DETERMINATION OF THE DAMPING CHARACTERISTICS OF A HYDRAULIC HOSE

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ABSTRACT

A method for simulating transient flow in a hose is proposed. The method makes it possible to account for both linear and nonlinear hose damping properties and has been used to investigate how different types of hysteresis can be recognized from measurements. It is shown that the proposed model is an adequate basis for investigating which properties one should promote in order to improve a hose's damping characteristics.

KEYWORDS

Transients
Hose
Damping
Hysteresis

NOMENCLATURE

a velocity of sound [m/s]
A cross section area [m²]
C_{tot} total deformation capacity [m⁵/N]
C_1 oil's deformation capacity [m⁵/N]
C_2 hose wall's deformation capacity [m⁵/N]
d inner pipe/hose diameter [m]
L total pipe/hose length [m]
H relation between amplitudes
Nc number of cords
Nx number of axial discretisations
p pressure [N/m²]
Q flow [m³/s]
R_C Coulomb-friction factor [N/m²]
R_L linearised friction factor [Nm²/m³/s]

R_{lin} linearised friction factor [Nm²/m³/s]
t time [s]
v velocity [m/s]
λ Darcy–Weisbach friction factor
ν oil kinematic viscosity [m²/s]
ρ oil density [kg/m³]

INTRODUCTION

Hydraulic hoses may be used to damp pulsations and noise. Two main damping mechanisms are present in all pipe flows: viscous friction and hysteresis. Viscous friction between the flow and the pipe wall depends on properties such as the fluid viscosity, pipe relative roughness, Reynolds number and the pulsation frequency. Such damping has been described by many authors e.g. refs [1], [2] and [3].

Viscous damping in laminar pipe flow is now well understood and predictable. When the flow is turbulent, the friction is more complex, and the damping can be predicted accurately only in certain cases.

In addition to the viscous damping, pulsations in pipes are damped because pipe walls have a built-in damping mechanism known as hysteresis. When a pressure peak propagates along a pipe, the walls expand in the region close to the peak. The pipe wall then stores potential energy, just like a strained spring. After the peak has passed, the pipe retracts, and the energy is transferred back to the fluid. Since no known material is perfectly elastic, some energy is always lost in the pipe wall. The regained energy amount is therefore less than the energy taken up by the pipe. Steel is nearly perfectly elastic, and in steel
pipes, the hysteresis can usually be neglected. This is not so in hydraulic hoses. The hose material's hysteresis properties contribute significantly to the damping of transients and noise.

Commercially available hoses consist of several cords, and unfortunately the hysteresis has been shown to be very complex, see refs. /4/, /5/, /6/ and /7/. A mathematical model of the damping in a hydraulic hose therefore has to account for different kinds of nonlinear material properties, as well as mechanisms due to interference between neighbouring cords. The model proposed here shows how some linear and nonlinear hose properties may be accounted for in the model. We also discuss how these properties could be determined from measurements of a particular hose.

A SIMPLE MODEL OF TRANSIENT PIPE FLOW

Most simulation algorithms for transient pipe flow are based on a continuous mathematical model consisting of partial differential equations. The numerical solution, which uses discrete equations, is usually done as a purely mathematical manipulation. Alternatively, one could discretize the physical system first, and then establish a mathematical model of the discrete system afterwards. Fig. 1 shows how such a physical discretization can be done.

The fluid is regarded as being composed of "plugs", each assumed to behave as a rigid body. The fluid deformation is assumed to be located between the plugs. For each plug, one ordinary differential equation, a momentum equation, is established. Accordingly, one continuity equation is established for each "intervening space". It is shown in ref. /2/ that such a physical discretization leads to the following system of equations.

Momentum equations:

\[ \frac{dv_i}{dt} = \frac{N_x}{L} \frac{1}{\rho} \left( p_{i-1} - p_i \right) - \frac{32 \nu}{d^2} V_i \]  \hspace{1cm} (1)

when the flow is laminar, and

\[ \frac{dv_i}{dt} = \frac{N_x}{L} \frac{1}{\rho} \left( p_{i-1} - p_i \right) - \frac{\lambda}{2d} \cdot V_i \cdot |V_i| \]  \hspace{1cm} (2)

Continuity equations:

\[ \frac{dN_x}{dt} = \frac{a^2 \rho}{L} \left( V_i - V_{i+1} \right) \]  \hspace{1cm} (3)

where

\[ a = \text{velocity of sound} \left[ \text{m/s} \right] \]

The oil- and pipe's total deformation capacity may be defined as

\[ C_{\text{tot}} = \frac{L \cdot A}{a^2 \rho} \]  \hspace{1cm} (4)

\[ A = \text{inner cross-section} \left[ \text{m}^2 \right] \]

Each intervening space's deformation capacity is given by:

\[ C = \frac{L \cdot A}{N_x \cdot a^2 \rho} \]  \hspace{1cm} (5)

Putting eqn. (5) into eqn. (3) yields:

\[ \frac{dp_i}{dt} = \frac{A}{C} \left| V_i - V_{i+1} \right| \]  \hspace{1cm} (6)

A SIMPLE MODIFICATION FOR LINEAR HYSTERESIS

Figs. 2 and 3 illustrate how the hysteresis could be included in the model. The "intervening spaces" previously described in connection with fig. 1, are now modified.

The deformation capacity is split into a term due to the oil's compressibility, \( C_{i} \).
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Fig. 2. Hydraulic analogy of the fluid in the hose. The oil's deformation capacity, $C_1$, is assumed to be perfectly elastic. The hose wall's deformation is not perfectly elastic, as indicated by the friction factor $R_1$.

Fig. 3. Mechanical analogy of linear hysteresis.

and a term due to hose wall's elasticity, $C_2$. The oil's deformation capacity is assumed to be perfectly elastic. The hose wall's deformation capacity is not, as indicated by the linear friction factor $R_L$.

Continuity in oil compression gives:

$$\frac{dp_1}{dt} = \frac{A}{C_1} |V_i - V_{i+1}| - \frac{Q_{\text{wall}}}{C_1}$$

(7)

$Q_{\text{wall}}$ = compression flow associated with expansion of the pipe [m$^3$/s]

In this model the pipe expansion is assumed to be opposed by linear friction, described by a friction factor $R_L$. The compression flow is therefore:

$$Q_{\text{wall}} = \frac{1}{R_L} (p_{1,i} - p_{2,i})$$

(8)

$p_{1,i}$ = actual pressure in the fluid [N/m$^2$]

$p_{2,i}$ = the pressure on the elastic part of the hose wall [N/m$^2$]

Substituting eqn. (8) into eqn. (7) now yields:

$$\frac{dp_{1,i}}{dt} = \frac{A}{C_1} (V_i - V_{i+1}) - \frac{1}{R_L C_1} (p_{1,i} - p_{2,i})$$

(9)

Continuity applied on the pipe wall's deformation gives:

$$\frac{dp_{2i}}{dt} = \frac{Q_{\text{wall}}}{C_2}$$

(10)

and hence

$$\frac{dp_{2i}}{dt} = \frac{1}{R_L C_2} (p_{1,i} - p_{2,i})$$

(11)

As previously shown the momentum equations are described by eqn. (1) and (2), but the oil pressure $p_1$ is now denoted by $p_{1,i}$.

**IMPROVED MODEL FOR LINEAR HYSTERESIS**

The simple linear-hysteresis model previously described is illustrated by fig. 3. The mechanical analogy for an intervening space is to replace the oil's compression by a spring, $C_1$. The hose wall elasticity $C_2$ is replaced by another spring, and the linear damper $R_L$ symbolizes the hose wall's viscoelastic damping.

Since hydraulic hoses generally consist of several cords, the hose walls elasticity and damping properties could be split into a number of separate parts. The mechanical analogy of such a hose is shown in fig. 4.

It can easily be shown that one new set of continuity equations arises for each cord. In intervening space no. $i$ we get:
Fig. 4. Improved linear hysteresis analogy. The hose wall is assumed to consist of several cords, each cord having an individual loss coefficient.

\[
\frac{dp_{ki}}{dt} = \frac{A}{C_i} (v_i - v_{i+1}) - \frac{1}{N_c+1} \sum_{k=2}^{N_c} \frac{1}{R_{k-1} C_k} (p_{i,k-1} - p_{k-1})
\]

where \( N_c \) = total number of cords.

For each cord:

\[
\frac{dp_{ki}}{dt} = \frac{1}{R_{k-1} C_k} (p_{k-1,i} - p_{k-1}) , \quad k=2,..,N_c
\]

In practice, it may be difficult to select relevant values for the constants \( C_1, C_2, \ldots, C_{N_c+1} \) and the corresponding damping factors. This problem is not further discussed here. Instead it should be noticed that the equations (12), (13) and (1) or (2) are general, and could be applied to any pipe or hose with any number of linearly viscoelastic cords.

**MODIFICATION FOR NONLINEAR HYSTERESIS**

If the hose wall's damping characteristics are nonlinear, the flow pattern becomes more complex. Generally, the wall's viscoelastic behaviour may be assumed to be exponential. Equation (8) is then modified to:

\[
Q_{\text{wall}} = \frac{1}{R_L} \left( \frac{p_{i,i} - p_{i,i+1}}{n_1} \right)^{\frac{1}{n_1}}
\]

where \( n_1 = \text{exponent describing the hose wall's viscoelastic behaviour.} \)

In the general case with a total number of \( N_c \) cords, equation (12) is rewritten as

\[
\frac{dp_{ki}}{dt} = \frac{A}{C_i} (v_i - v_{i+1}) - \frac{1}{N_c+1} \sum_{k=2}^{N_c} \frac{1}{R_{k-1} C_k} (p_{i,k-1} - p_{k-1})^{n_{k-1}}
\]

and hence equation 13 are rewritten as

\[
\frac{dp_{ki}}{dt} = \frac{1}{R_{k-1} C_k} (p_{i,k-1} - p_{k-1})^{n_{k-1}}
\]

where \( n_k = \text{viscoelastic factor between cord no. } k \) and cord no. \( k-1 \).

**DETERMINE WHICH HYSTERESIS-DAMPING MECHANISMS ARE PRESENT**

Eqn. (14) may be rewritten as

\[
(p_{i,i} - p_{i,i+1}) = R_L \cdot \left( Q_{\text{wall}}^{n_1} \right)^{n_1}
\]

The linearised damping factor for a single-cord hose could be determined by differentiating (17):

\[
\frac{dp_{i,i+1}}{dt} = \frac{1}{R_{\text{lin}}} \cdot \frac{dQ_{\text{wall}}^{n_1}}{dt} = n_1 R_L \cdot \left( Q_{\text{wall}}^{n_1} \right)^{n_1}
\]

When \( n_1 = 1 \), we have the previously described linear system. If \( n_1 > 1 \), we see that the linearized damping factor, \( R_{\text{lin}} \), increases with increasing flow, \( Q_{\text{wall}} \). Therefore increased signal amplitude, which yields increased flow, has to yield an increased value of \( R_{\text{lin}} \). The system becomes more "stiff" since the hose's expansion is delayed by its viscoelastic behaviour, and the velocity of sound is increased accordingly. This is an important result, because it indicates how we can easily determine which hysteresis-damping mechanisms are present. If the velocity of sound increases when the transient's amplitude is increased, we know that \( n_1 > 1 \).

If the velocity of sound decreases when the transients amplitude is increased, \( n_1 < 1 \).

**MODIFICATION FOR HYSTERESIS DUE TO DRY-FRICTION**

A situation of particular interest is when the cords have a dry- (Coulomb-) friction between each other. This is the same as letting \( n_1 = 0, N_c = 1 \), in eqns. (15) and (16). Dry-friction together with linear friction has been reported in hydraulic hoses /4/.

In the simplest situation with a single-cord hose the mechanical analogy of one intervening space is as in fig. 5. \( C_i \) is the oil's...
deformation capacity, $C$, the hose wall's deformation capacity, $R_L$ describes the linear part of the pipe wall's viscoelasticity, and $R_C$ is the dry-friction part of the pipe wall's viscoelasticity.

The compression flow caused by the wall is now:

$$Q_{\text{wall}} = \frac{1}{R_L} (p_{1,i} - p_{2,i} - R_C) \quad (19)$$

$R_C$ = minimum pressure needed to overcome the dry-friction [N/m²].

The dry-friction force has the opposite direction when the hose contracts compared to when it expands. Expansion occurs when the pressure increases sufficiently to overcome $R_C$. Otherwise, the hose doesn't expand. Generally, one may write

$$Q_{\text{wall}} = \frac{1}{R_L} (p_{1,i} - p_{2,i} - R_C) \quad \text{when} \quad (p_{1,i} - p_{2,i}) > R_C$$

$$Q_{\text{wall}} = 0 \quad \text{when} \quad |p_{1,i} - p_{2,i}| \leq R_C$$

$$Q_{\text{wall}} = \frac{1}{R_L} (p_{1,i} - p_{2,i} + R_C) \quad \text{when} \quad (p_{1,i} - p_{2,i}) < - R_C$$

The complete model is now described by eqns. (1), (2), (7) and (19) together with eqn. (20).

MEASUREMENTS

As previously described, the velocity of sound depends on the transient's amplitude if the hose's damping properties are nonlinear.

A simple hose was selected for testing in the laboratory, fig. 6. The measurements were carried out by introducing sinusoidal hydraulic pressure excitations at one end of the hose. The other end was kept closed at all times. The damping was determined by measuring the amplitude of the pressure signals at both hose ends. This was done for different amplitudes and frequencies. Some of the results are plotted in fig. 7. The relation $H$ between the two amplitudes can be seen to depend strongly on both angular frequency and amplitude at the inlet end. Since the damping depends on the amplitude, some of the damping mechanisms are nonlinear.

As a second step towards determining the hysteresis, we compared the measurements with simulations. This has been done in fig. 8.
in two parts.

Using \( a = 720 \text{ m/s} \), eqn. (4) leads to \( C = 3.4 \times 10^{-12} \text{ m}^5/\text{N} \). The velocity of sound in a rigid pipe is approximately 1400 m/s. The oil's total deformation capacity is therefore according to eqn. (4) \( C \text{ tot} = 8.9 \times 10^{-13} \text{ m}^5/\text{N} \).

The rest of the deformation capacity, \( C_2 = C \text{ tot} - C_1 = 2.5 \times 10^{-12} \text{ m}^5/\text{N} \), is due to the hose's elasticity.

In the simulation, if we select \( N_x = 40 \), the deformation capacity in each intervening space is \( C = 8.9 \times 10^{-13} \text{ m}^5/\text{N}/40 \).
\[
C = 2.2 \times 10^{-14} \text{ m}^5/\text{N}, \quad C_2 = 2.5 \times 10^{-12} \text{ m}^5/\text{N}/40
\]
\[
= 6.3 \times 10^{-14} \text{ m}^5/\text{N}.
\]

The damping factor \( R_L \) affects the velocity of propagation. If \( R_L = \infty \), the hose wall would obviously act as if it were rigid. The transients would then propagate at approximately 1400 m/s, corresponding to the oil's compressibility. If \( R_L = 0 \), similar reasoning tells us that the signals then would propagate at 720 m/s. The actual \( R_L \) would have to be somewhere in between these two extremes. The velocity of propagation for a signal of amplitude around 0.5 bar was measured as 830 m/s. By simulating with different values of \( R_L \), it could be shown that \( R_L = 7.8 \times 10^{10} \text{ Ns/m}^5 \), \( N_x = 40 \) yields this velocity of propagation. Using this, and by accounting for the hydraulic friction, curve 4 is established. It can be seen that this linear-hysteresis assumption strongly over—estimates the damping.

Finally, we investigated how the damping would be if all the hysteresis were of Coulomb type. This was done by neglecting the linear damping coefficient. For numerical reasons it was not possible to set \( R_L = 0 \). Instead, a very low value, \( R_L = 2 \times 10^9 \text{ Ns/m}^5 \), was selected. The correct velocity of propagation was now obtained by letting \( R_C = 0.24 \text{ bar} \). We also accounted for the frequency dependence of the hydraulic friction. The results are shown in fig. 9. These results agree better with the measurements in fig. 8 than when the hysteresis was neglected or assumed to be linear. It is also interesting to observe that the resonance peaks lie closer to eachother when the excitation amplitude is 0.9 bar. This means that the velocity of propagation decreases when the transient's amplitude is increased, which is the same tendency as one could observe in the measured curves. It is therefore very likely that the hysteresis damping is mainly of the Coulomb-type.

Full agreement between measurements and simulations was not obtained for all frequencies. There may be several reasons for this. It is possible that the Coulomb friction—factor \( R_C \) is not independent of the amplitude, as assumed here. Further, it is possible that \( R_C \) is also affected by the signal frequency. The deformation capacities
C and C are also uncertain. Here, C only included the oil's deformation capacity. Another possibility would be to add some of the hose wall's innermost layer to C, since the hysteresis probably occurs in the steel cord.

None of the above mentioned modifications have been tried out, but it is likely that better agreement between measurements and simulations could be obtained by further investigations.

CONCLUSIONS

Pulsation damping in hydraulic hoses is present not only because of the hydraulic damping: linear or nonlinear hysteresis damping due to the hose wall's viscoelastic and Coulomb-hysteresis properties are also present, and these damping mechanisms may well be dominant compared with the hydraulic damping.

The main mechanisms behind the hysteresis damping in hydraulic hoses is relatively well understood, but it is still difficult to quantify the hose properties needed to predict the damping accurately.

The proposed model should, however, form an adequate basis for investigating which properties one should promote in hydraulic hoses in order to improve their damping characteristics. For the hose investigated in figs. 8 and 9, linear hysteresis seems to be far more efficient at damping transients than Coulomb-type damping.

REFERENCES


