Optimum design of bearing/seal parts for hydraulic equipment on mixed lubrication

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ABSTRACT

Reasonable design criteria of the optimum size of bearing/seal parts of hydraulic equipment on mixed to fluid film lubrication are presented. We treat circular hydrostatic thrust bearings as the most fundamental model for the bearing/seal parts of hydraulic equipment. The basic equations considering metallic contact and eccentric loads are derived. The effects of the surface roughness, the ratio of pocket radius, and the radius of restrictors on the optimum size are clarified. The results are arranged by the concept of the ratio of hydrostatic balance. We show that the optimum criterion corresponding to the minimum power loss is given by the ratio of hydrostatic balance being close to unity.

KEYWORDS

Optimum design, Mixed lubrication, Hydrostatic bearings, Hydraulic equipment, Power loss

NOMENCLATURE

a = ratio of pocket radius $=R_1/R_2$	M = moment load
A = contact area	N = speed of rotation
d_e^* = separation $=d_e/\sigma^*$	p = pressure
E' = equivalent elastic modulus	p_r, p_s = pocket and supply pressure
f = coefficient of friction	Q = flow rate
$\bar{h}$ = film thickness $=h/H$	$\bar{r}, \theta$ = coordinate $=r/R_2, \theta$
$h_s = h/\sigma$	R_1, R_2 = pocket and outer radii
H = representative film thickness	$S_0 = 6\mu\omega(R_2/H)^2/p_s$
$\bar{L}$ = power loss $=L/[6\mu\omega^2R_2^3(R_2/H)^2]$	T = frictional torque
	W = load; load carrying capacity

$\bar{x}_w$ = eccentricity of load $= x_w/R_2$

$\bar{\alpha}$ = tilting angle $= \alpha R_2/H$

δ = roughness

ζ = ratio of hydrostatic balance

μ = fluid viscosity

σ = equivalent standard deviation of surface

τ = shear stress, time $= \omega t$

φ = azimuth of maximum tilting angle

Subscripts

a, f = asperity and fluid, respectively

1, 2 = stationary and rotator, respectively

INTRODUCTION

Hydraulic equipment is operated under the conditions of high-pressure and wide-speed ranges. From industry applying hydraulic equipment for automation and energy saving, much lighter and smaller equipment is requested strongly. Furthermore, from the viewpoint of fire prevention and environmental protection, low viscous fluids, such as tap water, seawater, and high water content fluid (HWCF) are promoted to use as hydraulic fluids. On the other hand, the computer-aided design (CAD) of hydraulic equipment is also required. Therefore, it is necessary to clarify the optimum conditions and the design criteria of the bearing/seal parts of hydraulic equipment.

As to the optimum design criteria of bearings, there are some studies on thrust bearings [1], journal bearings [2], and so forth. These optimum conditions are based on the maximum stiffness in the fluid film lubrication range. Then, it is irrational to apply directly

these criteria to the bearing/seal parts of hydraulic equipment.

With regard to the design criteria of the bearing/seal parts of hydraulic pumps and motors, there are some studies discussed the minimum power losses for slipper bearings on fluid film lubrication [3] and on mixed lubrication [4], and the maximum stiffness and the minimum leakage flow rate for pistons [5]. However, it is necessary to discuss much more generalized design criteria.

In this paper, we treat hydrostatic thrust bearings as the bearing/seal parts of hydraulic equipment on mixed to fluid film lubrication. We clarify numerically the optimum design criteria of the bearings for the surface roughness, the ratio of the pocket radius, and the radius of the restrictors.

BASIC EQUATIONS

Consider circular hydrostatic thrust bearings with a restrictor as shown in Fig. 1.

FIGURE 1 Schematic diagrams and coordinates

The mixed lubrication model [6] is based on both of the asperities contacting model proposed by Greenwood and Williamson (the GW model) [7] and the mean flow model proposed by Patir and Cheng (the PC model) [8], and also included the effects of adsorption of lubricants, elastohydrodynamic lubrication (EHL), and cavitation due to asperities. It agrees well with many experimental results, and it is shown that the reliability of the model is reasonable.

The assumptions in this paper are: i) the roughness δ_1 and δ_2 are the normal distributions with the standard deviations σ_1 and σ_2 , respectively, and $\delta = \delta_1 + \delta_2$, $\sigma^2 = \sigma_1^2 + \sigma_2^2$; ii) the roughness is isotropic; iii) the configuration of an asperity close to a summit is a sphere; iv) the reference plane is rigid; v) the effects of heat and elastic deformation [9] are negligible, and vi) fluid is incompressible and its viscosity remains constant.

Load carrying capacity

For an area ΔA_0 , the load carrying capacities ΔW_a , ΔW_{ae} , and ΔW_{ap} are

$$\left. \begin{aligned} \Delta W_a &= \Delta W_{ae} + \Delta W_{ap} \\ \Delta W_{ae} &= (2/3)E'(\eta\beta'\sigma^*)(\sigma^*/\beta')^{1/2} \\ &\quad \times [F_{3/2}(d_e^*) - F_{3/2}(d_e^* + w_p^*)] \Delta A_0 \\ \Delta W_{ap} &= \pi H_a(\eta\beta'\sigma^*)F_1(d_e^* + w_p^*) \Delta A_0 \end{aligned} \right\} \quad (1)$$

The load carrying capacity due to fluid is given by solving the Reynolds equation in the cylindrical coordinates $(\bar{r}, \theta, \bar{z})$ (see Appendix)

$$\begin{aligned} & \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\phi \bar{h}^3 \bar{r} \frac{\partial p^*}{\partial \bar{r}} \right) + \frac{1}{\bar{r}^2} \frac{\partial}{\partial \theta} \left(\phi \bar{h}^3 \frac{\partial p^*}{\partial \theta} \right) \\ &= 6\mu\omega \left(\frac{R_2}{H} \right)^2 \left(\frac{\partial \bar{h}_T^*}{\partial \theta} - \frac{\sigma}{H} \frac{\partial \phi_s}{\partial \theta} + 2 \frac{\partial \bar{h}_T^*}{\partial \tau} \right) \end{aligned} \quad (2)$$

where ϕ is a Couette shear factor and ϕ_s is a shear flow factor [8].

Friction torque

The friction torque due to asperities T_a is

$$T_a = \iint_{A_0} R_2 \bar{r} \Delta F_a \quad (3)$$

where

$$\begin{aligned} \Delta F_a &= \xi \tau_p [(\tau_s/\tau_p) \Delta A_e + \Delta A_p] \\ &\quad + (1-\xi) \tau_{ad} \Delta A_r \end{aligned} \quad (4)$$

$$\xi = \phi' \exp \left[- \frac{(h_{ad}/a_r)^m}{\Delta A_r / \Delta A_0} \right] \quad (5)$$

where $\phi' = (\text{coefficient of kinematic friction under boundary lubrication}) / (\text{that under dry friction}) = 0.27$ (for steel), $a_r = \text{mean radius of asperities contacting} = [\beta'\sigma^*F_1(d_e^*)/F_0(d_e^*)]^{1/2}$, $h_{ad} = \text{EHL film thickness of point contact}$, $\Delta A_r/\Delta A_0 = \text{ratio of contacting area under dry friction} = \pi(\eta\beta'\sigma^*)F_1(d_e^*)$, and $m = 0.67 + 1.4\sigma$ (σ : μm).

Following the PC model, the friction torque due to fluid acting on a rotator T_{f2} is

$$\begin{aligned} T_{f2} &= - \frac{\mu\omega R_2^4}{H} \int_0^{2\pi} \int_a^1 \frac{\bar{r}^3}{\bar{h}} \\ &\quad \times [\phi + (1-2\sigma_2^2/\sigma^2)\phi_s] d\bar{r} d\theta \end{aligned} \quad (6)$$

Flow rates and pocket pressure

In the case of an orifice restrictor, the flow rates through the restrictor Q_{in} and into the bearing/seal parts Q_s are

$$Q_{in} = \pi c_0 r_0^2 \sqrt{2(p_s - p_r)/\rho} \quad (7)$$

$$Q_s = -a_0 \left[\frac{p_r - p_e}{\log a} + \frac{3\mu\omega}{a_0} \left(\frac{R_2}{H} \right)^2 \frac{\partial c_0}{\partial \tau} \left(\frac{1-a^2}{\log a} + 2a^2 \right) \right] \quad (8)$$

$$- \bar{\alpha}^2 a_1 \left[\frac{A^2}{8} a^2 (a^2 - 1) + \frac{C}{4} a^2 \left(1 - \frac{2 \log a}{a^2 - 1} \right) \right]$$

The continuity equation of incompressible fluid is

$$Q_{in} = Q_s + \pi \omega a^2 H R_2^2 \frac{\partial \bar{h}_T}{\partial \tau} \quad (9)$$

The pocket pressure p_r is given by substituting Eqs.(7) and (8) into Eq.(9).

Power losses, coefficient of friction, and ratio of hydrostatic balance

The power loss L and the coefficient of friction f are defined by

$$L = p_s Q_s + \omega (T_a - T_f) \quad (10)$$

$$f = (T_a + T_f)/(-RW) \quad (11)$$

where R =mean radius $=(R_1+R_2)/2$.

For the bearing/seal parts of hydraulic equipment, the loads are mainly supported by the hydrostatic pressure. Now, we introduce the ratio of the load to the maximum hydrostatic load carrying capacity ζ defined by

$$\zeta = \frac{W}{\pi p_s R_2^2 (1-a^2)/(2 \log a)} \quad (12)$$

According to this definition, $\zeta < 1$ implies the fluid film lubrication and $\zeta > 1$ implies the mixed lubrication (in the case of metallic contact).

The examples of the numerical parameters are: $a=0.7$, c_0 =discharge coefficient=0.6, $E'=200$ GPa, $H_1=1.75$ GPa, $N=50$ s⁻¹, p_e =ambient pressure =0 MPa, $p_s=10$ MPa, r_0 =radius of restrictor=0.5 mm, $R_2=10$ mm, $\beta'=0.05$ mm, γ =viscosity-pressure coefficient=22.8 GPa⁻¹, $\eta\beta'\sigma^*=0.05$, $\mu=27.4$ mPa·s, ρ =density=860 kg/m³, $\sigma_1=\sigma_2=0.5$ μm, $\tau_{ad}=15$ MPa.

RESULTS AND DISCUSSIONS

Figure 2 shows the effects of the ratio of hydrostatic balance ζ on the film thickness h_s , the ratio of real contact area to apparent area A_r/A_0 , the power losses due to leakage flow rate $\bar{L}_Q$ and friction torque $\bar{L}_T$, and the ratio of pocket pressure p_r/p_s . h_s decreases with increasing ζ . For $\zeta \approx 1$, h_s changes remarkably because the asperities begin to contact and support the loads. For hydrostatic bearings, we can recognize that this is the boundary between the fluid film and mixed lubrication ranges. For $\zeta < 1$, h_s is very large because the bearing stiffness due to a restrictor is very small. A_r/A_0 increases constantly with ζ in the mixed lubrication range. p_r/p_s increases with ζ in the fluid film lubrication range and keeps nearly equal to unity in the mixed lubrication range. $\bar{L}$ is deeply dependent on $\bar{L}_Q$ and $\bar{L}_T$ for $\zeta < 1$ and $\zeta > 1$, respectively.

FIGURE 2 Effects of ζ FIGURE 3 Effects of σ_2/σ_1 ($\sigma_1=0.5 \mu\text{m}$) FIGURE 4 Effects of a FIGURE 5 Effects of r_c

Figure 3 shows the effects of roughness σ_2/σ_1 on the power loss $S_0\bar{L}$ and the coefficient of friction f . Any σ_2/σ_1 , $S_0\bar{L}$ in the case of $\zeta=1.5$ is the largest and that of $\zeta=1.1$ is the smallest. For $\zeta=1.1$, $S_0\bar{L}$ and f become the minimum at $\sigma_2/\sigma_1 \sim 0.5$. Therefore, this is the optimum design criterion for surface roughness.

Figure 4 shows the effects of the ratio of pocket radius a on $\bar{L}$ and f . $\bar{L}$ becomes the minimum at $\zeta \sim 1$ (see Fig. 2). For the same loads, the larger a , the smaller $\bar{L}$ and f on $\zeta > 1$.

Figure 5 shows the effects of the radius of restrictors r_c on $\bar{L}$ and h_{0s} . The smaller r_c , the smaller $\bar{L}$ on fluid film lubrication (no effects of r_c on mixed lubrication). However, considering the silting of the restrictors by contaminants in hydraulic fluids and the operating on mixed lubrication, we recommend to select the larger r_c .

CONCLUDING REMARKS

Reasonable design criteria of the optimum size of bearing/seal parts of hydraulic equipment

on mixed to fluid film lubrication are presented. The minimum power loss is given in the ratio of hydrostatic balance being unity in all cases. The size of a restrictor does not influence power losses on mixed lubrication. Therefore, we recommend the optimum design criterion that the ratio of hydrostatic balance is set to slightly larger than unity.

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APPENDIX

We apply the method of perturbation with the parameter $\bar{\alpha}$ to solve Eq.(1). The factors ϕ , ϕ_s , and the mean fluid film thickness (expectation) h_T^* are approximated by the polynomial functions of the fourth order of h_s , that is

$$\left. \begin{aligned} \phi &= A_0 + A_1 h_s + A_2 h_s^2 + A_3 h_s^3 + A_4 h_s^4 \\ \phi_s &= (\sigma_1^2 - \sigma_2^2) / \sigma^2 \\ &\quad \times (B_0 + B_1 h_s + B_2 h_s^2 + B_3 h_s^3 + B_4 h_s^4) \\ h_T^* &= h + \sigma (C_0 + C_1 h_s + C_2 h_s^2 + C_3 h_s^3 + C_4 h_s^4) \end{aligned} \right\} \quad (A1)$$

The pressure $p^* = p_0^* + \bar{\alpha} p_1^* + \dots$, where

$$\begin{aligned} p_0^* &= \frac{\log \bar{r}}{\log a} p_r + \left(1 - \frac{\log \bar{r}}{\log a} \right) p_e \\ &\quad + \frac{3\mu\omega}{a_0} \left(\frac{R_2}{H} \right)^2 \left[\frac{(1-a^2)\log \bar{r}}{\log a} + \bar{r}^2 - 1 \right] \frac{\partial c_0}{\partial \tau} \end{aligned} \quad (A2)$$

$$\begin{aligned} p_1^* &= \frac{A}{8} \left[\bar{r}^3 - (1+a^2)\bar{r} + \frac{a^2}{\bar{r}} \right] \cos(\varphi - \theta) \\ &\quad + \frac{B}{8} \left[\bar{r}^3 - (1+a^2)\bar{r} + \frac{a^2}{\bar{r}} \right] \sin(\varphi - \theta) \\ &\quad + \frac{C}{2} \left(\bar{n} \log \bar{r} - \frac{a^2 \log a}{a^2 - 1} \bar{r} + \frac{a^2 \log a}{a^2 - 1} \frac{1}{\bar{r}} \right) \\ &\quad \times \cos(\varphi - \theta) \end{aligned} \quad (A3)$$

The load carrying capacity W_f , and moment load carrying capacities M_{fx} and M_{fy} are

$$\begin{aligned} W_f &= R_2^2 \int_0^{2\pi} \int_a^1 p^* \bar{r} \left(1 - \frac{\Delta A_r}{\Delta A_0} \right) d\bar{r} d\theta \\ &\quad + \pi R_2^2 (a^2 p_r - p_e) \end{aligned} \quad (A4)$$

$$M_{fx} = R_2^3 \int_0^{2\pi} \int_a^1 p^* \bar{r}^2 \sin \theta \left(1 - \frac{\Delta A_r}{\Delta A_0} \right) d\bar{r} d\theta \quad (A5)$$

$$M_{fy} = -R_2^3 \int_0^{2\pi} \int_a^1 p^* \bar{r}^2 \cos \theta \left(1 - \frac{\Delta A_r}{\Delta A_0} \right) d\bar{r} d\theta \quad (A6)$$