On the frequency response characteristics of PWM high-speed on-off valves

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ABSTRACT

High-speed on-off valves are widely used in PWM electropneumatic servo systems and their characteristics are generally described by their on and off delay time. This paper focuses on establishing the relationships between their on-off switching behaviors and their frequency response characteristics. A method is proposed by which the frequency response characteristics for an analog PWM high-speed on-off valve at the points that the periods of the input signal are certain times of the carrier ones can be calculated based on its switching behaviors. Then a simple and direct describing function for a PWM high-speed on-off valve is established based on the simulation.

KEYWORDS

PWM, Frequency Response, High-Speed On-Off Valves

V_s = amplitude of carrier wave

φ_d = phase lag

NOMENCLATURE

A = amplitude of sine wave
 a_m = coefficient of Fourier series
 b_m = coefficient of Fourier series
 $f_T(t)$ = carrier wave
 T = period of sine wave
 Tc = period of carrier wave
 t_1 = open delay time
 t_2 = open moving time
 t_3 = close delay time
 t_4 = close moving time

INTRODUCTION

A high-speed on-off valve is the key element in a PWM fluid power servo system. It has great influence on the control accuracy, the bandwidth and the stability of the system. It is, therefore, necessary to study its static and dynamic behaviors.

Generally, the characteristics of a high-speed on-off valve is described by its on

and off delay time and the moving time of the valve piston respectively. These four parameters reflect its performances completely and precisely. Especially, they are clear in physical meaning and easy to be obtained by experiments. The static characteristics of a PWM high-speed on-off valve, that is the relationship between the output duty ratio and the input one has, already been widely studied. The influences of its switching behavior and the carrier frequency on the static characteristics have been discovered^{[1][2]}. So this paper focuses on the dynamic one.

The frequency domain method is commonly used in the design and the correction of a fluid power servo system. The premise for utilizing this method is that transfer functions or the describing functions of all the elements are known. The describing function of the PWM high-speed on-off valve, therefore, must be discovered in order to use the frequency domain method. As mentioned above, its characteristics is described by the four time domain parameters, and no direct relationship between them can be seen. This paper tries to propose a method by which the relationship between them can be discovered. Based on this method, the influences of the switching behavior of the valve and the carrier frequency on its frequency response characteristics are fully studied. And some conclusions are obtained, which are useful for the designer to identify the high-speed on-off valve correctly during his designing.

A natural sampling analog pulse width modulator with both edges modulated can be divided into three parts, which are an adder, a relay and a carrier wave which is generally a triangular wave^[3]. Fig 1 shows the construction of a PWM high-speed on-off valve, where $f_T(t)$ is the triangular wave with a period T_c and an amplitude V_s .

The transfer characteristics of the pulse width modulator can be expressed as equation (1).

$$U = \text{sgn}[E + f_T(t)] \quad (1)$$

Fig 1. The construction of PWM high-speed on-off valve

CALCULATION METHOD OF THE DESCRIBING FUNCTION

Assumption

Working range of the valve: The PWM high-speed on-off valve works in the linear modulated range in the normal working state. So only the describing function in this range is studied.

If the amplitude of the input sine wave is A , the maximum and the minimum duty ratio can be given as follows:

$$\tau_{max} = \frac{A}{2V_s} + \frac{1}{2} \quad (2)$$

$$\tau_{min} = -\frac{A}{2V_s} + \frac{1}{2} \quad (3)$$

If the high-speed on-off valve works in the linear modulated range, the following inequalities must be satisfied^[2].

$$\tau_{max} < 1 - (\tau_3 + \tau_4) \quad (4)$$

$$\tau_{min} > \tau_1 + \tau_2 \quad (5)$$

The amplitude of the input sine wave, therefore, must satisfy the following inequality.

$$A < V_s \cdot \text{Max}\{[1 - 2(\tau_1 + \tau_2)], [1 - 2(\tau_3 + \tau_4)]\} \quad (6)$$

The Period of the Input Signal: It is known from equation (1) that $U(t)$ is a periodic signal with a period of T if and only if the ratio between T and the period of the carrier

wave Tc is an integer. For the Fourier series method is used in the calculation of the describing function, only those at the points that $U(t)$ is a periodic function can be calculated.

Switching Behavior of the High-Speed On-Off Valves: It can be easily verified that when $T = nTc$ and the inequality (6) are satisfied, there will be $2n-1$ roots in the interval $[0, T)$ for equation (7).

$$f_T(t) + A \sin \omega t = 0 \quad (7)$$

Without loss generality, assume they are $t_0, t_1, t_2, \dots, t_{2n-2}$. Furthermore, assume $U(t)$ changes from 0 to h at the points t_{2i} and return to 0 at the points t_{2i+1} . The width of the positive pulse is, therefore, given by equation (8).

$$\Delta t_i = t_{2i+1} - t_{2i} \quad (8)$$

The piston of the on-off valve cannot follow the pulse strictly and at any time because of its inertia and the delay of the solenoid. Generally, its switching behavior is described by parameters t_1, t_2, t_3, t_4 as Fig 2.

Figure 2. Switching behavior of high-speed on-off valve

During the calculation the assumption that the motion of the piston in the open and close processes is an uniform speed one. The accuracy of the approximation has already been verified by many researches. Thus, the motion of the piston can be given as follows:

$$x_v = \begin{cases} 0 & t_{2i} \leq t \leq t_{2i} + t_1 \\ \frac{x_m}{t_2} (t - t_{2i} - t_1) & t_{2i} + t_1 \leq t \leq t_{2i} + t_1 + t_2 \\ x_m & t_{2i} + t_1 + t_2 \leq t \leq t_{2i+1} + t_3 \\ -\frac{x_m}{t_4} (t - t_{2i+1} - t_3 - t_4) & t_{2i+1} + t_3 \leq t \leq t_{2i+1} + t_3 + t_4 \\ 0 & t_{2i+1} + t_3 + t_4 \leq t \leq t_{2i+2} \end{cases} \quad (9)$$

Calculation Method

When a high-speed on-off valve works in PWM mode, its piston must follow $U(t)$ and work in the synchronized forced oscillation state. If $U(t)$ is a periodic function, X_v will also be a periodic one with the same period as $U(t)$. X_v , therefore, can be expanded into a Fourier series in the interval $[0, T]$. It will converge to X_v because the convergent condition is satisfied. The coefficients of the Fourier series are given as follows.

$$a_m = \frac{2}{T} \int_0^T x_v(t) \cos m \omega t dt \quad (10)$$

$$b_m = \frac{2}{T} \int_0^T x_v(t) \sin m \omega t dt \quad (11)$$

Both $X_v(t)$ and $\sin \omega t$ are with a period of T , therefore, there exists the equation (12).

$$\int_0^{t_0} x_v(t) \sin m \omega t dt = \int_T^{T+t_0} x_v(t) \sin m \omega t dt \quad (12)$$

There also exists the equation (13) for the same reason.

$$\int_0^{t_0} x_v(t) \cos m \omega t dt = \int_T^{T+t_0} x_v(t) \cos m \omega t dt \quad (13)$$

The expression of a_m and b_m then can be changed to equation (14) and (15).

$$a_m = \frac{2}{T} \int_{t_0}^{T+t_0} x_v(t) \cos m \omega t dt \quad (14)$$

$$b_m = \frac{2}{T} \int_{t_0}^{T+t_0} x_v(t) \sin m \omega t dt \quad (15)$$

The a_m and b_m can be obtained by substituting the equation (9) into (14) and (15) respectively.

$$a_m = \frac{2}{T} \sum_{n=1}^{\infty} \left\{ \sum_{i=0}^{n-1} \frac{x_m}{t_2 (m\omega)^2} [\cos m \omega (t_{2i} + t_1 + t_2) - \cos m \omega (t_{2i} + t_1)] \right. \\ \left. - \frac{x_m}{t_4 (m\omega)^2} [\cos m \omega (t_{2i+1} + t_3 + t_4) - \cos m \omega (t_{2i+1} + t_3)] \right\} \quad (16)$$

$$b_m = \frac{2}{T} \sum_{n=1}^{\infty} \left\{ \sum_{i=0}^{n-1} \frac{x_{rm}}{t_2(m\omega)^2} [\sin m\omega(t_{2i} + t_1 + t_2) - \sin m\omega(t_{2i} + t_1)] \right. \\ \left. - \frac{x_{rm}}{t_4(m\omega)^2} [\sin m\omega(t_{2i+1} + t_3 + t_4) - \sin m\omega(t_{2i+1} + t_3)] \right\} \quad (17)$$

Because the describing function is determined by a_1 and b_1 , the amplitude and phase frequency response characteristics can be obtained from the equation (18) and (19).

$$A_i(\omega) = 20 \log \frac{\sqrt{a_1^2 + b_1^2}}{A} \quad (18)$$

$$\theta_i(\omega) = \arctg \frac{a_1}{b_1} \quad (19)$$

The remaining problem is how to determine the roots $t_0, t_1, t_2, \dots, t_{2n-2}$ of equation (7). It is true that there is no way to express them in equations. However, with the help of a computer and by means of the half interval search algorithm it is very simple to find them. The describing function of an analog PWM high-speed on-off valve, thus, can be obtained.

SIMULATION RESEARCH

The influence of the carrier frequency, the initial phase of the input sine wave and the switching behavior of the high-speed on-off valve on the frequency response characteristics are studied by means of the calculation method.

First, the influence of the initial phase of the input sine wave is studied. During the calculation, the carrier frequency is 100Hz, the amplitude of the carrier wave V_s is 10V, the amplitude of the input sine wave A is 4V and $t_1 = t_3 = 1\text{ms}$, $t_2 = t_4 = 0.3\text{ms}$. Fig 3 illustrates the simulation results when the initial phase are $0^\circ, 90^\circ, 180^\circ$ and 270° respectively. It can be seen that the initial phase of the input has no influence on both the amplitude bandwidth

and the phase lag when $\omega \leq \frac{\omega_c}{4}$.

Figure 3. Frequency response characteristic at different initial phase

Secondly, the influence of the carrier frequency is studied. During the calculation, the initial phase of the sine wave is 0° , the amplitude of the carrier wave V_s is 10V, the amplitude of the input sine wave A is 4V and $t_1 = t_3 = 1\text{ms}$, $t_2 = t_4 = 0.3\text{ms}$. Fig 4 shows the simulation results when the carrier frequency is 50Hz, 100Hz and 200Hz respectively. It can be seen that the bandwidth is $\frac{f_c}{4}$, where f_c is the carrier frequency.

Figure 4. The relationship between bandwidth and carrier frequency

Thirdly, the influence of the switching behavior of the high-speed on-off valve is studied. During the calculation, the carrier frequency is 100Hz, the amplitude of the carrier wave V_s is 10V, the amplitude of the input sine wave A is 4V and the initial phase of the input sine wave is 0° . Fig 5 illustrates the

simulation results with different switching behaviors. In the first curve the four parameters are $t_1 = t_3 = 1\text{ms}$, $t_2 = t_4 = 0.3\text{m}$. In the second curve $t_1 = 0.4\text{ms}$, $t_3 = 0.5\text{ms}$ and $t_2 = t_4 = 0.3\text{m}$. In the third curve $t_1 = 1.5\text{ms}$, $t_3 = 2.0\text{ms}$ and $t_2 = t_4 = 0.3\text{m}$. It can be seen that the switching behavior just affect the phase lag and has no influence on the amplitude bandwidth. From large amounts of simulation results it is discovered that the phase lag can be expressed by equation (20) when $T < 4T_c$.

$$\varphi_d = \frac{2t_1 + 2t_3 + t_2 + t_4}{4} \omega \quad (20)$$

CONCLUSIONS

On the basis of the simulation the following conclusions can be drawn.

1. The bandwidth of a PWM high-speed on-off valve working in the linear modulated range is $\frac{f_c}{4}$.

2. The switching behavior of the high-speed on-off valve only affect the phase lag.
3. The describing function of a PWM high-speed on-off valve can be simplified to a pure delay when the period of the input sine wave satisfies $T < 4T_c$.

Figure 5. Frequency response with different switching behaviors

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