DYNAMICS OF AN ELECTRO-HYDRAULIC VELOCITY SERVO WITH STICK-SLIP FRICTION

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ABSTRACT

Electrohydraulic servos exhibit nonlinear behavior whenever subjected to dynamic inputs due to the presence of several significant nonlinearities in the servovalve and hydraulic motor. Some mechanical nonlinearities are often responsible for performance limitations as expressed by speed of response, accuracy and stability. This paper deals with the dynamic characteristics of electrohydraulic velocity servos in order to throw more light on the complicated role played by stick-slip friction of the actuator.

KEYWORDS

Electrohydraulic servosystem
stick - slip friction

NOMENCLATURE

B viscous damping coefficient, N.m.s/rad
C_d valve discharge coefficient
d_s spool diameter, m
e_{ss} steady-state error
I servovalve operating current, A
J motor and load inertia, kg.m²
K_a servoamplifier gain, A/v
K_d pressure transducer constant, v/Pa
K_h bulk modulus of oil, N/m²
K_i integral controller gain
K_f pressure feedback gain
K_s valve coefficient m²/s. Pa^{1/2}
K_t tachogenerator constant, v/rad/s
K_v servovalve gain, m/A
K_w velocity feedback gain

L_e leakage coefficient, m⁵/N.s
M_p maximum overshoot in speed
N motor speed, rpm
N_r reference speed, 60 V_i/2 π K_t, rpm
P_l load pressure, N/m²
P_s supply pressure, N/m²
Q motor flowrate, m³/s
t time, s
t_s starting time, s
T_f motor friction, N.m
T_l external torque, N.m
T_s settling time defined at 2 % variation, s
T_{sl} slip friction, N.m
T_{st} stick friction, N.m
 $\bar{u}$ input vector
V_c volume of oil in motor and hoses, m³
V_i input voltage to the system, v
V_m motor displacement, m³/rad
 $\bar{x}$ state vector
x_s valve spool displacement, m
 $\bar{\alpha}$ parameter vector
β valve deadband, A
ζ valve damping ratio
 $\dot{\theta}$ angular speed, rad/s
Δθ a very finite range of speed, rad/s
ω_v valve natural frequency, rad/s

INTRODUCTION

Electrohydraulic servos exhibit nonlinear behavior whenever subjected to

dynamic inputs due to the presence of several significant nonlinearities in the servovalve and hydraulic motor. Some mechanical nonlinearities are often responsible for performance limitations as expressed by speed of response, accuracy and stability. The influence of coulomb - friction on the step response of an electrohydraulic position servo was clarified (1). The response could be effectively damped if coulomb - friction is introduced in the actuator, while the system may not be asymptotically stable.

Electrohydraulic velocity servo has received less attention in the literature and most studies concerned with speed control of hydraulic motor are still of open loop type (2). In this paper a feedback speed control servosystem is studied, where a rotary hydraulic motor is driven by an electrohydraulic servovalve. In this study two major mechanical nonlinearities are considered: (a) stick-slip friction which is present in the actuator, and (b) valve deadband which definitely affects system dynamics with low amplitude inputs.

The objective of the present paper is to investigate whether stick-slip friction contributes to stability, damping and speed of response or not.

STRUCTURE OF CONTROL SYSTEM

The block diagram of the hydraulic velocity servo analysed in this paper is represented by Fig. 1. The control valve which is a standard type 32-Moog servovalve (3), is connected to a five-axial piston type hydraulic motor. The feedback action is implemented by using a tachogenerator and a pressure transducer to measure the shaft speed and load pressure.

MATHEMATICAL MODEL

Taking into account the pressure/flow nonlinearity of the servovalve, the steady-state valve characteristics are given by (1) as

$$Q = K_s x_s \operatorname{sgn} [P_s - (\operatorname{sgn} x_s) P_l] \cdot \sqrt{|P_s - (\operatorname{sgn} x_s) P_l|} \quad (1)$$

where

$$K_s = C_d \pi d_s \sqrt{1/\rho}$$

is the valve coefficient.

The valve dynamics is represented by an equivalent second order differential equation, while taking into consideration the deadband of the servovalve, Fig. 2, as

$$\ddot{x}_s + 2 \zeta_v \omega_v \dot{x}_s + \omega_v^2 x_s = \omega_v^2 K_v I_0 \quad (2)$$

with

$$I_0 = \begin{cases} I + \beta & \text{when } I < -\beta \\ 0 & \text{when } -\beta \leq I \leq +\beta \\ I - \beta & \text{when } I > +\beta \end{cases}$$

where I is the input current of servovalve and β is the deadband. The hydraulic motor characteristics are essentially the same as given in (1) and are represented by

$$Q = V_m \dot{\theta} + (V_c/4 K_h) \dot{P}_l + L_e P_l \quad (3)$$

The nonlinear behavior of actuator friction is responsible for performance limitations. Exact simulation of the nonlinear behavior of friction in the vicinity of a zero velocity is difficult. The friction torque is simulated by a stick-slip friction law as shown in Fig. 3. Outside the small region $-\Delta\dot{\theta} < \dot{\theta} < \Delta\dot{\theta}$, the friction torque T_f is an arbitrary function of $\dot{\theta}$. Inside this small region, surrounding $\dot{\theta} = 0$, $\dot{\theta}$ is considered to be zero and T_f is determined by other torques in the system in such a way that $\dot{\theta}$ remains in the region until the breakaway value is reached. The value of the stick-slip friction for positive $\dot{\theta}$ is simply given by

$$T_f = \begin{cases} T_{st} & \text{for } 0 \leq \dot{\theta} < \Delta\dot{\theta} \\ T_{sl} & \text{for } \dot{\theta} \geq \Delta\dot{\theta} \end{cases} \quad (4)$$

where T_{sl} is the slip friction, T_{st} is the stick friction, and $\dot{\theta}$ is the motor speed.

The equation of the load is described by

$$P_l V_m = J \ddot{\theta} + B \dot{\theta} + T_f \operatorname{sgn}(\dot{\theta}) + T_l \quad (5)$$

where T_f is the friction torque and T_l is the external torque.

Control Strategy

The major feedback loop is formed by applying a tachogenerator to measure the motor speed and generates a feedback signal. On the other hand a pressure transducer is adopted to measure the load pressure and using the generated signal to form the minor loop which contributes significant damping effect to the system in order to improve its dynamic response. The action of the feedback is described by the following relations, Fig. 1:

$$e_1(t) = V_i(t) - K_w K_t \dot{\theta} \quad (6)$$

$$e_2(t) = e_1(t) - K_l K_d P_l \quad (7)$$

Integral controller is adopted for controlling the motor speed with step input, the control action is defined by the following equation

$$I = K_i K_a \int_0^t e_2(t) dt \quad (8)$$

State - Space Model

The state space analysis can be used to analyse systems with multi-nonlinearities, and they are typical in electrohydraulic servosystems. It is a valuable tool to evaluate system transient with multiple input.

The state variable $\bar{x}$, the input variable vector $\bar{u}$, and parameter vector $\bar{\alpha}$ will be defined as:

deadband, Eq. (2), which makes the structure dependent on the state variable x_5 and correspondingly on x_4 . Therefore the auxiliary variables λ and η are defined according to:

$$\begin{aligned}\lambda &= 1, \quad \eta = -1 && \text{for } x_4 < -\beta \\ \lambda &= 0, \quad \eta = 1 && \text{for } -\beta \leq x_4 \leq +\beta \\ \lambda &= 1, \quad \eta = 1 && \text{for } x_4 > +\beta\end{aligned}$$

The first order state equations (10) were solved simultaneously using a fourth-order Runge-Kutta method with step size of $\Delta t = 0.1$ ms. The simulation program was executed at Assiut University computer centre on the ICL 2955 machine.

Table 1: System parameters

parameter	value
$\alpha_1 = 4 K_h/V_c$, N/m ²	2.7317×10^{14}
$\alpha_2 = K_s$, m ² /s.Pa ^{1/2}	3.0×10^{-4}
$\alpha_3 = V_m$, m ³ /rad	0.716×10^{-6}
$\alpha_4 = L_e$, m ⁵ /N.s	2.8×10^{-11}
$\alpha_5 = \omega_v$, rad/s	1005
$\alpha_6 = \zeta_v$	0.55
$\alpha_7 = K_v$, m/A	0.0375
$\alpha_8 = K_\omega K_t$, v/rad/s	1.0×0.026
$\alpha_9 = K_l K_d$, v/Pa	0.0×10^{-6}
$\alpha_{10} = K_i K_a$, A/v	1.0×0.01
$\alpha_{11} = J$, kg.m ²	3.4×10^{-3}
$\alpha_{12} = B$, N.m.s/rad	2.95×10^{-3}
β , A	0.0001
P_s , Pa	7.0×10^6
T_l , N.m	0.0

RESULTS AND DISCUSSIONS

In this paper the load pressure gain K_l and the external torque T_l are set equal to zero, in order to highlight the effect of nonlinear friction on the system response.

Simulated results are obtained under different operational scenarios to clarify the effect of friction on stability and speed of response of the servosystem. Fig. 4 presents results for step increase in speed to 500 rpm when the stick-slip friction, with a zero velocity range of $\Delta \dot{\theta} = 0.001$ rad/s, is considered or not. It is clearly indicated that the effect of friction is to increase the starting time (dynamic dead zone), t_s , where it is increased from 0.01 s to 0.06 s when friction is considered. When the zero velocity range is increased to 0.01 rad/s and further to 0.1 rad/s, the response is almost not affected except the dynamic dead zone which is increased to 0.09 s (results are very close to those in Fig. 4). This indicates that at high speeds, the zero

velocity range is not a critical parameter and hence the effect of motor sticktion is negligible whereas the slip friction is still effective and is responsible for the delay in the response. However the friction torque has a significant effect on the load pressure response as clearly displayed in Fig. 5. The steady-state characteristics of the servo is considerably affected where the magnitude of the load pressure P_l is much higher with friction, which is necessary to overcome friction at first before driving the inertia load. The zero velocity range has also negligible effect on the load pressure.

The dynamics of the servodrive at low operating speed of 50 rpm is investigated to clarify the influence of the nonlinear friction on the response, since low speed operation is important to enable direct connection of the servodrive to loads such as machine tool lead screws without intermediate gear reduction. Results are presented in Fig. 6 and Fig. 7 for the system speed and load pressure respectively. Results represented by curve (a) correspond to a zero velocity range of $\Delta \dot{\theta} = 0.001$ rad/s with friction torque of $T_{st} = 0.75$ N.m and $T_{sl} = 0.5$ N.m, however identical results were obtained when the zero velocity range is increased to $\Delta \dot{\theta} = 0.01$ rad/s or when sticktion is neglected; namely $T_{st} = T_{sl} = 0.5$ N.m. These indicate that up to this zero velocity range, 0.01 rad/s, the mechanism of friction has not shown any influence for the sticktion part of motor nonlinear friction. Therefore the zero velocity range is increased to $\Delta \dot{\theta} = 0.1$ rad/s and the simulated results are represented by curve (b) in Fig. 6 and Fig. 7, where the influence of actuator sticktion is showed up. Comparisons between the results obtained with sticktion for a zero velocity range of $\Delta \dot{\theta} = 0.1$ rad/s and those obtained for $\Delta \dot{\theta} = 0.001 - 0.01$ rad/s, as displayed by curves (b) and (a) respectively, indicate that the sticktion has a dominant effect on the transient characteristics of the servo-system. The maximum overshoot, M_p , in speed is increased from about 62 % to 98 %, the starting time is increased from 0.61 s to 0.88 s, and the settling time, T_s , from 1.76 s to 2.3 s as motor sticktion is introduced, Fig. 6. In other words the transient response is seriously deteriorated due to the destabilizing effect of sticktion, however steady-state response is not affected by the presence of motor sticktion. When the motor friction is removed, $T_f = 0$, the response of the actuator speed is significantly improved where the starting time is decreased from 0.6 s to 0.1 s as predicted from curves (a) and (c) in Fig. 6 respectively. It should be indicated that motor friction does not contribute any damping effect to the system for step increase inspeed. This is because the motor velocity does not change sign while the input signal is changing stepwise from zero, and hence there is no change of the coulomb friction torque with velocity to provide the system damping. The load pressure is considerably decreased to a value sufficient to drive the inertia load

Fig. 4: Effect of friction on speed for $N_r = 500$ rpm ($V_i = 1.361$ v)

Fig. 5: Effect of friction on load pressure for $N_r = 500$ rpm ($V_i = 1.361$ v)

as friction is removed, Fig. 7 curve (c). A further improvement in performance can be achieved by removing the servovalve dead-band, where the velocity servo started smoothly even at such low operating speeds as displayed by results of curve (d) which were obtained with $T_f = 0$, $\beta = 0$, Fig. 6.

Results presented so far in this paper are obtained with step increase in reference speed, N_r . It is of interest to obtain the system response due to step decrease in speed in order to throw more light on the complicated role played by motor dry friction. The transient response of the system due to step decrease in the reference speed from 500 rpm to 50 rpm is displayed in Fig. 8 and Fig. 9 for the motor speed and load pressure respectively. Comparing results obtained with friction to those obtained without friction, Fig. 8, indicates that the motor dry friction has a remarkable damping effect. As the speed of

the hydraulic motor is reduced from 500 rpm to 50 rpm, two sign changes in motor speed were achieved which motivate the dry friction mechanism to deliver an excess amount of damping to the system. Fig. 9 clearly indicates that nonlinear friction has a filtering effect on the load pressure response.

CONCLUSIONS

The dynamic response of an electro-hydraulic speed control servosystem is analysed in order to throw more light on the complicated role played by the actuator nonlinear friction. The following conclusions are derived from the simulated results :

- 1- For step increase in motor speed with large amplitude, high operational speed, the actuator sticktion has a negligible effect on the speed characteristics

Fig. 6: Effect of friction on speed for $N_r = 50$ rpm ($V_i = 0.136$ v)

Fig. 7: Effect of friction on load pressure for $N_r = 50$ rpm ($V_i = 0.136$ v)

Fig. 8: Effect of friction on response for step decrease in reference speed.

Fig. 9: Effect of friction on pressure for step decrease in reference speed.

whereas slip friction (coulomb friction) is responsible for the major delay in the response. On the other hand the load pressure is considerably increased in magnitude with stick-slip friction.

- 2- The role played by motor friction at low speeds, for step increase in reference speed, is rather complicated. The transient response is seriously deteriorated due to the destabilizing effect of motor sticktion which showed up when the zero velocity range is increased to 0.1 rad/s. The slip friction (coulomb friction) increases the starting time.

It should be indicated that for step increase in motor speed, the nonlinear friction does not contribute any damping effect to the system.

- 3- A remarkable damping effect is realized with friction as motor speed is reduced suddenly from 500 rpm to 50 rpm, where two sign changes in speed were achieved and motivated the dry friction mechanism to deliver an excess amount of damping.

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