The Effect of Nonlinear Characteristics on Pneumatic Servo Systems
(Lap Condition Effect on Sinusoidal Action)

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ABSTRACT

Pneumatic servo systems are used on fatigue machines and robots. Although, those systems have nonlinear elements which make the motion control difficult. On the sinusoidal movements, the non-linearity of the servo valves, especially the lap condition of the valves, is considered to cause a bad effect on the position control. We have already proposed a new method to measure the static characteristics of servo valves including the lap condition using an isothermal chamber. In this paper, firstly the lap condition of a servo valve is measured by the proposed method. Secondly, the effect of the lap condition of the servo valve during sinusoidal position control on a pneumatic cylinder is investigated. Finally, we compensate the effect of the lap condition of the valve using repetitive control.

KEY WORDS

Pneumatic system, Position control, Servo valve, Lap condition, Repetitive control

NOMENCLATURE

<table>
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<tr>
<th>Symbol</th>
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<tr>
<td>A</td>
<td>cross section area of cylinder [m²]</td>
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<td>B</td>
<td>friction coefficient [Ns/m]</td>
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<td>f</td>
<td>frequency [Hz]</td>
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<tr>
<td>G, G1, G2</td>
<td>mass flow rate [Kg/s]</td>
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<td>k</td>
<td>proportional constant [Kg/(s Pa mm²)]</td>
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<tr>
<td>k, k'</td>
<td>position feedback gain [V/m]</td>
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<tr>
<td>Y0</td>
<td>room temperature [K]</td>
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<td>L</td>
<td>time delay [s]</td>
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<tr>
<td>m</td>
<td>mass [Kg]</td>
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<tr>
<td>P1, P2</td>
<td>pressure [Pa]</td>
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<td>P0</td>
<td>initial pressure [Pa]</td>
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<tr>
<td>P_s</td>
<td>supply pressure [Pa]</td>
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<td>R</td>
<td>ideal gas constant [Pa m²/(Kg K)]</td>
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<tr>
<td>S_e</td>
<td>effective area of tested valve [mm²]</td>
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<tr>
<td>T_v</td>
<td>velocity feedback gain [V/(m/s)]</td>
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<tr>
<td>T_a</td>
<td>acceleration feedback gain [V/(m/s²)]</td>
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<tr>
<td>u</td>
<td>input voltage to valve [V]</td>
</tr>
<tr>
<td>x, x_r</td>
<td>displacement of piston [m]</td>
</tr>
<tr>
<td>x_r</td>
<td>reference displacement [m]</td>
</tr>
<tr>
<td>∂0</td>
<td>room temperature [K]</td>
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1. INTRODUCTION

Pneumatic cylinders are used as actuators on fatigue machines and robots. The pneumatic actuators are capable of providing high power output levels at a relatively low cost. Also the pneumatic actuators have advantages of clean energy. Recently, pneumatic servo valves having high dynamic responses have been developed and become to use[1]. Although, the pneumatic systems have nonlinear elements which make the motion control difficult. On the sinusoidal movements, the non-linearity of the servo valves, especially the lap condition of the valves is considered to cause a bad effect on the position control. We have proposed a new method to measure the static characteristics of servo valves including the lap condition using an isothermal chamber. The isothermal chamber is a chamber in which steel wool is stuffed to make the heat transfer area larger and can almost realize isothermal condition. We have already shown that steady and unsteady flow rates of air could be measured with the chamber[2].

In this paper, firstly the lap condition of a servo valve used to drive a pneumatic cylinder is measured by the proposed method. Secondly, the effect of the lap condition of the servo valve during sinusoidal position control on the pneumatic cylinder is investigated. Finally, we compensate the effect of the lap condition of the valve using repetitive control.

2. CHARACTERISTIC MEASUREMENT OF SERVO VALVES

2.1 Method

Fig. 1 shows the test circuit for 3 ports valves. The apparatus consists of a chamber used to restrain the change of the supply pressure, the tested valve, the function generator, the isothermal chamber and the pressure sensor. It was already shown in our previous study[2] that both steady and unsteady flow rate of air can be obtained only by measuring the pressure change in the isothermal chamber. The measured pressure data was taken into a personal computer through an AD converter.

Method used to measure the input-output characteristic is as follows: At first, we set the supply pressure at 600[kPa]. Then, the input voltage, which was a sinusoidal signal, is given from the function generator to the tested valve. The amplitude of the voltage is from the neutral to the maximum point's width. The frequency of the input voltage is selected at the value which must not be effected by the dynamic response of the valve. When the input voltage is given to the tested valve, the pressure in the isothermal chamber changes sinusoidally.

At this time, we must measure the input voltage and the pressure in the isothermal chamber at least one period. The measured data was taken into the personal computer in the real time through the AD converter. At this time, the sampling time was selected to get at least a hundred point on one cycle. After the measurement, the flow rate G which passed through the valve could be given by differentiating the pressure.

In this study, the differentiation was done in the following procedure. At first, the measured data was smoothed using a low pass filter. The cut off frequency of the filter was set at 12 times larger value than that of the data which would not show any delay in phase. The noise level of the pressure transducer was smaller than the resolution of the AD converter. Then, the smoothed data was differentiated with respect to the time using 5 points of the data. The effective area of the tested valve is obtained from the flow rate and the pressure. The following equation which is defined in the ISO standards is used to calculate the effective area.

\[
G = k_S P_d \left( \frac{273}{\theta_0} \right)^{0 < x < b}
\]

\[
G = k_S P_d \left( 1 - \frac{x - b}{1 - b} \right)^2 \left( \frac{273}{\theta_0} \right)^{x > b}
\]

(1)
In Eq.(1), $P_u$ indicates the upstream pressure and $P_d$ indicates the downstream pressure. The input-output characteristic can be obtained by plotting the input voltage and effective area $S_e$. Although, the tested valve has 5 ports, only 3 ports were used. We have already proved that the proposed method is useful by comparing the results obtained by the proposed method with the results obtained by the ISO method[2].

2.2 Static characteristic of the tested valve

The static characteristic of the tested servo valve was measured by the proposed method. The frequency of the inputted sinusoidal voltage was set at 1 [Hz]. The input – output characteristics of the tested valve obtained from the method is shown in Fig.2.

The solid lines on Fig.2 are the results obtained by the proposed method. The black dots are the results measured by the method defined in the ISO standards[3] using the area type flowmeter of which the accuracy is 2%. It is known from Fig.2 that the results obtained from the proposed method and the defined method show good agreement. The effectiveness of the proposed method is clear. Even when the tested valve has overlap characteristic, the proposed method can measure the characteristic very well. The overlap characteristic of the tested servo valve is considered to cause a bad effect on the sinusoidal motion on the pneumatic servo system.

![Fig.2 Input-output characteristic of the tested servo valve]

3. PNEUMATIC SERVO SYSTEM

3.1 Experimental apparatus

Fig.1 shows the experimental apparatus for the position control of the pneumatic servo system. The apparatus consists of a pneumatic cylinder, a servo valve, an encoder and a computer. The full stroke of the cylinder is 300 [mm] and the inside radius is 32 [mm]. The servo valve has dynamic response of about 100 [Hz]. The weight of mass installed on the cylinder is 3 [kg]. The supply pressure was set at 600 [kPa]. The position of the cylinder was measured by the encoder and was taken into computer through an AD converter. The control signals were calculated in the computer which were given to the servo valve through a DA converter. The experiments were done against sinusoidal position movement.

3.2 Linearized model

In order to derive the model of the pneumatic servo systems, which is useful on designing the controller, the following assumptions are made: (1) the thermofluid processes are adiabatic, (2) the bandwidth of the valve is much higher than that of the closed loop system, (3) the characteristic of the valve can be linearized, (4) the cylinder is symmetric, (5) initial condition is at the mid stroke, (6) Coulomb friction is negligible. As a result, the energy, the linearized flow and the kinematics equation become

\begin{align}
G_1 &= \frac{1}{R\theta_0} \left( \frac{P_0 A}{\kappa} \frac{dx}{dt} + \frac{V_0}{\kappa} \frac{dP_1}{dt} \right) \\
G_2 &= \frac{1}{R\theta_0} \left( -\frac{P_0 A}{\kappa} \frac{dx}{dt} + \frac{V_0}{\kappa} \frac{dP_2}{dt} \right)
\end{align}
\[ G_1 = k_u u - k_P P_1 \]  
\[ G_2 = -k_u u - k_P P_2 \]  
\[ m \ddot{x} + B \dot{x} = A(P_1 - P_2) \]

respectively. From Eq.(2) to Eq.(4), the transfer function of the pneumatic servo cylinder from \( u \) to \( x \) can be written as

\[
G(s) = \frac{2 \cdot \frac{\kappa}{m V_0} \cdot k_e \cdot R \theta_0 \cdot A}{s \left( s^2 + \left( B \frac{\kappa R \theta_0}{m V_0} \right) s + \frac{\kappa}{m V_0} \left( B R \theta_0 k_e + 2 P_s A^2 \right) \right)} = \frac{c}{s(s^2 + as + b)} = \frac{x}{u} \]

It is known that the transfer function of the pneumatic servo cylinder is written as the 3rd order system as shown in Eq.(6)[4]. The coefficient of linearized valve characteristic \( k_p \) and \( k_u \) was determined from the experimental result shown in Fig.2. The friction coefficient \( B \) was determined experimentally. The calculated results of each parameter on Eq.(6) become \( a = 19.8, b = 1263, \) and \( c = 180. \)

### 3.3 Frequency response of the open loop system

To investigate the accuracy of linearized model shown in Eq.(6), the frequency response of the pneumatic servo cylinder was measured. The sinusoidal signal was input to the servo valve and the displacement of the cylinder was measured. Hence, the cylinder was moved on the open loop system. As the servo valve have the over lap characteristic, the displacement is not a perfect sinusoidal motion. Although, gains and phases were measured.

The experimental results were summarized on the bode diagram shown in Fig.4. The upper figure shows the gain and the lower figure shows the phase. The solid lines show the gain and the phase of the linearized model indicated on Eq.(6). The black dots show the experimental results. Although, the pneumatic system has nonlinear elements, the linearized model and the experimental results show good agreements. It is clear from Fig.4 that the linearized model can represent the pneumatic servo system very well. Therefore, the linearized transfer function Eq.(6) is useful on designing the controller.

### 4. DESIGN OF CONTROLLER

#### 4.1 PDD\(^2\) control

At first, PDD\(^2\) controller was used for the motion control, i.e., piston position, piston velocity and piston acceleration are used for state feedback. The information of the velocity and the acceleration was obtained by differentiating the positional signal. Fig.5 shows the block diagram of the control system. The repetitive control part in Fig.5 will discuss later. The control signal becomes

\[ u = K_p (x_r - x) + T_v \dot{x} + T_a x \]  

at the PDD\(^2\) control. Therefore, the closed loop transfer function from \( x_r \) to \( x \) becomes

\[ \frac{x}{x_r} = \frac{c K_p}{s(s^2 + (c T_v + a)s + (c T_a + b)) + c K_p} \]

Optimal gains are selected by applying the following method. Firstly, the loop gain \( K_p \) is fixed at any constant value. It is known that a gain-phase margin diagram of 3rd order system is useful to estimate dynamic control performance.
By introducing dimensionless parameter $\alpha$ and $\beta$ into the transfer function Eq.(8), the next equation is obtained.

\[
\begin{align*}
\frac{x}{x_r} &= \frac{1}{s^3 + \alpha s^2 + \beta s + 1} \\
T_v &= K_p \alpha - \frac{a}{c} \\
T_a &= K_p \frac{b}{c}
\end{align*}
\]  
(9)

We know that $T_v$ and $T_a$ are the function of $K_p$, $\alpha$ and $\beta$. Therefore, the minor loop gains are obtained by selecting $\alpha = 2$ and $\beta = 3$ which realize desirable phase and gain margin for 3rd order system. We fixed the loop gain $K_p$ at first. Then, $T_v$ and $T_a$ are calculated from Eq.(9) which are 3.04[V/s/m] and 0.02[V s^2/m] respectively.

4.2 Repetitive control

Secondly, repetitive controller together with PDD^2 controller was adopted to improve the motion control. The block diagram of the control system is shown in Fig.5. The parameters of PDD^2 control part are same as that of calculated above. It is known from the small-gain theorem that repetitive controller will remain stable if

\[
\frac{F(s)}{1 + C(s)G'(s)} < 1
\]  
(10)

It is clear that the settling time will become shortest when $C(s)G'(s) = 1$. As we already know the parameters of $G'(s)$, we selected $F(s) = 10/(s+10)$ and $C(s) = (1+0.015s)(1+0.02s)$ in Fig.5.

5. EXPERIMENTAL RESULTS AND DISCUSSIONS

5.1 Sinusoidal motion control using PDD^2 Control

The sinusoidal motions of the pneumatic cylinder were investigated when the PDD^2 controller was used. Fig.6 shows the results when the target displacement was $0.15 + 0.05 \sin(2 \pi t)$ [m]. The upper figure shows the displacement and the lower figure shows the input voltage to the servo valve. The lateral axis shows the time from the beginning of the experiment. The broken line shows the reference displacement and the solid line shows the experimental result.

Fig.6 Frequency response using PDD^2 control
($K_p=111$[V/m], $T_v=3.04$[Vs/m], $T_a=0.02$[Vs^2/m], $f=1$[Hz])

Fig.7 Frequency response using PDD^2 control
($K_p=111$[V/m], $T_v=3.04$[Vs/m], $T_a=0.02$[Vs^2/m], $f=0.5$[Hz])
It is clear from Fig.6 that even the input signal changes gradually, the displacement does not follow to the reference displacement and keep constant position at the top and the bottom of the sinusoidal curves. Therefore, little delay in phase is seen. This is because of the over lap characteristic of the servo valve and the friction of the cylinder. Fig.7 shows the result when \( x_r = 0.15 + 0.05 \sin(\pi t) \) [m]. Only the displacement figure is shown. Same tendency as the results at the frequency of 1 [Hz] could be observed.

5.2 Sinusoidal motion control using PDD\(^2\)+Repetitive Control

It became clear from Fig.6 and Fig.7 that PPD\(^2\) control could not realize smooth sinusoidal movement on the pneumatic servo cylinder because of the over lap characteristics of the servo valve. Then, we used repetitive control together with PDD\(^2\) control to improve the movement. The block diagram of the system is shown in Fig.5. The repetitive control gives the difference from the target position, which is a cycle before this cycle, to the feedback loop repetitively. Hence, the movement could be modified and follow the target displacement as we desired after several cycles.

The experimental result at the frequency of 1 [Hz] is shown in Fig.8. The target displacement is same as Fig.6. First cycle in Fig.8 is 2nd cycle from the beginning of the repetitive control. The displacement of the experimental result is larger in amplitude than the target displacement. Even though, after several cycles, the result follow the target displacement very well. The difference at the top and the bottom, which is observed in Fig.6, is compensated in Fig.8. Fig.9 shows the results at the frequency of 0.5 [Hz]. It is clear that the repetitive control is useful to compensate the lap condition of valve on the sinusoidal motion on pneumatic servo cylinders.

6. CONCLUSIONS

The conclusions of this research are as follows:
1. It is known from the experiments that even the pneumatic servo cylinder has nonlinear characteristics, the linearized model can represent the system.
2. It is known that the over lap characteristic of the servo valve causes a bad effect on the sinusoidal motion on the pneumatic cylinder.
3. Using the repetitive control together with PDD\(^2\) control, the lap condition effect of valves can be compensated and we can realize smooth sinusoidal movement.

REFERENCES