CHAOTIC OSCILLATIONS OCCURRING IN A HYDRAULIC CIRCUIT (Digital Simulation and Experimental Study)

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ABSTRACT

This paper deals with chaotic oscillations occurring in a hydraulic circuit consisting of a poppet valve and a pipeline terminating at a constant pressure tank. The circuit with a long pipeline becomes unstable and various kinds of oscillations are self-excited. In this study, it is shown by digital simulations that irregular solutions are obtained from a deterministic mathematical model for the dynamics of the circuit in addition to periodic or almost periodic oscillations. The irregular solutions which are considered as a kind of chaotic phenomena are classified into (a) period-doubling type, (b) Ruelle-Takens type, and (c) intermittent type.

Experimental results are qualitatively in good agreement with theoretical ones.

KEYWORDS

hydraulic circuit; poppet valve; pipe line; stability; chaotic oscillation

NOMENCLATURE

A = cross-sectional area of pipe
C = flow coefficient ($= c_p \pi d_p \sin \alpha \sqrt{2/\rho}$)
 c_p = discharge coefficient of poppet valve
c = sound velocity in fluid ($= \sqrt{\beta/\rho}$)
d = poppet valve discharge coefficient
e = restitution coefficient
F = flow force acting on poppet
fi = frequency of the mode i ($= \omega_i/2\pi$, $i = 0, 1, 2, 3, \dots$)
Fo = initially compressed force of spring ($= \frac{1}{2} \pi d_p^2 P_{si} = KX_i$)
K = spring stiffness
Kh = hydraulic spring stiffness
L = pipeline length
m = mass of poppet
Pc = valve chamber pressure
Pco = steady state value of Pc

Ps = pressure in reservoir (supply pressure)
p = perturbed pressure from Pc
Qc = flow through poppet valve
Qco = steady state value of Qc
q = perturbed flow from Qc
s = Laplace operator
si = characteristic root ($= \sigma_i + j\omega_i$, $i = 0, 1, 2, \dots$)
T = time for sound to travel length of pipe line ($= L/c$)
t = time
X = valve displacement
Xo = steady state value of X
Xi = initially compressed spring length
x = perturbed valve of X from Xo
Zo = characteristic impedance of pipeline ($= \rho c/A$)
 α = half angle of poppet
 β = bulk modulus of working fluid
 δ = viscous damping coefficient
 ρ = density of working fluid
 ζ = damping ratio of poppet valve
 ω_n = undamped natural frequency of poppet valve

INTRODUCTION

Poppet valves are extensively used as a pressure regulating valve in hydraulic circuits. The poppet valve circuit having a long pipe line sometimes becomes unstable and periodic or almost periodic self-excited oscillations develop [1],[2]. In addition, it is also known that violent irregular oscillations occur, and that they disturb normal operations of the hydraulic control system. However, few investigations have been devoted to the phenomenon because of difficulty of nonlinear analysis. Thus, the mechanism of occurrence of irregular oscillations has not been clear yet.

In this paper, the authors investigate nonlinear oscillations occurring in the unstable poppet valve circuit by a digital simulation using R-K-G method.

A mathematical model of the system is expressed as deterministic differential-difference equations which contain time delay terms due to the pipeline transmission delay and various nonlinearities, i.e., pressure-flow characteristic of the valve, valve flow force and collision of the poppet with the valve seat.

Transition of self-excited oscillations with pipeline length is examined by digital simulations and, in particular, irregular oscillatory solutions occurring in the deterministic system, i.e., chaotic phenomena, are investigated in detail.

Experiments are carried out and the results are qualitatively compared with the theoretical results.

GOVERNING EQUATIONS

Governing equations describing dynamic behaviors of a poppet valve circuit treated here shown in Fig. 1 are derived on referring to the schematic model.

The pressure-flow characteristic of the poppet valve is

$$Q_c = C X \sqrt{P_c} \quad \text{for } P_c \geq 0, \quad (1a)$$

and putting in calculation as follows:

$$Q_c = 0 \quad \text{and} \quad P_c = 0 \quad \text{for } P_c < 0, \quad (1b)$$

where $C = c_p \pi d_p \sin \alpha \sqrt{2/\rho}$. Since we have to analyse large amplitude oscillatory motions, the discharge coefficient of the poppet valve c_p should be treated as variable with the valve opening. Thus, c_p is approximated on the basis of measured value as follows:

$$\left. \begin{aligned} c_p &= 5.75 X & \text{for } 0 \leq X \leq 0.122 \text{ mm} \\ c_p &= 0.7 & \text{for } 0.122 \text{ mm} < X \end{aligned} \right\} \quad (2)$$

The equation of motion for the poppet supported by a spring is

$$m\ddot{X} + \delta\dot{X} + KX = F - F_o \quad (3)$$

Fig. 1 Schematic diagram of a poppet valve circuit

where

$$F = \frac{1}{4} \pi d_p^2 P_c [1 - 4c_p (X/d_p) \sin 2\alpha] \quad (4)$$

$$F_o = \frac{1}{4} \pi d_p^2 P_{si} = KX_i \quad (5)$$

Under assumptions for the fluid in the pipe being compressible and lossless, the equation of motion expressed by perturbed pressure and flow at the downstream end is

$$p(t) + Z_o q(t) = -p(t-2T) + Z_o q(t-2T).$$

This can also be rewritten by P_c and Q_c .

$$P_c(t) + Z_o Q_c(t) = E(t-2T), \quad (6)$$

where $E(t-2T) = -P_c(t-2T) + Z_o Q_c(t-2T) + 2P_s$.

When self-excited oscillations develop in the circuit, the poppet valve often collides against the valve seat. A condition of collision should be given in order to obtain exact valve motions. Let $[x(t^-)]$ and $[x(t^+)]$ be the poppet velocity immediately before and after the collision at the valve seat respectively, the condition of the collision is written as follows:

$$[X(t^+)] = -e[X(t^-)] \quad (7)$$

where e is the restitution coefficient.

METHOD OF DIGITAL SIMULATION

It is difficult to solve Eqs. (1)–(7) analytically because of its nonlinearity. Thus, the authors make an attempt to solve them numerically by Runge-Kutta-Gill method. For that purpose, putting $X_1 = X$ and $X_2 = \dot{X}$, the equations can be rearranged as follows:

$$\dot{X} = f(X, P_c) \quad (8)$$

where

$$\left. \begin{aligned} X &= (X_1, X_2)^T, \quad f = (f_1, f_2)^T \\ f_1 &= X_2, \quad f_2 = [-KX_1 - KX_i - \delta X_2 + F]/m \end{aligned} \right\} \quad (9)$$

P_c in Eq. (8) should be solved as a function of X and t . The delay term $E(t-2T)$ in Eq. (6), which has already been calculated before time $2T$, is considered as a known function of time. The solution is

$$\left. \begin{aligned} P_c &= [2E + (Z_o C X)^2 - \sqrt{D}]/2 \\ &\quad \text{for } P_c \geq 0, \text{ or } E \geq 0, \\ P_c &= 0 \quad \text{for } P_c < 0, \text{ or } E < 0, \end{aligned} \right\} \quad (10)$$

where $D = [2E + (Z_o C X)^2]^2 - 4E$. Substituting Eq. (10) into Eq. (8), $f(X, P_c)$ can be represented as

$$\dot{X} = f(X, t-2T). \quad (11)$$

Equation (11) is solved under the following initial conditions:

$X_1 = X_o, \quad X_2 = 10.0 \text{ cm/s}$ at $t = 0$. The time function $E(t-2T)$ for the time interval $0 \leq t \leq 2T$ is calculated by using steady state values P_s and Q_{co} , and $E(t-2T)$ for $t > 2T$ is calculated by using P_c and Q_c

having already been computed before time $2T$.

In calculation, time when the poppet valve collides against its valve seat is evaluated by means of Lagrange's interpolation formula of order 4 to make truncation errors of the estimation the same order as those of R-K-G method $O(h^5)$, where h is the meshsize of time.

The values of system parameters used in the calculation are shown in Table 1.

Table 1 System parameters

$m = 8.33 \times 10^{-2}$	kg,	$d_p = 5.0 \times 10^{-3}$	m
$\delta = 6.11$	N/s,	$K = 2.71 \times 10^4$	N/m
$\rho = 0.856 \times 10^3$	kg/m ³ ,	$A = 6.22 \times 10^{-5}$	m ²
$c = 1316$	m/s,	$\alpha = \pi/4$	rad
$P_{si} = 1.96$	MPa,	$X_o = 0.08$	mm

Before the numerical simulation, it is worthwhile to consider stability of the system and to calculate linear frequencies for the linearised system of Eq. (1)–(6).

Denoting perturbed variables for P_c , Q_c and X from a steady state as p , q and x respectively, linearized equations are

$$q = a_x x + a_p p$$

$$p + Zoq = -p(t-2T) + Zoq(t-2T) \quad (12)$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = K_p p$$

where

$$a_x = (\partial Q_c / \partial X)_o, \quad a_p = (\partial Q_c / \partial P_c)_o$$

$$K_h = \pi c_p d_p P_{co} \sin 2\alpha$$

$$K_p = \frac{1}{2} \pi d_p^2 [1 - 4c_p (X_o/d_p) \sin 2\alpha] / m$$

$$K_n = K + K_h, \quad \omega_n = \sqrt{K_n/m}, \quad \zeta = \delta / (2\sqrt{K_n m})$$

The characteristic equation for Eq. (12) is

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)(1 + \lambda \tanh Ts) + \eta \lambda \tanh Ts = 0 \quad (13)$$

where s is Laplace operator, $\lambda = Zo a_p$ and

$$\eta = K_p a_x / m a_p.$$

Characteristic roots $s_i (= \sigma_i + j\omega_i)$ for Eq. (13) correspond to oscillatory modes of the system with linear frequencies f_i , which are close to i -th natural frequencies of the pipeline $[(2i-1)(c/2L)]$, $i = 1, 2, \dots$. On the other hand, the frequency f_o is related to the poppet valve motion.

Change of linear frequencies with pipeline length L is shown in Fig. 2 by solid curves (unstable portions) and broken curves (stable portions). Two horizontal broken lines represent critical frequencies of stability, every mode of the system being unstable between them. The lower critical frequency is nearly equal to the undamped frequency of the poppet ω_n .

Instability of the system starts from the pipe length $L = 0.416$ m and the lowest mode with the frequency f_1 is self-excited. With increase of L , higher modes become unstable successively. Frequencies decrease with increase of L and after reaching the lower critical frequency, they are again stabilized successively.

As seen in Fig. 2, the number of unstable modes increases with L . Not all unstable modes of oscillations can occur, but only some of them selectively occur. At present, however, it is impossible to predict theoretically what modes can occur under a given condition.

DIGITAL SIMULATION RESULTS

Results of the digital simulation are shown in Fig. 2 together with linear frequencies mentioned in the previous chapter. In Fig. 3 and Fig. 4, transition of oscillations is shown by plane trajectories. Oscillatory solutions obtained are considerably verified because of influence of nonlinearity in the system. The oscillations occurring in the system are plotted by symbols $\circ, \bullet, \square, \dots$ in Fig. 2.

Fig. 2 Frequency changes of oscillations with pipeline length L (Simulation results)

Fig. 3 Phase plane trajectories of oscillatory solutions obtained in the range (ii) of the pipe length L .

Fig. 4 Phase plane trajectories of oscillatory solutions obtained in the range (iii) of the pipe length L .

The meanings of the symbols are explained in referring to phase plane trajectories illustrated in Fig. 3 and Fig. 4 as follows:

- = period-one motion of the mode $[f_1]$. This type of motion has a sinusoidal wave form with the frequency f_1 . Fig. 3 (a)
- = period-two motion which is $1/2$ -subharmonic oscillation. Fig. 3 (b)
- = period-three motion, i.e., $1/3$ -subharmonic oscillation. Fig. 3 (h).
- ⊙ = period-four motion. Fig. 3 (f).
- ▽ = period-five motion.
- = period-six motion. Fig. 3(c)
- ⊖ = period-eight motion. Fig. 3(e)
- ▲ = almost periodic motion, where two modes of oscillation with immensurable frequency ratio f_i/f_j occur simultaneously. Fig. 4(b) (APO)
- Δ = irregular motion which can be considered as a kind of chaos. Fig. 3 (d),(g) and Fig. 4 (d) (CO)
- ⊘ = intermittent chaotic motion. (ICO)
- ⊙ = damped oscillation in the stable system. (DO)

As seen in Fig. 2, oscillations self-excited between the pipeline length $L = 0 \sim 7.5$ m are as follows:

- (i) $L = 0 \sim 0.67$ m stable
- (ii) $L = 0.67 \sim 2.78$ m mode f_1
- (iii) $L = 2.78 \sim 3.05$ m modes f_1 and f_2
- (iv) $L = 3.05 \sim 5.50$ m mode f_2
- (v) $L = 5.50 \sim 7.50$ m modes f_2 and f_3

Fig. 5 Frequency change of the oscillation mode $[f_1]$ occurring in the range (i)

Fig. 6 Oscillatory wave forms and their frequency spectra in the range (i)

A part of the range (ii) illustrated in Fig. 2 is partially enlarged in Fig. 5. In this range, only a single mode of oscillation $[f]$ occurs. However, wave forms change variously with L . Some examples for typical wave forms and their power spectra are illustrated in Fig. 6 (a)–(d).

As seen in Fig. 2, immediately after instability of the system, a period-one motion of the mode $[f_1]$ [Fig. 6(a)] occurs which is a typical self-excited oscillation occurring in a unstable system. It changes to a period-two motion from $L=0.67$ m [Fig. 6 (b)] and to a period-six motion from $L=0.85$ m [Fig. 6(c)] successively, and then becomes irregular from $L=0.85$ m [Fig. 6(d)] The irregularity is sustained till $L=0.875$ m. In concurrence with appearance of the irregularity, the power spectra change from discrete types to continuous ones having broad band width.

These changes of oscillatory wave forms are similar to a chaotic phenomenon appearing after the period-doubling bifurcation discussed by Feigenbaum [3] and Moon et al. [4].

This kind of irregular motions appears again in the range $L=0.913 \sim 0.925$ m via period-eight and -four motions with elongation of the pipe length. After that, the oscillation returns to a period-one motion via period-five, period-three and period-two motions successively.

There is another route to irregularity in the range (iii). The oscillation of the mode $[f_2]$ becomes unstable at $L = 1.22$ m, but it can not be excited till $L = 2.78$ m. At this pipe length, the oscillation $[f]$ emerges for the first time and builds up an almost periodic oscillation together with the mode $[f_1]$, which lasts to $L = 2.84$ m and then the oscillation becomes irregular between $L = 2.84 \sim 3.60$ m.

This route to an irregular motion is analogous to a scenario given by Ruelle and Takens [3].

A enlarged figure of f vs. L in the range (iii) is shown in Fig. 7 and oscillatory wave forms and their spectra are shown in Fig. 8(a)–(d). Frequency ratio f_2/f_1 is, in general, commensurable and wave forms of almost periodic oscillations are so complicated that it looks irregular at first sight. The almost periodic oscillation, however, can be distinguished from

Fig. 7 Frequency changes of oscillation modes $[f_1]$ and $[f_2]$ in the range (iii)

Fig. 8 Oscillatory wave forms and their frequency spectra in the range (iii)

irregular ones by knowing if the frequency spectrum is discrete, or not.

The power spectrum of an almost periodic oscillation in Fig. 8 (b) has lots of peak frequencies, all of which can be expressed as linear combinations of f_1 and f_2 with integer coefficients, i.e., $f = mf_1 + nf_2$ where m and n are integers. The relations satisfied by f_1 and f_2 in Fig. 8 (b) are listed in Table 2.

Table 2 Combination frequencies

No.	Mode	Freq. Hz
1	$f_2 - f_1$	156.3
*2	f_1	*219.7
3	$3f_1 - f_2$	283.1
*4	f_2	*376.0
5	$2f_1$	439.4
6	$3(f_2 - f_1)$	468.9
7	$4f_1 - f_2$	502.8
8	$2(3f_1 - f_2)$	566.2
9	$f_1 + f_2$	595.7
10	$3f_1$	659.1
11	$2f_1 + f_2$	814.6

On the other hand, such relations can not be found in power spectra of irregular oscillations.

The almost periodic oscillation turns to irregular oscillations through a phase locking state, where ratio of two modal frequencies is fixed to a simple value $f_2/f_1 = 7/4$. This phenomenon is similar to one pointed out by Gollub et al.[5] who has studied Ruelle-Takens type chaotic motions in Rayleigh-Benard convection flow.

This type of transition to chaotic oscillations is also observed in the range $L = 5.5 \sim 6.3$ m, where the modes $[f_2]$ and $[f_3]$ are excited simultaneously.

In addition to chaotic motions studied so far, a different type of irregular motion is also observed in the range of the pipe length $L = 4.06 \sim 4.45$ m, whose wave form and power spectrum are shown in Fig. 9. The oscillation is apparently sinusoidal with a frequency f_2 , but accompanies beats in its wave form. Consequently, the FFT spectrum has side bands around frequencies f_2 and its integral multiples $2f_2, 3f_2, \dots$.

Fig. 9 An oscillatory wave form accompanying beats and its frequency spectrum

The phenomenon is similar to intermittent type chaos pointed out by Maneville and Pameau [3], who have posited that this type of chaos occurs when the system is close to a periodic motion, but experiences about burst of chaotic transient.

EXPERIMENTAL RESULTS

The test setup to confirm occurrence of chaotic motions predicted by numerical simulations is illustrated in Fig. 10.

Working fluid whose pressure is regulated by a relief valve is supplied to a poppet valve ④ through a constant pressure reservoir and a pipeline ③. The flow from the poppet valve is exhausted to a oil tank in atmospheric pressure.

Measured quantities are the poppet valve displacement X and the poppet chamber pressure P_c . The volt meter is used to measure the valve opening at steady states. Wave forms of oscillatory motions are monitored by a synchroscope, and they are recorded and analysed by FFT analyser.

Fig. 10 Test setup

Parameters of the poppet valve circuit tested are identical to those in Table 1. The supply pressure in the reservoir is so adjusted that the valve lift may become $X = 0.08$ mm. In the experiment, transition of oscillatory modes with change of pipe length L is investigated. The pipe length is varied over the range $L = 0.4 \sim 5.5$ m.

Measured frequency changes with the pipe length L are shown in Fig. 11 together with linear frequencies (solid or broken curves). Experimental results are indicated by symbols which have the same meaning as those in Fig. 2. The ranges (i)~(v), which represent what modes of oscillations are excited, are a little different in limiting pipe lengths from the simulation.

Diversity of oscillations occurring in the actual system is similar to results of the simulation, and all types of irregular motions, that is, (a) period-doubling type, (b) Ruelle-Takens type, and (c) intermittent type can also be observed in this experiment. But the relation between the oscillation type and L gives fairly good agreement with results of the simulation. Disagreements are considered to be mainly caused by ignorance of the viscous loss in the pipeline. The upper critical frequency of instability, for example, is lowered owing to the viscous friction in the actual system as seen in Fig. 11.

Typical examples of experimental wave forms and their frequency spectra are shown in Fig. 12 (a) ~ (f). Figure 12 (a), (b) and (c) indicate a route to a chaos of the type (a) in the range of the relatively short pipe length.

Figure 12 (d) indicates the chaotic motion of the type (b) at $L = 2.80$ m, which has two peak frequencies f_1 and f_2 in the frequency spectrum. Neither almost periodic oscillation nor phase locking phenomenon is observed in this range of the pipe length in the experiment.

On the other hand, almost periodic oscillations consisting of the modes $[f_2]$ and $[f_3]$ occur in the range $L > 5.00$ m.

Fig. 11 Frequency changes of oscillations with pipeline length L (Experimental results)

Fig. 12 Oscillatory wave forms and their frequency spectra
(Experimental results)

Figures (a) (c): oscillations in the range (ii), figure (d): a chaotic oscillation in the range (iii), figure (e) : an almost periodic oscillation in the range (v), and figure (f) : an oscillation with beats in the range (iv).

corresponding to the range (v) in the simulation. The wave form in Fig. 12 (e) shows an almost periodic oscillation in this range. Prominent peak frequencies of the spectrum coincide with linear combination of f_2 and f_3 . Measured peak frequencies and calculated ones are compared in Table 3. The motion changes to a chaotic motion when the pipe length L still more increases.

Table 3 Combination frequencies

No.	Meas. f Hz	Mode $mf_2 + nf_3$	Cal. f Hz	Error $\bar{f} - f$
1	68.4	$3f_3 - 4f_2$	65.0	-3.4
2	102.0	$f_3 - f_2$	101.6	-0.4
3	171.9	$5f_2 - 3f_3$	174.8	+2.9
*4	239.8	f_2	*239.8	
5	307.9	$3f_3 - 3f_2$	304.8	-3.1
*6	341.4	f_3	*341.4	
7	411.6	$6f_2 - 3f_3$	414.6	-3.0
8	479.4	$2f_2$	479.6	+0.2
9	547.7	$3f_3 - 2f_2$	544.6	-3.1
10	581.2	$f_2 + f_3$	581.2	0.0
11	684.9	$2f_3$	682.8	-2.1
12	718.9	$3f_2$	719.4	+0.5
13	787.4	$3f_3 - f_2$	784.4	-3.0
14	958.2	$4f_2$	959.2	+1.0

Figure 12 (f) shows the chaotic motion of type (c), which has beat in the oscillatory wave, whose spectrum shows side bands as seen in Fig. 12 (f). The distinctive feature is analogous to the intermittent chaos in Fig. 7.

CONCLUSIONS

Oscillatory motions in an unstable poppet valve are studied by digital simulations and experiments.

It is pointed out that three different types of irregular solutions, i.e.,

- (a) period doubling type chaos,
- (b) Ruelle-Takens type chaos

and (c) intermittent type chaos, can be obtained from deterministic governing equations describing dynamic behaviors of the poppet valve circuit which has marked nonlinearities including collision of the poppet against the valve seat.

It is confirmed experimentally that those irregular motions obtained by simulations can occur in the actual system.

REFERENCES

- [1] Funk, J. E.: Poppet Valve Stability, Trans. ASME, Ser. D, 76-5(1964), p. 207.
- [2] Kasai, K.: On the Stability of a Poppet Valve with an Elastic Support, Bulletin of JSME, 11-48(1968), p. 1068.
- [3] Eckmann, J. P.: Roads to Turbulence in Dissipative Dynamical Systems, Review of Modern Physics, 53-4(1981), p. 643.
- [4] Moon, F. C. et al.: Chaotic Vibrations of a Beam with Non-linear Boundary Conditions, Int. J. Nonlinear Mechanics, 18-6(1983), p. 465.
- [5] Gollub, J. P et al.: Many Routes to Turbulent Convection, J. Fluid Mechanics, 100(1980), p. 449.