Some theoretical aspects and recent developments in pneumatic positioning systems

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ABSTRACT

The brief review of control laws, proposed or susceptible to proposition given in this paper, is based on the structure of linear and non-linear models adopted for the pneumatic positioning system which is under consideration. Attention is brought to relating and if possible, to justifying the control laws presented for the right field of control theory. A comparison is made between the theoretical possibilities of each control law. Also some questions open to discussion and some industrial applications are mentioned.

NOTATIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>piston area</td>
<td>m²</td>
</tr>
<tr>
<td>A_c</td>
<td>controlled area</td>
<td>m²</td>
</tr>
<tr>
<td>A_l</td>
<td>leakage area</td>
<td>m²</td>
</tr>
<tr>
<td>A_ij</td>
<td>orifice area</td>
<td>m²</td>
</tr>
<tr>
<td>i</td>
<td>upstream index</td>
<td></td>
</tr>
<tr>
<td>j</td>
<td>downstream index</td>
<td></td>
</tr>
<tr>
<td>a_i</td>
<td>n by n matrix</td>
<td></td>
</tr>
<tr>
<td>b_viss</td>
<td>viscous coefficient</td>
<td>kg.s⁻¹</td>
</tr>
<tr>
<td>C_pp</td>
<td>pressure flow coefficient</td>
<td></td>
</tr>
<tr>
<td>C_pn</td>
<td>&quot;</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>cylinder half stroke</td>
<td>m</td>
</tr>
<tr>
<td>F</td>
<td>force</td>
<td>N</td>
</tr>
<tr>
<td>F_p</td>
<td>perturbation force</td>
<td>N</td>
</tr>
<tr>
<td>F_sec</td>
<td>dry friction force</td>
<td>N</td>
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</tbody>
</table>

The Lie derivative of a function $h$ along a vector field $f$ is given by:

$$\mathcal{L}_f h(x) = \sum_{i=1}^{n} \frac{\partial h}{\partial x_i} f_i(x)$$

and the iterated Lie derivative is given by:

$$\mathcal{L}_{f_k} h = \mathcal{L}_{f_{k-1}} \mathcal{L}_{f_{k-2}} \ldots \mathcal{L}_f h \quad \text{k times}$$

1. INTRODUCTION

In order to study the problem posed by the position control of a pneumatic ram by an electropneumatic servovalve, researchers have adopted schematically and sometimes successively two principle approaches. The first approach is based on the choice of an a priori control structure and an experimental exploration of the influence of the control law parameters. The second approach consists of using the results of one or several fields of control theory in order to determine the control law completely. In this paper we propose discussing the recent theoretical and pratical advances in the position control of electropneumatic systems and more particularly in the light of recent results in non-linear control theory.

We shall being by recalling firstly the assumptions necessary to establish a basic non-linear state model, secondly the non-linear state model based on these assumptions, thirdly different linearized state models which will enable us to explore certain dynamic behaviors and to justify the choice of such control laws (preferably one to another). Then we shall show the possibilities offered by non-linear control theory, alone or in combination with other kinds of control which have been researched and those which are open to discussion either in a non-linear context or associated to other kinds of control.

2. DESCRIPTION OF THE SYSTEM UNDER CONSIDERATION

In order to have at ones disposition a general system framework, we suppose that:

- the electropneumatic system under consideration (figure 1) is composed of: a rodless, double acting, linear pneumatic cylinder whose position in controlled by two three-way single stage electropneumatic servovalves.
- the pneumatic ram drives an inertial load
- the system is controlled by a microprocessor
- all sensors necessary are available
3. STATE MODELS

3.1 MODELLING ASSUMPTIONS

Usually a non-linear state model is obtained by assuming that [1][2][3]:

- air is a perfect gas,
- pressures and temperatures are homogeneous in each chamber,
- the chamber process is adiabatic,
- in each chamber the temperature changes are negligible with regard to the source temperature $T_s$, however this hypothesis is not obligatory
- in each chamber the gas kinetic energy is negligible,
- the cylinder leakage mass flow rates are negligible,
- the chosen servovalves are identical and their dynamic is negligible,
- the viscous friction is characterized by the coefficient $b_{vis}$
- the dry friction effort is designated by $F_{sec}$
- the perturbation effort is $F_p$.

3.2 NON LINEAR STATE MODEL

By choosing as state variables the chamber pressures $P_p$ and $P_n$, the piston velocity $\dot{y}$ and the piston position $y$, and, as input variable, the servovalve current $i$, we can obtain a non-linear state model under the form [4]:

$$\dot{x} = f(x, u)$$

(1)

defined as below.

$$\begin{align*}
\frac{dP_p}{dt} &= \frac{\gamma r T_s}{V_p(y)} \left[ m_{sp}(i,P_p) - m_{pe}(i,P_p) - \frac{P_p A}{r T_s} \dot{y} \right] \\
\frac{dP_n}{dt} &= \frac{\gamma r T_s}{V_n(y)} \left[ m_{sn}(i,P_n) - m_{ne}(i,P_n) + \frac{P_n A}{r T_s} \dot{y} \right] \\
\frac{dy}{dt} &= \dot{y} \\
\frac{d\dot{y}}{dt} &= A_p P_p - A_n P_n - b_{vis} \dot{y} - F_{sec} - F_p
\end{align*}$$

(2)

The mass flow rates $m_{sp}$, $m_{pe}$, $m_{sn}$ and $m_{ne}$ can be expressed under the following form:

$$
m_{sp}(i,P_p) - m_{pe}(i,P_p) = A_{sp}(i) D(P_s,P_p) - A_{pe}(i) D(P_p,P_e)
$$

$$
m_{sn}(i,P_n) - m_{ne}(i,P_n) = A_{sn}(i) D(P_s,P_n) - A_{ne}(i) D(P_n,P_e)
$$

In non-linear control theory, important results have been developed for non-linear analytic systems, i.e. for systems[5] which are linear with the input. Then the behavior of these systems is
described by a non-linear state equation of the following form:
\[
\dot{x} = f_s(x) + g(x)u \\
y = h(x)
\] (3)

In order to apply the non-linear control theory, Richard [6] [7] has proposed decomposing the area of each servovalve orifice into a constant leakage area \( A_1 \) and a current controlled algebraic area \( A_c \) defined by the relations:
\[
A_c(i) = \begin{cases} 
A_{sp}(i) - A_1 & A_1 = A_{pe}(i_0) \text{ if } i \geq i_0 \\
-A_{sp}(i) + A_1 & A_1 = A_{ne}(i_0) \text{ if } i < i_0 
\end{cases}
\]
(4)

By assuming that the two three-way servovalves are identical and by introducing the proposed decomposition into the expressions (2) of mass flow rates, \( \dot{m}_{sp} - \dot{m}_{pe} \) and \( \dot{m}_{sn} - \dot{m}_{ne} \) can be written:
\[
\dot{m}_{sp}(i,P_p) - \dot{m}_{pe}(i,P_p) = \dot{m}_{p}(i,P_p) = -\dot{m}_{ip}(P_p) + q_{p}(P_p)A_c(i) \\
\dot{m}_{sn}(i,P_n) - \dot{m}_{ne}(i,P_n) = \dot{m}_{n}(i,P_n) = -\dot{m}_{in}(P_n) - q_{n}(P_n)A_c(i)
\]

with
\[
q_{p}(P_p) = \begin{cases} 
D(P_p, P_p) & A_c \geq 0 \\
D(P_p, P_e) & A_c < 0 
\end{cases}
\]
(5)
\[
q_{n}(P_n) = \begin{cases} 
D(P_n, P_n) & A_c \geq 0 \\
D(P_n, P_p) & A_c < 0 
\end{cases}
\]

The equations (4) can be interpreted by two schemes corresponding respectively to a positive value and a negative value of \( A_c \).

By introducing the relation (4) into the state equation (1), one can obtain a non-linear state model of the desired form:
\[
\dot{x} = f_s(x) + g(x)A_c
\]
(6)

If the dry friction and the perturbation effort are negligible, the components of the vector fields \( f, g \) are defined by the following relations [6] [7]:
\[
f_s(x) = \begin{bmatrix} 
\gamma r T_s V_p(y) \left[ -\dot{m}_{ip}(P_p) - \frac{P_p A}{r T_s} y \right] \\
\gamma r T_s V_n(y) \left[ -\dot{m}_{in}(P_n) + \frac{P_n A}{r T_s} y \right] \\
y \\
\dot{y} \\
A \frac{P_p - A P_n - b_{via}}{M} y \\
\gamma r T_s V_p(y) q_{p}(P_p) \\
\gamma r T_s V_n(y) q_{n}(P_n) \\
0 \\
0
\end{bmatrix}
\]
(7)

The output variable is the piston position \( y \).

3.3 SET OF EQUILIBRIUM POINTS
The set \( E \) of equilibrium points is the set of singular points for which the second member of the relation (6) is self-cancelling i.e. the set of points defined as follows [4]
\[
E = \{(x_0, u_0) \in \mathbb{R}^n \times \mathbb{R}, f(x_0, u_0) = 0\}
\]
(8)

In the case of an electropneumatic system and by introducing the mass flow rate in each orifice, this set \( E \) of equilibrium points is characterized by the following set of non-linear algebraic equations (for dry friction and perturbation force equal to zero):
The solution of this system is unique, well known and easily physically interpretable. Notice that the existence of a dry friction force implies firstly that it is not possible to determine the chamber pressures at the equilibrium independently from the system history, secondly that in certain conditions a limit cycle could appear.

3.4 TANGENT LINEARIZED STATE MODEL

For a non-linear system, studying the properties of the tangent linearized state model is, in most cases, a way of improving comprehension of the dynamic behavior. This possibility is, and has been, largely used. The tangent linearized state model is obtained by a Taylor series development limited to the first order and calculated about an equilibrium state determined by neglecting the dry friction force.

By designating the changes in any variable by a superscript * added to the corresponding letter, this tangent linearized state model has for a general expression

\[ x^* = F(x_0,u_0)x^* + G(x_0,u_0)u^* \]  

(10)

where

\[ F(x_0,u_0) = \frac{\partial f(x,u)}{\partial x} \bigg|_{x=x_0 \atop u=u_0} \quad \text{and} \quad G(x_0,u_0) = \frac{\partial g(x,u)}{\partial u} \bigg|_{x=x_0 \atop u=u_0} \]

Then, in our case, the tangent linearised state model for a given piston position has the well known expression given below [1]:

\[
\begin{align*}
\dot{\dot{y}_0} &= 0 \\
AP_{p0} - AP_{n0} - b_{vis} \dot{y}_0 &= 0 \\
\dot{m}_{sp}(P_{p0},i_0) - \dot{m}_{pe}(P_{p0},i_0) - \frac{P_{p0}}{r_{Ts}} A \dot{y}_0 &= 0 \\
\dot{m}_{sn}(P_{n0},i_0) - \dot{m}_{ne}(P_{n0},i_0) + \frac{P_{n0}}{r_{Ts}} A \dot{y}_0 &= 0 \\
\end{align*}
\]

(9)

The block diagram shown in figure 4 is associated to the linearized state model (11).

\[
\begin{bmatrix}
\frac{dy}{dt}
\end{bmatrix} =
\begin{bmatrix}
p_{p}^* \\
p_{n}^* \\
A \\
M
\end{bmatrix}
\begin{bmatrix}
\dot{y}
\end{bmatrix}
+ \begin{bmatrix}
\gamma r T_s \frac{G_{ip}}{V_{po}} \\
0 \\
0
\end{bmatrix} i^* + \begin{bmatrix}
0 \\
0 \\
-\frac{1}{M}
\end{bmatrix}
\]

(11)

with:

\[
C_{pp} = \begin{bmatrix}
\frac{\partial \dot{m}_{sp}}{\partial P_{p0}} & \frac{\partial \dot{m}_{pe}}{\partial P_{p0}} \\
\frac{\partial \dot{m}_{sp}}{\partial P_{n0}} & \frac{\partial \dot{m}_{pe}}{\partial P_{n0}} \\
\end{bmatrix}, \quad C_{pn} = \begin{bmatrix}
\frac{\partial \dot{m}_{sn}}{\partial P_{p0}} & \frac{\partial \dot{m}_{ne}}{\partial P_{p0}} \\
\frac{\partial \dot{m}_{sn}}{\partial P_{n0}} & \frac{\partial \dot{m}_{ne}}{\partial P_{n0}} \\
\end{bmatrix}
\]

\[
G_{ip} = \begin{bmatrix}
\frac{\partial \dot{m}_{sp}}{\partial i} \\
\frac{\partial \dot{m}_{pe}}{\partial i}
\end{bmatrix}, \quad G_{in} = \begin{bmatrix}
\frac{\partial \dot{m}_{sn}}{\partial i} \\
\frac{\partial \dot{m}_{ne}}{\partial i}
\end{bmatrix}
\]

One can easily deduce from the value of the state matrix coefficients, the expression of the time constant of each chamber and the natural cylinder pulsation, then:

\[
\tau_p = \frac{V_{po}}{\gamma r T_s C_{pp}}, \quad \tau_n = \frac{V_{no}}{\gamma r T_s C_{pn}}
\]

\[
\omega = \sqrt{\frac{\gamma A^2 (P_{p0} + P_{n0})}{M (V_{po} + V_{no})}}
\]

(12)

with \( V_{p0} = V_{pi} + A \dot{y}_0 \), \( V_{n0} = V_{ni} + A \dot{y}_0 \)

These expressions clearly show that:
- the natural pulsation is minimal for the mid-stroke piston position.
- the time constants \( \tau_p \) and \( \tau_n \) are functions of the piston position.

The block diagram shown in figure 4 is associated to the linearized state model (11).
3.5 REDUCED TANGENT LINEARIZED
STATE MODEL

A state feedback can be easily deduced from a previous tangent linearised state model. However in most cases the state feedback is based on a reduced model. Many authors used the reduction proposed by Shearer in the case of a symmetric ram, a symmetric four way servovalve and a mid stroke position as equilibrium position. Because of the supposed perfect symmetry, this reduction consists of combining the two state equations, related to the chamber pressures, in order to obtain a single state equation associated with the acceleration. Kellal et al. [8] have shown that this reduction is possible for any piston position if the state matrix coefficients $F_{11}$ and $F_{22}$ are replaced by their half sum. The frequency responses shown in figure 5 enable us to appreciate the validity of this approximation. The reduced tangent linearized state model obtained by Kellal's et al. [8] procedure, has the following form:

\[
\frac{d}{dt} \begin{bmatrix} y^* \\ \dot{y}^* \\ \ddot{y}^* \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\omega^2 & \frac{b_{vis}}{M\tau_m} - \frac{1}{\tau_m} \frac{b_{vis}}{M} \end{bmatrix} \begin{bmatrix} y^* \\ \dot{y}^* \\ \ddot{y}^* \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}
\]

(13)

The curves given in figure 6 related to a particular ram, clearly show the variations of the principal parameters with the desired equilibrium position and suggest the adoption of gain scheduling with this position. Furthermore Richard [6] has shown that this reduction is associated to an observability loss.
4. CONTROL LAWS

Now we are going to examine how these different state models are related to different control laws which have been proposed or could be proposed. The diagram given in figure 7 summarizes the relations between some control laws and the different state models presented in the first part of the paper.
FIGURE 8: State feedback

This diagram does not give an exhaustive list of possible control laws but indicates different possibilities, certain of these control laws are well known and are not presented in this paper, we shall only recall the corresponding references.

4.1 CONTROL LAWS BASED ON THE REDUCED LINEARIZED STATE MODEL

4.1.1 State Feedback

The earlier control laws used either the reduced linearized state model and a state feedback, i.e. position, velocity and acceleration feedbacks as proposed by Shearer [1] then by Burrows [3] or are based on a simplified linearized model [9] [10] [11] [12] [13]. This kind of control law was initially adopted by the manufacturers of the first electropneumatic positioning systems such as Martonair [13] and Gas.

Let us note that it is possible to design a state feedback control law by using a fourth order linearized state model, however one can add at least one pressure sensor. Let us recall that the mass flow pressure coefficients C_{pp} and C_{pn} become very small when the spool reaches the closed position and consequently these coefficients are often neglected. Let us underline that with this assumption, the obtained linearized state model does not represent the state behavior of the system any more, particularly if there is dry friction or perturbation effort. Furthermore the mass flow gains are estimated in the linear part of the corresponding characteristics. This estimation leads to an increase of the steady state error for a step input. For this reason certain manufacturers, such as Kolvenbach, have introduced the inverse characteristic of the orifice area in order to improve this steady state error.

The state feedback can be calculated by using pole placement, optimal control or experimental study of the dynamic behavior for every other equilibrium position. However as the closed loop state feedback matrix eigenvalues are related to the open loop natural frequency, which is a function of the desired equilibrium position, the state matrix coefficient related to the position has a limited value. So for a given dry friction, this limited value leads to a steady state position error too high in relation to the possibilities of the ram for any equilibrium position other than the central position. Obviously this kind of control law is not robust with respect to large load mass changes.
4.1.2 Adaptive Control Law Based on the Linearized State Model

In order to improve the dynamic behavior of pneumatic positioning systems in the case of parameter changes, for example inertial load, and supply pressure changes, several researchers proposed using a model reference adaptive control law based on the linearized state model. Zhou and Lu [19] applied this kind of control to an electropneumatic positioning system in the case of a linear actuator. Dransfield and Fok [20] studied a system using a reversible vane air motor. Araki et al. [21] proposed a model reference adaptive control with the constant trace algorithm.

A classical direct adaptive linear control law is used in the three cases. This kind of control law requires that the system has stable zero. Instability problems can appear during the transient phase of adaption. These problems are related to choice of the sampling time.

The block diagram corresponding to this control law [19], given in figure 9 shows that the state feedback coefficients are adjusted by the adaptive algorithm in order to ensure that the state error tends to zero.

4.1.3 Adaptive Variable Structure Control Law

Another way of improving the robustness of the control law has been proposed by Noritsugu and Wada [22] in the case of Placemate’s pneumatic robot. These authors assumed that:

- the translation load movement is governed by a classical second order differential equation related to the position error $x$:
  \[ \dot{x} + px = u \]
- an output feedback is used
- the equation of switching line is $s = cx + x$ ($c > 0$)
- the control input for a proportional servo-system is given by:
  \[ \dot{x} + px = u = \begin{cases} -\alpha x : xs > 0 \\ -\beta x : xs \leq 0 \end{cases} \]

In this context they proposed an adaptive variable structure control which switches the control from $-\alpha x$ to $-\beta x$ when the state trajectory intersects the switching line, this is the usual procedure. The adaption procedure consists of increasing the slope $c$ of the switching line until the moment when the slope of the tangent to the state trajectory becomes greater than the slope associated to the corresponding state vector. For a proportional servo system this procedure causes the state trajectory to approach the dominant eigenvector as shown figure 10. These authors studied several other cases, i.e. a relay servo system and a servo system with saturation and applied their control law in discrete time to an on-off electropneumatic robot and obtained interesting results. One can imagine that by taking into account the specificities of the pneumatic actuator, it would be probably possible to obtain a supplementary improvement both of the dynamic and static behavior.
4.2 CONTROL LAW BASED ON A FAMILY OF REDUCED LINEARIZED STATE MODELS

We have shown in the first part of this paper that a tangent linearized state model can be calculated for each equilibrium position $y_0=q$ belonging to the cylinder stroke. Furthermore one can deduce from this model a reduced linearized state model. One can consider [14] that the two corresponding families of linearized state models can be parametrized by a parameter $q$, in this case the desired piston position $y_e$. Then one can calculate a static feedback control law in continuous time by using a general method such as pole placement or optimal control. As certain parameters of the linearized state model are a function of the piston position, the coefficients of the state feedback matrix will be a function of this piston position. The linearized state model, being valid only in the neighborhood of the corresponding equilibrium point, it is necessary to determine a new state-feedback matrix for each new desired piston position.

In fact as the control law is implemented on a microcomputer, the control law must be calculated in discrete time and one must deduce the discrete-time state model from the continuous time linearized state model. For a given sampling time and a given desired piston position the discretization procedure leads to a discrete tangent linearized model of the following form:

$$
\begin{align*}
\xi_{k+1} &= \alpha(q)\xi_k + \beta(q)u_k \\
y_k &= \gamma\xi_k
\end{align*}
$$

with $\xi_k = [y_k \; \dot{y}_k \; \ddot{y}_k]^T$

By choosing a set of $N$ desired piston positions belonging to the interval corresponding to the piston stroke and by introducing an extended state vector $x_k = [\xi_k \; 1]^T$, one can obtain a unique state affine model whose state and control matrix coefficients are polynomial functions of the desired position $q$. This state affine model can be written [15]:

$$
\begin{align*}
x_{k+1} &= \begin{bmatrix} a_{0}+a_{1}q+...+a_{n}q^{n} & 0_{n,1} \\
0_{1,n} & 1 \\
0_{n,n} & b_0+b_1q+...+b_{n}q^{n} \\
0 & 0_{1,1}
\end{bmatrix} u_k \\
x_k &= [\xi_k \; 1]^T
\end{align*}
$$

The degree $\sigma_i$ of each matricial polynomial function appearing in this state affine model results from a compromise between the calculus complexity and the state affine model quality. The different coefficients of each polynomial function are obtained by a matricial regression in the least square sense.

This state affine representation justifies the introduction of a state affine control law (figure 11) which will be developed in the case of a pneumatic positioning system.

In this system the state feedback control law has the following expression by choosing the family of reduced linearized state models:

$$i_k = \gamma(y_e)\xi_k + D(y_e)y_e$$

In order to obtain a static gain equal to one, the control law becomes:

$$i_k = [\gamma_p(y_e), \gamma_v(y_e), \gamma_a(y_e)] [y_k \; \dot{y}_k \; \ddot{y}_k]^T + \gamma_p(y_e)y_ek$$
By using the same method as before, one can try to determine a affine control law of the following form:

\[
\begin{align*}
    i_k &= \sum_{j=0}^{c_k} y_c c_{c_2j}(1) y_k + \sum_{j=0}^{c_k} y_c c_{c_2j}(2) \dot{y}_k \\
    &+ \sum_{j=0}^{c_k} y_c c_{c_2j}(3) \ddot{y}_k + \sum_{j=0}^{c_k} y_c c_{c_2j}(1) y_c
    \end{align*}
\] (16)

From these expressions, it can be seen that a polynomial function of the desired piston position \( y_c \) has been chosen for each coefficient of the state-feedback control law. For each position \( y_c \) belonging to the \( N \) chosen position, one can calculate the state feedback matrix coefficients \( \gamma_p, \gamma_v, \gamma_a \) using a classical approach and suitable software, for example MATLAB. Then the coefficients \( c_{c_2j}(1), c_{c_2j}(2) \) and \( c_{c_2j}(3) \) can be obtained from the set of \( N \) following equations by using a matricial regression:

\[
\begin{align*}
    \gamma_p(y_c) &= \sum_{j=0}^{c_k} k_{c_2j}(1) y^i_c \\
    \gamma_v(y_c) &= \sum_{j=0}^{c_k} k_{c_2j}(2) y^i_c \quad \text{for } y_c \in \{y_{\text{min}}, ..., y_{\text{max}}\} \\
    \gamma_a(y_c) &= \sum_{j=0}^{c_k} k_{c_2j}(3) y^i_c
    \end{align*}
\] (17)

Then the state affine control law can be written:

\[
i_k = K_p (y_c - y_k) + K_v \dot{y}_k + K_a \ddot{y}_k
\]

with:

\[
K_p = \sum_{j=0}^{c_k} k_{c_2j}(1) y^i_c, \quad K_v = \sum_{j=0}^{c_k} k_{c_2j}(2) y^i_c, \quad K_a = \sum_{j=0}^{c_k} k_{c_2j}(3) y^i_c
\] (18)

The control laws proposed by Tapio Virvalo [16], Det [17] and Moore et al. [18] can be classified in this kind of control.

4.3 NON-LINEAR CONTROL LAWS BASED ON INPUT-OUTPUT LINEARIZATION

The most straightforward way of specifying the requirements with a positioning system, whatever the chosen technology, is to specify either the desired output dynamic in the case of point to point control or the time history of the system output for tracking purposes. It has been shown that the order of the desired point to point dynamic or the derivability order of the output time history in the tracking case is equal to the minimum number of integrators between the input and the output of the system. So for a pneumatic positioning system, either one can fix a third order point to point output dynamic or one needs the time history of position, velocity, acceleration and jerk. In other words jerk is the minimum order time derivative whose time history can be specified by a piecewise constant function.

Non-linear control theory [5] provides a systematic method of calculating the corresponding input vector in the case of non-linear analytic systems defined by expression (3). For these systems if there are the same number of inputs as outputs and if all the output characteristic indexes \( r_i \) (i=1,..,m) are defined, a necessary and sufficient condition of the existence of a linearising and decoupling control law is that the decoupling matrix is invertible[5].

The characteristic indexes are a list of integers strictly positive defined, for \( f = f_a \), by:

\[
r_i = \min \{k | \exists j \in \{1, ..., m\} \text{ that } L^{k-1}_{y_i} f_j = 0\} \quad i=1, ..., p
\]
This expression implies the calculus of several Lie derivatives whose formal expression is recalled in the notations.

The decoupling matrix is defined by:
\[
\Delta(x) = \begin{bmatrix} \Delta_i(x) \end{bmatrix}
\]
\[
\Delta_i(x) = L_{ij} L_{ij}^{-1} h_i(x) \quad i=1,\ldots,m, j=1,\ldots,m
\]

The linearizing and decoupling control law has the following expression:
\[
u(x) = \Delta^{-1}(x) \left[ v - \Delta_0(x) \right]
\]
with \( \Delta_0(x) \) vector of natural feedbacks defined by:
\[
\Delta_0(x) = \begin{bmatrix} \Delta_0_1(x), \ldots, \Delta_0_m(x) \end{bmatrix}
\]
\[
\Delta_0(x) = L_i h_i(x) \quad i = 1,\ldots,m
\]

The control law enables us to transform the system into \( m \) decoupled systems composed of \( r_i \) integrators if:
\[
\sum_{i=1}^{m} r_i = n
\]

In most cases this equality is not verified and the control law introduces a unobservable system whose stability must be proved.

4.3.1 Application to a Pneumatic Positioning System

Another way of calculating the linearizing control law consists of calculating the successive time derivatives of each output until the input appears in the expression of the time derivative of the output under consideration [6] [7]. The order of this time derivative is equal to the index related to this output. Then by setting, for each of the last time derivative calculated, the desired value, one can obtain a system of linear algebraic equations whose solution gives the desired system input vector. By applying this procedure to the case of a pneumatic positioning system we can show successively that:

- the third time derivative of the position is a function of the input \( A_c \), i.e. the jerk is the first position time derivative which can be equal to a constant piecewise time function.
- the expression of this time derivative can be written:
\[
\begin{align*}
\frac{d^3y}{dt^3} &= A_T s_{y} \left[ -r_{n} p \frac{AP}{r_{T}} y \right] \\
&- \frac{A_T s_{y}}{M V_{p}} \left[ -r_{n} p + \frac{A_{p}}{r_{T}} y \right] \\
&- b_{v} \left[ \frac{A_{p}}{M} p - b_{v_{y}} y \right] \\
&+ \frac{A_T s_{y}}{M} \left[ q_{p} (p) + q_{n} (p) \right] A_{c} \\
&+ \frac{M}{M} \left[ V_{p} (y) + V_{n} (y) \right] A_{c}
\end{align*}
\]

- the coefficient of \( A_c \) is equal to \( \Delta_0(x) \)
\[
\begin{align*}
\Delta_0(x) &= \frac{A_T s_{y}}{M V_{p}} \left[ -r_{n} p \frac{AP}{r_{T}} y \right] \\
&- \frac{A_T s_{y}}{M V_{n}} \left[ -r_{n} p + \frac{A_{p}}{r_{T}} y \right] \\
&- b_{v} \left[ \frac{A_{p}}{M} p - b_{v_{y}} y \right] \\
&+ \frac{A_T s_{y}}{M} \left[ q_{p} (p) + q_{n} (p) \right] A_{c} \\
&+ \frac{M}{M} \left[ V_{p} (y) + V_{n} (y) \right] A_{c}
\end{align*}
\]

- the sum of the terms which are not functions of \( A_c \) is equal to \( \Delta_0(x) \)

- each \( \Delta_0(x) \) term corresponds to a natural feedback of the system which is calculated in order to compare it to the jerk reference.

- in order to make the unobservable part appear, one can choose as new state vector:
\[
[z] = \begin{bmatrix} y & L_{y} y & L_{y}^{2} y & P_{p} \end{bmatrix} = \begin{bmatrix} y & y & y & P_{p} \end{bmatrix}
\]

then the corresponding state representation is:
\[
\begin{align*}
dy &= \dot{y} \\
d\dot{y} &= \ddot{y} \\
d\ddot{y} &= J \\
d\dot{P}_{p} &= \frac{y r_{T}}{V_{p} V_{n}} \left[ \frac{1}{q_{p}} - \frac{m_{p} q_{n} - m_{n} q_{p}}{V_{p} + V_{n}} \right] \\
&+ \frac{A_{T}}{r_{T}} \left( -P_{p} q_{n} + P_{n} q_{p} \right) + q_{p} \left[ \frac{b_{v} V_{n} \ddot{y}}{y r_{T} s_{y}} + J \right]
\end{align*}
\]

with
\[
a = \frac{A_T s_{y}}{M}
\]

- the unobservable part is locally stable
- the block diagram of the closed loop system which is equivalent to a triple integrator is given in figure 12.
FIGURE 12: Input output linearized system

Then there are two possibilities for controlling this triple integrator which have respectively as objectives, point to point control and tracking. This involves the introduction of an outer loop.

4.3.1.1 Point to point control

By using a state feedback it is possible to transform the triple integrator into a third order system with a given dynamic behavior:

\[ \ddot{y} + \alpha_2 \dot{y} + \alpha_1 \dot{y} + \alpha_0 y = \alpha_0 y_c \]

where \( y_c \) is the desired position.

A suitable choice of the coefficient \( \alpha_i \) enables us to obtain either a constant dynamic behavior (\( \alpha_i \) are constants) or a variable dynamic behavior, for example, if each coefficient \( \alpha_i \) is a function of the position, one can profit from the natural dynamic properties of the cylinder [15]. The corresponding control scheme is given in figure 13.

![Control diagram](image)

FIGURE 13: Non-linear point to point control law

On this figure the following notation are used:

\[
\begin{align*}
\theta(P_p, P_n, y) &= \frac{q_p(P_p)}{V_p(y)} + \frac{q_n(P_n)}{V_n(y)} \\
d_i(P_p, P_n, y) &= \frac{\dot{m}_p(P_p)}{V_p(y)} - \frac{\dot{m}_n(P_n)}{V_n(y)} \\
d_u(P_p, P_n, y, \dot{y}) &= \frac{A \dot{y}}{r T_s} \left[ \frac{P_p}{V_p(y)} + \frac{P_n}{V_n(y)} \right] \\
d_r(P_p, P_n, \dot{y}) &= \frac{\text{bvis}}{\gamma r T_s A} \left[ \alpha(P_p - P_p) \cdot \text{bvis} \dot{y} \right]
\end{align*}
\]

(23)

One can notice that the input current of the servovalve is obtained by using the inverse function \( A_c(i) \) which is deduced from experimental characteristics.

The control algorithms previously proposed either by Miyata and Hanafusa [23] or by Scholz [24] can be related to this kind of non-linear control though they are not directly deduced from non-linear control theory.
4.3.1.2 Tracking

In certain cases the user wants to define the time history of the position, velocity, acceleration and jerk, so it is preferable to choose a tracking control law.

Starting from the state equation of the triple integrator

\[ \dot{x}(t) = Fx(t) + Gu(t) \]

this tracking control law involves the use of [6] [7]:

- a first state feedback \(-kx\) for specifying the class of admissible trajectories which leads to a new state equation:

\[ \dot{x}(t) = Fx(t) + Gu(t) \]
\[ y(t) = Hx(t) \]

- a state equation which defines the reference state trajectory \(x_r(t)\) and has the following form:

\[ x_r(t) = Fx_r(t) + Gu_r(t) \]
\[ y_r(t) = Hx_r(t) \]

- a state feedback on error \(-k^* x^*\) defined from the error state equation:

\[ \dot{x}^*(t) = Fx^*(t) + Gu^*(t) \]
\[ y^*(t) = Hx^*(t) \]

with

\[ x^*(t) = x(t) - x_r(t), u^*(t) = u(t) - u_r(t) \]

- an input of the triple integrator \(u_0(t)\) having the following expression:

\[ u_0(t) = u_r(t) - k^* x^* - kx \quad (24) \]

The use of state feedback on error enables us set the dynamic behavior of the error state. The block diagram corresponding to this control law is given in figure 14. Richard and Scavarda [7], Lin [15] have published experimental results on point to point control and tracking control.

4.3.2 Some Possible Improvements

Non-linear control theory provides a powerful tool for designing new control laws, however the robustness of this kind of control has not yet been widely studied. Certain results obtained with an electrical actuator indicate that an improvement to the robustness can be obtained by combining non-linear control either with adaptive control or with variable structure control.

As in the case of a linear system, two adaptive control schemes can be studied corresponding to direct and indirect non-linear adaptive control. The principle of these control laws is given in this paper for continuous time.

4.3.2.1 Direct non-linear adaptive control

The aim of this non-linear adaptive control law is to obtain a tracking state error between a desired state trajectory and the real state trajectory to converge asymptotically to zero when one or several process parameters are unknown. The non-linear state model must be linear with respect to the unknown parameters \(\Phi_i\). This control scheme needs:

- an overparametrization due to the presence of parameter products in the expression of the
linearizing control law. Each parameter product must be considered as a new unknown parameter. Then the parameter number increases fast with the relative index.

- a stable zero dynamic
- verification of Lipschitz conditions by certain functions
- invertibility of the decoupling matrix.

However this control scheme does not require a parameter convergence.

The block diagram corresponding to this control law is given in figure 15. This figure shows:

- The linearizing inner loop whose natural feedback $\Delta_0$ and inverse "decoupling scalar" $\Delta^{-1}$ are considered as non-linear with respect of the state vector $x$ and linear with respect to the unknown parameters $\Psi_i$. These terms are calculated by using the estimated values $\hat{\Psi}_c$ of unknown parameters.

- A linear outer loop which uses a reference model, a tracking state error feedback and a suitable non-linear change of base in order to obtain the output and its required time derivatives. This loop fixes the dynamic behavior of the tracking state error.

- An adaptive loop which adjusts the unknown parameters to assure an asymptotic convergence to zero of the tracking state error.

Figure 15: Direct non-linear adaptive control law

4.3.2.2 Indirect non-linear adaptive control law

As with the previous non-linear adaptive control law this law requires the linearity of the non-linear state model with respect to the unknown parameters. This law needs parametric convergence but presents the advantage of no overparametrization.

On the block diagram given in figure 16 it can be shown that:

- The structures of the inner linearizing loop and of the outer loop which set the dynamic behavior of the tracking state error are the same as those of the previous adaptive control.
- The adaptive loop uses
- a prediction state model of the process which needs a state error feedback
- an adaptation algorithm which adjusts the model parameters in order to obtain a state prediction error which tends to zero

\[ r(t) = y_r \]

- a block which calculates the parameters whose linearizing control law and base change they need.

FIGURE 16: Indirect non-linear adaptive control

5. CONTROL LAW POSSIBILITIES

For each kind of control, the table shown in figure 17 gives indications about the theoretical possibilities of taking into account (+) or of decreasing the effects (*) of the different difficulties which are inherent in electropneumatic positioning systems.

To establish this table we have distinguished the non-linearity function of position from the non-linearities modelled. The first can be easily compensated as the piston position is known. The second generally requires the measurement of all state variables.

As the piston velocity and rate of pressure changes can be high, we have supposed that only non-linear control is able to compensate for the function of position non-linearities, and, more generally, the modelled non-linearities. Due to these possible high piston velocities and to the dry friction changes due to piston position and chamber
pressure, we have considered that linear and non-linear adaptive control laws do not compensate for dry friction. Theoretically dry friction can be taken into account with non-linear control by assuming that the law representing dry friction as a function of position, velocity, chamber pressures, is known. It seems that the only effective possibility for decreasing the dry friction effect is given by the use either of adaptive variable structure control or of self learning control. Adaptive variable structure has a similar effect to that of dither. Self learning control [25] [26] is a good solution in the case of repetitive cycles. This solution is used by the German manufacturer Gas (now Mannesmann Rexroth).

An efficient way of improving the precision of a pneumatic positioning system consists of introducing the inverse characteristic of each orifice area as a function of the servo valve current. Several authors [13] [27] have proposed adding an integrator to improve the static error. However this solution slows the system responses. Let us notice that the manufacturer Origa changes the state feedback matrix coefficients near the equilibrium point using a velocity criterium. This control law may introduce instability problems.

The sensor number has been determined by assuming that analog or numerical derivation is used to obtain velocity and acceleration.

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FIGURE 17: Control law comparison

Obviously the cost in terms of CPU units is high for non-linear control combined to adaptive control or adaptive variable structure control. The seriousness of this disadvantage decreases with the increase of microcomputer rapidity.
6. CONCLUSION

In this paper we try to show how the control algorithms proposed for a pneumatic positioning system can be related to the properties of the different linear and non-linear adaptive models for these systems. We have shown that non-linear control theory provides a powerful tool for understanding the behavior of the pneumatic positioning systems. For calculating a control law in order to fix either the output dynamic behavior in the case of point to point control or the error dynamic behavior in the case of tracking. An important research work remains to be carried out to evaluate the relative weight of each term of the non-linear control law for different cylinder sizes and to obtain simplified control laws. More particularly it would probably be interesting to compare the control laws deduced, from experimentation, by researchers of Aachen University, particularly Scholz [24], and theoretical non-linear control laws. The implementation cost of non-linear control depends on the pressure sensor prices. The rapid developments of such sensors for the car industry should bring a solution to this problem.

The non-linear control theory enables us to compensate the modelled non-linearities and to solve the decoupling problems in a non-linear context, it does not enable us to take into account the parameter changes such as mass or pressure supply changes. So if non-linear control laws are to be efficient, they must be completed by coupling them to other control laws obtained by using adaptive control theory or variable structure theory. Studying and comparing the advantages and disadvantages of such coupling is still an unresolved question.

Recently another method has appeared with fuzzy control. The first research [28] concerns either the automatic adjustment of the state feedback matrix coefficients, or the introduction of new control strategies [29] for improving the accuracy. An evaluation of the possibilities of this kind of control, used either alone, or coupled with another control, must be carried out. State observers have not been presented in this paper and it must be apparent that new problems are associated to non-linear state observers.

REFERENCES


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