APPLICATION OF THE MODEL REFERENCE ADAPTIVE CONTROL WITH THE CONSTANT TRACE ALGORITHM TO A PNEUMATIC SERVO

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ABSTRACT

The constant adaptation gain algorithm has been often applied to the MRAC of electro-hydraulic servo systems. This paper describes the application of the MRAC with the constant trace algorithm to a pneumatic servo system. The response of the MRAC of the plant without feedback is compared with that of the MRAC of the plant with feedback, whose response was rather smooth and stable. It is clear that the power spectra of control inputs up of both MRAC systems, obtained by FFT, differ remarkably. The effectiveness of the constant trace algorithm in the MRAC was shown by the experiments of the sudden changes of the feedback gain of the plant during the control. It is shown that the limiter of the control input is very effective to stabilize the MRAC system where the response speed of a reference model is much faster than that of the plant.

KEYWORDS

MRAC, pneumatic servo, constant trace algorithm, power spectrum, limitation of control input.

 NOMENCLATURE

AM: piston effective area
ai, bi: parameters of plant
ai, bi: estimated parameter
aM1, bM1: parameters of model
d: dead time
e: adaptation error signal
F: adaptation gain matrix
Fc: Coulomb friction
M: mass of piston including sensor
P: estimated parameter vector
up: control input (plant input)
yM: output of reference model
y: plant output (piston displacement)
ζ: damping factor
ϕT: plant variable vector
ω3: break point frequency of plant [rad/s]
ω4: resonance frequency of plant [rad/s]
ωM1: break point frequency of model [rad/s]
ωM2: resonance frequency of model [rad/s]

1. INTRODUCTION

The dynamic characteristics of pneumatic or hydraulic servo systems are apt to change with the change of the load. Even though the characteristics of a plant may change, the model reference adaptive control (abbreviated MRAC hereafter) can control the output of the plant to follow the output of a reference model. Recently, many researches on the MRAC of electro-hydraulic servo systems have been reported. The characteristics of an MRAC system (abbreviated MRACS hereafter) largely depend upon the adaptation algorithm of the MRAC. In the most researches mentioned above, a constant adaptation algorithm or an adaptation algorithm, in which the adaptation gain decreases with the lapse of time, has been used.

The friction of a pneumatic actuator will change with the dwelling time of the piston, and the motion of the actuator is apt to be influenced by the load and disturbances. Generally, a pneumatic source, a pneumatic servovalve and a pneumatic system are all limited in the capacity. The compressibility of compressed air is very large comparing with that of hydraulic fluid. It is very difficult to realize an especially high performance pneumatic servo by a conventional feedback control method. And so, the main purpose of this paper is to realize higher performance of a pneumatic servo by MRAC, which uses the constant trace algorithm of an adaptation matrix, where the signal of the piston displacement is used in the algorithm. The effectiveness of the feedback in the pneumatic plant and amplitude
limitation of the control input \( u_p \) is studied experimentally.

2. MATHEMATICAL MODELS OF THE PNEUMATIC SERVO AND THE REFERENCE MODEL

Fig. 1 shows the plant (the pneumatic system) used in the experiments. For the design of MRACS, it is generally assumed that the order and the relative order of the mathematical model of the plant are known.

![Plant (the pneumatic system)](image)

Fig. 2 Frequency response of the plant

But, in the practical plant, there may be many other elements, and so, it is very difficult to decide strictly the order of the mathematical model of the plant. Then, the transfer function of the plant were decided from the experimental results of a frequency response [Fig. 2. Feedback gain=0.3 (potentiometer gain)x0.138 [V/cm] (sensor gain of piston displacement), without load]. From Fig. 2, the transfer function is approximated by the following 3rd order system.

\[
G_p(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z^2 + a_2 z + a_3}
\]  

(2)

The order of the plant may be estimated to be 3, and then the order of the reference model is also considered to be 3. The following 3rd order system is used here for the transfer function of the reference model.

\[
G_M(s) = \frac{\omega_M^2}{(s + \omega_M)(s^2 + 2\omega_M s + \omega_M^2)}
\]  

(3)

The \( \omega_M \) and \( \omega_M^2 \) are decided from the response speed of the reference model. The damping factor \( \zeta \) is fixed constant to be unity, in order that the step response of the reference model does not overshoot.

Two reference models are prepared here. The response speed of the Model 1 is lower than that of the plant, and the response speed of the Model 2 is much higher than that of the plant.

Model 1: \( 2\omega_M^1 = 0.4 \) [Hz], \( 2\omega_M^2 = 20 \) [Hz], \( \zeta = 1 \)

Model 2: \( 2\omega_M^1 = 5 \) [Hz], \( 2\omega_M^2 = 20 \) [Hz], \( \zeta = 1 \)

Unless there is no special indication, Model 1 was used in the experiment. The pulse transfer function of Eq.(3) is given by

\[
G_M(z) = \frac{b_M z^2 + b_1 z + b_2}{z^2 + a_1 z^2 + a_2 z + a_3}
\]  

(4)

3. CONSTRUCTION OF MRACS

Fig. 3 shows the block diagram of the MRACS used here. The pulse transfer function, Eq.(2), is rewritten as follows.

\[
A(z^{-1})y_p(k) = z^{-d}B(z^{-1})u_p(k)
\]

(5)

The \( u_p(k) \) and \( y_p(k) \) are the control input and the output of the plant at time \( k \). \( a_i \) and \( b_i \) are unknown parameters, and \( d \) is a known positive integer.
is assumed. Assuming the reference model to be 3rd order system, we get Eq.(6) from Eq.(4).

\[ A_M(z^{-1})Y_M(k) = z^{-d}B_M(z^{-1})u_M(k) \]
\[ A_M(z^{-1}) = a_0 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} \]
\[ B_M(z^{-1}) = b_0 + b_1z^{-1} + b_2z^{-2} \]
where, \( u_M(k) \) is a finite input of the reference model, and \( A_M(z^{-1}) \) is an asymptotically stable polynomial. The \( y_M(k+d) \) in Fig.3 is derived from Eq.(6) as follows.

\[ y_M(k+d) = \frac{B_M(z^{-1})}{A_M(z^{-1})} u_M(k) \]  

Eq.(7) shows that the \( y_M(k+d) \) is the output of the transfer function \( \frac{B_M(z^{-1})}{A_M(z^{-1})} \) for input \( u_M(k) \), where no future output signal is required. The aim of the design of MRACS is to make an algorithm which generates the control input \( u_p(k) \), by which \( y_n(k) \) goes to \( y_M(k) \) asymptotically.

The plant variable vector is

\[ \Phi^T = [\hat{u}_p(k), \hat{e}_0(k)] \]
\[ = [u_p(k), u_p(k-1), u_p(k-2), y_p(k), y_p(k-1), y_p(k-2)] \]

The estimation parameter vector is

\[ \hat{\hat{P}}(k) = \{ \hat{\hat{e}}_0(k), \hat{\hat{e}}_1(k), \hat{\hat{e}}_2(k), \hat{\hat{e}}_3(k) \} \]

Adaptation algorithm makes \( e^*(k) \) to approach 0.

\[ e^*(k) = \frac{D(z^{-1})y_p(k) - \hat{\hat{P}}^T(k-1)\Phi(k-d)}{1 + \Phi^T(k-d)F(k-1)\Phi(k-d)} \]
\[ = \frac{\hat{\hat{e}}(k)}{1 + \Phi^T(k-d)F(k-1)\Phi(k-d)} \]

The control input \( u_p(k) \) is calculated from Eq.(11)

\[ u_p(k) = \frac{y_M(k+d) - \hat{\hat{P}}^T(k)\Phi(k)}{\hat{\hat{e}}_0(k)} \]

where, the parameter adaptation algorithm, which will keep the trace of adaptation gain matrix to be constant, is given by

\[ \hat{P}(k) = \hat{P}(k-1) + \frac{F(k-1)\Phi(k-d)\hat{e}(k)}{1 + \Phi^T(k-d)F(k-1)\Phi(k-d)} \]

4. EXPERIMENTAL SETUP

The following elements are used in the experiments.

Pneumatic servovalve: Remodeled from the electro-hydraulic servovalve (3F-30L-39, Tokyo Precision Instruments) and compensated with two soft springs attached to both sides of the spool (spool diameter: 8). Servo amplifier: Input \( \pm 1[V] \), Output current \( \pm 25[mA] \)

Cylinder: Inside diameter 60, Rod diameter 16, stroke 150 [mm], Piston effective area \( A_M = 10.6 [cm^2] \), moving mass \( M_M = 117 [kg] \).

Piston displacement sensor: A capacitance type micrometer.

Pressure sensor: PGM-10KE (Kyowa Electronic Instruments Co., Ltd.)

Control microprocessor: NEC-PC9801F, 8087NDP

D/A converter: AB98-05 (12 bits, Adtek System Science)

5. EXPERIMENTAL RESULTS

Here the constant trace of the adaptation gain matrix, \( F \), is used for the MRAC. The initial values of the estimation parameter vector, Eq.(9), were set as \( \hat{\hat{P}}^T(0) = [1, 0, 0, 0, 0, 0] \).

5.1 Effectiveness of the feedback loop in the plant

Usually, the loop transfer function of a servo system is a type 1 system to eliminate the steady-state position error. Fortunately, an actuator used in the servo system has integral characteristic, and so the loop transfer function becomes type 1 system. And so, the input signal to the actuator is nearly the derivative of the output signal of the actuator for a lower frequency range. So, the input signal has higher frequency components than the output signal. In the same way, in an MRACS, the control input \( u_p \) to a plant without feedback, in which such an actuator is used, has higher frequency components than the output \( y_p \) of the plant.
The theory of the MRACS used in this research is derived with an assumption that the plant is linear. There is no problem for the MRACS whose plant is linear. But the plant of the pneumatic servo, shown in Fig.1, is remarkably nonlinear; i.e., the flow versus pressure characteristics of the servovalve, Coulomb friction of the pneumatic cylinder. So, there may be some difference between the theory and the practical system. The higher frequency components of \( u_p \) of the MRACS may sometime have unfavorable effects to the system.

In the case of the plant, which has a feedback loop and is a type 0 system, the input signal of the plant has almost the same frequency components as the output signal for a lower frequency range.

Here, rectangular-input responses of MRAC systems, whose plant had and had not a feedback loop, were carried out. The reference model, Model 1, was used. The frequency of the rectangular reference input \( u_r \) was about 0.1[Hz]. Fig.4 shows the response wave forms of the MRAC of the plant without feedback (which is abbreviated the open-loop plant hereafter) and the power spectrum of \( u_r \) after the identification has proceeded. Fig.5 indicates the similar results of the MRAC of the plant with feedback (which is abbreviated the closed-loop plant hereafter) 0.5x0.138[V/cm]. The power spectra of Fig.4(b) and Fig.5(b) have large values at about 0.1[Hz] because the input rectangular wave was about 0.1[Hz]. The power spectrum shown in Fig.4(b) (the open-loop plant) contains higher frequency components than that shown in Fig.5(b) (the closed-loop plant).

In the case of the open-loop plant (MRACS), \( u_p \) is nearly the derivative of \( y_p \) (a little deformed rectangular waves), and so \( u_p \) has a higher frequency components [Fig.4(b)]. When the initial value of the adaptation matrix was large or the response speed of the reference model was high, \( y_p \) of the open-loop plant had sometimes high frequency oscillation superposed on the fundamental rectangular wave as the identification proceeded.

In the case of the closed-loop plant (MRACS), \( u_p \) is almost proportional to \( y_p \) (a little deformed rectangular waves), and then \( u_p \) has little higher frequency components [Fig.5(b)].

As long as the response speed of the reference model is not higher than that of the closed-loop plant itself, \( y_p \) of the MRACS with the closed-loop plant was very smooth and stable.
5.2 Effects of adaptation gain on the MRACS

In the MRACS, it is well known that the convergence speed of the estimation of parameters increases as the initial value of the adaptation gain increases.

\[
\text{Frequency [Hz]} \quad \text{Power spectrum of } u_p \text{ in Fig. 6(a)}
\]

Experiments of rectangular-input response of the MRACS with the closed-loop plant for the initial value of the adaptation gain 0.01, 0.03, and 0.05 were carried out (Fig. 6, I: unit matrix), from which we can see that the convergence speed increases as the initial value increases. But, in the case of the rectangular reference input \( u_m \), for \( F > 0.1 \), the MRACS became unstable and ran away to one side (\( b_0(k) = 0 \) and \( u_0(k) \rightarrow \infty \)). Fig. 6(d) shows the power spectrum of \( u_p \) in Fig. 6(c) after the identification has proceeded. The magnitudes of higher frequency components of it are very small. In the case of the sinusoidal reference input \( u_m \), the MRACS was still stable for the initial value of 0.5 (Maximum value, Fig. 7). And the power spectrum of \( u_p \) in Fig. 7(a) contains few frequency components except fundamental one [Fig. 7(b)].

\[
\text{Frequency [Hz]} \quad \text{Power spectrum of } u_p \text{ in Fig. 7(a)}
\]

But, in the case of the rectangular-input response of the MRACS with the open-loop plant, the MRACS became unstable for the initial value greater than 0.03. The power spectrum of \( u_p \) in Fig. 4(a) at \( F(0) = 0.03 \) contains higher frequency components [Fig. 4(b)].

As the magnitudes of power spectrum of \( u_p \) were smaller at higher frequencies, the MRACS was stable at a larger initial value of adaptation matrix, \( F(0) \). This may be caused by the fact that the plant is nonlinear and \( u_p \) is not sufficiently rich.
5.3 Effect of the sudden frequency change of sinusoidal reference input $u_M$ on the MRACS

Fig. 8(a) shows the experimental results of the MRACS for the sudden changes of $u_M$ (sinusoidal wave) from 0.1 to 0.5 [Hz] and from 0.5 to 0.1 [Hz] during the control. The power spectra of $u_p$ for intervals $T_1$, $T_2$ and $T_3$ in Fig. 8(a) are shown in Fig. 8(b), (c) and (d) respectively. Though $u_M$ is 0.1 [Hz] for

![Diagram](image)

(a) System responses

![Diagram](image)

(b) Power spectrum of $u_p$ for the time interval $T_1$

![Diagram](image)

(c) Power spectrum of $u_p$ for the time interval $T_2$

and for $u_M$ is 0.5 [Hz] for $u_p$, we can see a little difference in the power spectra shown in Fig. 8(b) and (d). When the frequency changed from 0.5 to 0.1 [Hz], the power spectrum of $u_p$ contained the fundamental component 0.1 [Hz] and some higher frequency components around 0.8 [Hz]. And, in Fig. 8(a), the 0.8 [Hz] component was superposed on the system response 0.1 [Hz] sinusoidal wave.

5.4 Effects of the amplitude limitation of the control input on the system response

As the capacities of a pneumatic source and a servovalve are finite and the compressibility of air is large, so there is a limitation of the response speed of a pneumatic system. In the experiment, the piston followed up almost exactly the output of the reference model when the response speed of the model was slower than that of the MRACS. But, when the response speed of the model became faster (Fig. 9), there remained a large error at the switching instants and so the spikes of the control input $u_p$ increased almost in proportion to time. In the results, the control input saturated and sometimes the system became unstable. In order to avoid such an instability, a limiter of the control input $u_p$ was introduced.

$$u_{pc} = \begin{cases} u_p(k) : |u_p(k)| \leq R \\ R \cdot \text{sgn}(u_p(k)) : |u_p(k)| > R \end{cases}$$

The $u_{pc}$ is calculated by Eq. (11), and $u_{pc}$ instead of $u_p$ is substituted into the equation of $\phi_p(k)$. The $R$ is the limitation value of the control input and is a known value. Fig. 10 shows the same experiment as Fig. 9 except the limiter with $R=0.375$ [V]. It was cleared experimentally that the
lelimiter is very useful to stabilize such an unstable MRACS. The magnitudes of power spectrum at higher frequencies in Fig.9(b) are a little larger than that in Fig.10(b).

Fig.9 Response of the system with the faster model (Model 2) and without the limitation of the control input for the rectangular reference input

(a) System response

(b) Power spectrum of $u_p$ in Fig.9(a)

5.5 Effect of the change of a plant characteristics on the MRACS

Fig.11 shows the behavior of the MRACS for the sudden change of the potentiometer gain from 0.5 to 0.3 at the time shown by the mark. In the case of a conventional feedback, the plant output will increases as the feedback gain decreases. But, in the case of MRAC, the amplitude of the plant (piston) output $y_p$ kept almost constant by decreasing the control input $u$ corresponding to the sudden decrease of the feedback gain.

Fig.11 System response for a sudden change of the feedback gain at the time shown by an arrow

(a) System response

6. CONCLUSION

The followings are cleared from the experiments of the MRAC of a pneumatic servo with a constant trace of the adaptation matrix.
1) In the case of the MRAC of the plant without feedback loop, the power spectrum of the control input $u_P$ contains higher frequency components for rectangular reference input $u_M$. The system response is apt to be underdamped. In the case of the MRAC of the plant with the feedback loop, the magnitude of the power spectrum of $u_a$ are very small at higher frequencies. The system response is rather smooth and stable. The MRAC of the plant with a feedback loop is attractive not only from the stability and characteristic but also the practical application. For example, in the case of the ran away of the computer in an MRACS, a switching from the digital control to an analog feedback control is possible.

2) The limiter of the control input $u_a$ is very useful to eliminate the spike signals in $u_P$ and to stabilize the MRACS.

3) The MRAC was very effective control method even though the feedback gain of the plant changed suddenly.

ACKNOWLEDGEMENTS

The authors wish to thank the student, Mr.Y.Yamakawa, for his co-operation in the experiments and preparing the manuscript.

REFERENCE


