Feasibility study of sliding mode control in the operation of a variable displacement axial piston pump

Z. YOU, R.T. BURTON, and P.R. UKRAINETZ
Department of Mechanical Engineering
University of Saskatchewan
Saskatoon, Sask. Canada
S7N 0W0

ABSTRACT

The performance and the versatility of a variable displacement axial piston pump are, to a large extent, determined by its controller. Upgrading of existing controllers is considered one of the major means of improving the characteristics of the pump. Sliding mode control (SMC) is proposed here for the control of such a pump. The control scheme, while showing a great potential for increasing the robustness of the control system, substantially increases the difficulties of system implementation both theoretically and physically. The success of the application will be evident when these difficulties are resolved. Pump controllers consisting of different hardware structures and actuators are considered in this paper. The feasibility of these controllers, when designed using SMC theory, is studied; necessary conditions for the use of SMC in the control of such pumps are established. Systems that are deemed to be theoretically feasible are simulated, and issues on implementation of these control systems are discussed.

NOMENCLATURE

*C* = constant related to spring force
*Cl* = line capacitance
*c1,2,3* = sliding surface coefficients
*Dq* = a term from load flow
*Dt* = constant related to torque uncertainty
*Pa* = pressure acting on the control piston
*Pp* = output pressure of the pump
*Ps* = constant pressure supply
*P0* = return line pressure
*P* = distance of the states from the sliding surface
*u* = control input

*V* = I/O linearized control input
*Vi* = state variables
*yd1,2,3,4* = desired output and its derivatives
*y1,2,3,4* = output and its derivatives
*θ* = swashplate angle
*ωn* = desired natural frequency
*ωl* = corner frequency
*ζn* = desired damping ratio

INTRODUCTION

Variable displacement axial piston pumps have found widespread applications in the fluid power...
industry. One important reason for this is their use of various hydro-mechanical controllers that cause the pump output to match different load characteristics more efficiently and effectively. The design of these controllers, however, is often based on compromises and thus their performances are very operating condition dependent. For the case of the pressure compensated pump shown in FIGURE 1, this usually means high sensitivity to load flow disturbance and a deterioration in control performance when the system is operating away from the tuned operating point and parameters.

Much work has been done in the past years to improve pump performance. Recent studies feature the use of computer control and application of modern control techniques[1,2]. However powerful these novel control schemes are, considerable problems with the theoretical and practical implementation of these schemes exist. In this study, a sliding mode controller (SMC) is examined as applied to a pressure compensated pump; in particular, this study investigates the feasibility of SMC when using various types of hydro-mechanical actuators on the pump.

SLIDING MODE CONTROL

Sliding Mode Control (SMC) systems, also known as Variable Structure Control (VSC) systems, are a class of discontinuous feedback control systems that have been used in the control of a wide range of processes[3]. For example, SMC has been used for the control of a proportional valve[4].

Essentially, a SMC system has its dynamic behavior defined through a sliding surface which is, mathematically, an error dynamic equation. A discontinuous control law is designed to steer the state trajectory to the sliding surface and then stay on it (this is said to be in sliding mode). While in the sliding mode, the system remains insensitive to parameter variations and disturbances.

Design of Sliding Surface

A sliding surface as a constraint to system dynamics is normally a subspace of one dimension less than the system state space[3]. For a fourth order state space model, the sliding surface equation can be written as

\[ S = c_1 (y_{d1} - y_1) + c_2 (y_{d2} - y_2) + c_3 (y_{d3} - y_3) + y_{d4} - y_4 \]  

(1)

The coefficients are chosen according to the desired response and are independent of system parameters. In this study, this was accomplished by splitting the third order surface equation as follows:

\[ S = c_1 y_{d1} - (\omega_1 + \frac{d}{dt})(\omega_1^2 + 2\zeta\omega_1 \frac{d}{dt} + \frac{d^2}{dt^2}) y_1 \]  

(2)

In most applications, a pressure compensated pump controller functions as a pressure regulating device against the load flow disturbances as opposed to pressure tracking. The desired output is assumed to be constant. By deliberately assigning a relatively large value to \( \omega_1 \) in eq. (2), the second order term becomes a dominant factor in the system response. The sliding surface coefficients can thus be determined from the required system bandwidth and damping ratio by comparing eq. (1) with eq. (2).

Design of Control Law

In a SMC system, the problem of control of system performance is, in a way, transformed into a problem of sliding mode stability. Ideally, the sliding surface \( S \) equals zero during a sliding mode. To guarantee that the system states reach the sliding mode from off the surface in finite time, and subsequently stay on the sliding surface, the control law has to be designed so that
\[ \dot{S} \leq -\eta \text{Sign}(S) \quad \text{for} \quad S \neq 0 \quad (3) \]

holds true during the large majority of the control process. \( \eta \) (a positive value) is a measure of the speed at which the system states converge to the sliding surface.

**SYSTEM MODEL**

It is convenient to leave most of the original regulation system unchanged while the controller is being modified. The control scheme in FIGURE 2 is based on this consideration.

![Figure 2. SMC system schematic](image)

As one can see, the built-in control piston is retained to be the actuator of the control system. A high speed on-off valve or a servo valve is used in place of the original pressure controller known as a compensator. Output from the microcomputer is used to command the control system. An arbitrary load flow is considered and the controller is designed to regulate the pressure in the full range of the pump flow. For this feasibility study, the dynamics of the servo valve are assumed to be negligible. The state space model is obtained by first mathematically modelling the pump using conventional equations [1]. By assigning the state variables as

\[ X_1 = P_p; \quad X_2 = \theta; \quad X_3 = \frac{d\theta}{dt}; \quad X_4 = P_a \]

the system model can be defined in the following state space equation form:

\[
\begin{align*}
\frac{dX_1}{dt} &= a_{12}X_2 + D_q + a_{1u}\sqrt{X_1 - X_4} \quad u \geq 0 \\
\frac{dX_1}{dt} &= a_{12}X_2 + D_q \quad u < 0 \\
\frac{dX_2}{dt} &= X_3 \\
\frac{dX_3}{dt} &= a_{32}X_2 + a_{33}X_3 + a_{34}X_4 + D_I + C \\
\frac{dX_4}{dt} &= a_{40}X_3 + a_{43}X_4 + a_{4u}\sqrt{X_1 - X_4} \quad u \geq 0 \\
\frac{dX_4}{dt} &= a_{40}X_3 + a_{43}X_4 + a_{4u}\sqrt{X_4 - P_0} \quad u < 0
\end{align*}
\]

and

\[ y_1 = X_1 = P_p \quad (5) \]

These system state equations are nonlinear, of non-canonical form, and have a discontinuity with respect to control \( u \). The objective is to design a robust control that will perform according to the design specifications in spite of the existence of modeling error, parameter variations, and load flow disturbances in the system.

**TENTATIVE SMC CONTROLLER DESIGN**

Systems with a non-canonical form of the nonlinear state equations have been little explored as far as application of SMC theory. One approach to this problem is the use of the I/O linearization technique [5].

Practically, I/O linearization can be performed by differentiating the output variable with respect to time until the first appearance of the input \( u \). The minimum number of times of differentiation required to retrieve input \( u \) from the state equation is called the relative degree of \( r \). If \( r \) equals the system order number, the linearization is called input-state linearization and the system is said to be completely linearizable.

I/O linearization was performed on eqs. (4) and (5). Two sets of results were obtained due to
the discontinuity of the system. The I/O linearized system for \( u \geq 0 \) can be represented by

\[
\frac{dy_4}{dt} = a_{12}X_2 + D_q + V \\
u = \frac{V}{a_{12}X_1 - X_4} \quad u \geq 0
\]

while for \( u < 0 \), the linearized system becomes

\[
\frac{dy_4}{dt} = y_2, \quad \frac{dy_2}{dt} = y_3, \quad \frac{dy_3}{dt} = y_4 \quad \text{and} \\
u = \frac{V}{a_{12}a_{34}a_{44}X_4 - P_0} \\ u < 0
\]

Note: D is a term consisting of the flow disturbance as well as its derivatives.

The linearized model results in two entirely different sliding surfaces. This requires the system states to be able to jump between the two sliding surfaces almost instantly to obtain the desired responses (two different specifications indeed). This is certainly unrealistic physically. Moreover, the stability of a control system whose design is based on a partially I/O linearized model is not guaranteed [5]. Modification has to be made to avoid these theoretical and practical difficulties.

**SYSTEM MODIFICATION AND CONTROLLER DESIGN**

Examining the initial control system structure and its dynamic model, it was found that the initial control scheme results in two discontinuities in the system dynamics. One results from the use of the pump's own flow for destroking, and another results from the spring biased actuator. Since continuity is generally required by I/O linearization and SMC control, the model has to be modified for the use of these techniques.

One may consider the use of a double acting cylinder a promising solution; however, for pump stroking, this results in a positive feedback by drawing flow from the pump when the supply is already less than the load flow requirement. The use of the pump's own flow to actuate the swashplate is, indeed, the major cause of the difficulties. The control system is, hence, changed to accommodate an external pressure source. The external pressure source is shown in FIGURE 2 by the dotted line. The state space equations are changed slightly to account for this. The input-state linearized model may now written in the form of eq. (8).

The sliding plane for this system is still a third order error differential equation as given previously. There are, however, two possible ways to construct this sliding surface. One is to use the measurable state variables and to find the output derivatives through the use of state equations. This works, however, only if the state equations used in the calculation do not involve any major parameter uncertainties. An immediate limitation to this is that the line capacitance has to be fixed to assure the accuracy of the derived results. One has to know the load flow and its derivatives to the second order since normally load flow is not a constant.

The second method directly uses pressure output and its derivatives. This requires a state observer or filters to acquire all the required information. This makes the control system less dependent on the system model so that the parameter and disturbance invariant nature of this control method would be sustained. This second method is applied in this study.

Sliding surface equation coefficients are determined from the appropriate frequency response and only limited by the available control input that the system can provide.

Assume a control that ensures the state variables move along the sliding surface. It follows that

\[
S = 0
\]

Differentiating eq. (1) and combining the result with eq. (8), control V can be found to be

\[
V = -c_1y_2 - c_2y_3 - c_3y_4 - (a_{34}a_{43} + a_{32})y_3 \\
- a_{34}y_4 - a_{42}a_{34}a_{44}X_4 - D
\]

This is called equivalent control which actually is the mean control input to the plant. \( X_4 \) is a measured auxiliary state variable; the use of it has simplified the control algorithm to a large extent.
A discontinuous portion is added to the mean control input such that

\[ V = -c_1y_2 - c_2y_3 - c_3y_4 -(a_4a_5 + a_2) y_3 - a_3y_4 - a_7a_8 X_4 + \eta \cdot Sign(S) + \eta_d \cdot Sign(S) \]

where \( \eta_d \geq |\eta| \) accounts for those terms involving load flow and its derivatives whose exact values are very difficult to evaluate. \( \eta \) as a normal sliding mode control term secures the reaching and existence of a sliding mode. It should be noted that a large \( \eta \), though, increases control system robustness to modeling error and disturbances, requires stronger control input, and may cause system saturation. It also means high control authority and may induce so called chattering in the output which is highly undesirable.

Once \( V \) is found, the control input to the servo valve can be evaluated by equations similar to eq. (9). Once the polarity of \( V \) is known, the proper equation to use is determined. The discontinuity in the model can be easily handled. A continuous and uniform sliding surface can be shared by two different sets of state equations.

**SIMULATION STUDY**

The SMC system was simulated to examine its performance under various operating conditions, including that of changing plant parameters.

In normal operation of a pressure compensated pump, the pressure setting is usually set to be constant. To test the system stability in the case of a setting change, a pressure response to a step input in pressure setting is shown in FIGURE 3.

The system responds quickly (.05 sec.) to the input step with no overshoot. This is just the system specification defined by the sliding plane equation. The responses only change slightly when line capacitance is varied. The corresponding control signal (FIGURE 4) shows switching of the control law at a very high speed.

A common operating condition is that of variation in the load flow requirement which is, from a control point of view, a load disturbance. This was simulated by a change of load valve opening. The first simulation assumed a sinusoidal type of disturbance signal. The opening was adjusted to induce a load flow change of 25% of rated flow peak to peak at a frequency of 20 Hz. Good disturbance rejection performance is demonstrated by FIGURE 5. A moderate pressure variation was maintained.

![Figure 3. Pressure responses to a step input in pressure setting under different line capacitances](image)

Figure 3. Pressure responses to a step input in pressure setting under different line capacitances

![Figure 4. Control input for the step response using designed line capacitance value](image)

Figure 4. Control input for the step response using designed line capacitance value

A step type of valve opening change was then considered. This again caused a load flow change of 25% of the rated flow. A moderate pressure overshoot is exhibited, and the system only takes a very short period of time to make all the necessary corrections. This is shown in FIGURE 6.

Line capacitance is the most uncertain variable; it is difficult to estimate and is system dependent. The step response for line capacitance values different than the design value, was
investigated. Robustness of the control can be seen in FIGURE 6 for these cases.

Simulation results show that the SMC system can operate robustly under bounded parameter variations and load disturbances. The control requires a high sampling rate and fast actuator. The chattering in control may cause severe mechanical wear to the servo valve. All of these limit the application of this control method. The use of an electrical motor as actuator is being investigated as an alternative solution[6].

CONCLUSIONS

This study shows that sliding mode control of a pressure compensated pump can be achieved with minimum modification. By use of the I/O linearization technique, the sliding mode control system is designed without the limitation of a small operating range.

REFERENCES


