\]

\[
\sum_{k \in E_K} \sum_{t} \sum_{j} \sum_{\ell \in E_K} \sum_{m \in D_{\ell}} \left( y_{ikmjt} \right) \leq \text{cap}_{m}w_{mt} \quad \forall m \in U, t \tag{22}
\]

\[
y_{ikmjt} \in \mathbb{R}^+ \quad \forall i,j,k \text{ and } m \in U, t \tag{23}
\]

It worth to note that all variables in upper level problem take discrete values while the variables in lower level problem has continuous variables. The nature of the model variables plays an important role in reformulation techniques of bi-level problems. In the bi-level reformulation literature there are many papers studying models having different kind of variables in upper level and lower level problem theoretically and practically.

**SOLUTION STRATEGY**

In particular, bi-level programs are of substantial importance due to the fact that they have been widely applied to Stackelberg game settings that involve two interacting players in different levels, pursuing different objectives, but using a set of common resources. The goal of a bi-level optimization technique is then to first find the lower level optimal solutions and then search for the optimal solution for the upper level optimization task. The
broad adoption of bi-level frameworks has only been limited by computational challenges; optimization problems with a bi-level nature are hard to solve. A bi-level program with a convex and regular lower level can be transformed in to a single level optimization problem using its optimality conditions. We used Karush-Kuhn-Tucker (KKT) condition to reformulate bi-level problem to a single level problem to be able to solve the problem and get results.

ILLUSTRATIVE EXAMPLE

Fig. 1(a) and (b) depict two feasible solution of an instance where \( N = 10, l = 4, p = 2, q = 2 \). This example was studied in \( T = 4 \), but here just the solution for \( t = 1 \) and \( t = 3 \) are depicted. The circles are markets and the green hexagonal are used to refer to the companies structured hubs, the yellow one is for rival’s hubs and the dark green is for the hubs that are utilized by both our company and rival. One can track the flow by the colors of the edges, also the shapes trapezoid, rectangle, and triangle refer to different kind of vehicles. These vehicles differ in capacity and other efficiency factors. In Fig.1(a) the rival’s hub is located in node 5 and the demand of node 4 is just covered by this rival. Hub 10 is utilizing by both companies, and hub 6 is managed by our company. For nodes that carry more flow the bigger and some times more than one vehicle is assigned. In Fig.1(b) because of predicted increasing demand the company decides to have an extension plan and increase the number of hubs to 4 and also buy more vehicles. In this model because the company does not have any information about rival’s plan no changes is happened in his hubs. In this period the company has also entered in the market number 4 and the flow of this market is carried by hub 3. The hubs are interconnected and a huge amount of flow is traversing through them. It should be noted that in this illustration the distance between nodes in not considered and this is just a schematic try to have a view of the problem.

Fig.1. two feasible solution of an instance where \( N = 10, l = 4, p = 2, q = 2 \).

CONCLUSIONS

We have proposed a mathematical model of mixed integer bi-level dynamic hub location problem for freight transportation. The KKT conditions are used to reformulate the problem into a single-level problem. The numerical results show the validity of the proposed model.

REFERENCES