A DESIGN TECHNIQUE OF HYDRAULIC SERVO ACTUATOR SYSTEMS FOR EFFECTIVE POWER TRANSMISSION

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ABSTRACT

This paper presents a methodology on how the drive characteristic curve should effectively enclose the load locus on the non-dimensional force-velocity plane, when the sinusoidal movement of a hydraulic actuator against the external force is applied to the load with a mass, damper, and spring. Based on the geometrical configuration, it has been analytically shown that the optimum design specification of the hydraulic servovalve and cylinder can simply be determined by the non-dimensional parameters corresponding to the load condition.

KEY WORDS

Electro-hydraulic servo mechanism, Hydraulic actuator, Load locus, Drive characteristics, Efficiency

NOMENCLATURE

\[ U : \] Driving velocity
\[ u : \] Load velocity
\[ y : \] Piston displacement
\[ \phi : \] Non-dimensional parameter given by Eq.(11)
\[ \eta : \] Overall efficiency
\[ \lambda : \] Bias rate
\[ \nu : \] Margin ratio
\[ \omega : \] Angular frequency

subscripts:
\[ m : \] Maximum value
\[ p : \] Value at or near peak power

superscript: \[ * : \] Non-dimensional value

INTRODUCTION

Electro-hydraulic servo mechanisms have the capability of transmitting high power at quick response. In the servo synthesis, it is necessary to determine the design
specification of the hydraulic servo, such as the supply pressure and flow capacity into the hydraulic flow control servovalve, and the actuator size, so that the power requirement satisfies the desired dynamic characteristics.

In general, a force versus velocity chart has been employed to make sure that the specification is suitable [1-3]. When the chart is drawn for the economic design, a drive characteristic curve of the hydraulic actuator should effectively enclose the load locus with as little overlap as possible. It seems to be troublesome to adjust the configuration of drive curve along the two coordinates so as to match each maximum power condition, as well as not to bulge at other points.

This paper conducts a non-dimensional force versus velocity diagram in order to illustrate how the load locus should lie wholly within the drive curve. Though a lot of drives with various kinds of loads can be considered such as vibratory and industrial machines, flight control surfaces and so on, it is assumed that a sinusoidal motion with a piston stroke and angular frequency, which also corresponds to the bandwidth in a system response is applied into the load. The drive curve is graphically selected on account of the effective and well matched enclosure to the load locus. Finally, as the stall force and runaway velocity of the drive characteristics can be expressed by non-dimensional parameters which are defined by these load conditions, the optimum design specification will be immediately determined.

LOAD LOCUS

Figure 1 shows a schematic diagram of the hydraulic servo actuator system. It consists of a critical lapped servovalve and a double-rod cylinder, which is connected to loads with a mass, a viscous damper, a spring and an external force. The basic equation on the load model is given by:

\[
f = M \frac{d^2 y}{dt^2} + B \frac{dy}{dt} + ky + f_o \text{sign}\left(\frac{dy}{dt}\right)
\]  

where, \(M\), \(B\), \(k\), and \(f_o\) respectively represent load mass, damping coefficient, spring stiffness and external force against the cylinder velocity.

When the load is driven by the sinusoidal motion with the angular frequency \(\omega\) and piston stroke \(a\) as follows:

\[
y = a \sin(\omega t)
\]

the load force \(f\) and velocity \(u\) yield:

\[
f = -a(M\omega^2 - k)\sin(\omega t) + B a \omega \cos(\omega t) + f_o \text{sign}(u)
\]

\[
u = a \omega \cos(\omega t)
\]

Provided the external force is zero \((f_o=0)\), the relationship between the load force \(f\) and velocity \(u\) is obtained from Eqs. (3) and (4) as follows:

\[
u^2 + \alpha u^2 - 2\beta fu = \gamma
\]

\[
\alpha = \omega^2 / \{(B\omega)^2 + (M\omega^2 - k)^2\}
\]

\[
\beta = B\omega^2 / \{(B\omega)^2 + (M\omega^2 - k)^2\}
\]

\[
\gamma = \{a\omega(M\omega^2 - k)\}^2 / \{(B\omega)^2 + (M\omega^2 - k)^2\}
\]

By taking the horizontal coordinate \(f\) and the vertical coordinate \(u\), the load locus with elliptical characteristic can be drawn by rotating the major axis to the anticlockwise direction \(\theta\) on the \(f-u\) plane as follows:

\[
\theta = (1/2) \tan^{-1}\{2\beta / (1 - \alpha)\}
\]
The maximum load force \( f_m \) and velocity \( u_m \) are given by differentiating Eqs.(3) and (4) with respect to time respectively as follows:

\[
\begin{align*}
    f_m &= a \sqrt{B \omega^2 + (M \omega^2 - k)^2} \\
    u_m &= a \omega
\end{align*}
\]  

(7)  
(8)

Now a non-dimensional approach will be employed to generalize the load characteristics in Eq.(5). The load force \( f \) and load velocity \( u \) are respectively nondimensionalised by using the maximum values \( f_m \) and \( u_m \):

\[
\begin{align*}
    f^* &= f / f_m = \cos(\alpha + \phi) \\
    u^* &= u / u_m = \cos(\alpha) \\
    \phi &= \tan^{-1}\{(M \omega^2 - k) / B \omega \}
\end{align*}
\]  

(9)  
(10)  
(11)

It should be noted that the parameter \( \phi \) depends on the load conditions. Therefore, the characteristics equation shown in Eq.(5) is normalized by using Eq.(7) to Eq.(10):  

\[
\begin{align*}
    u^* + f^* - 2 \cos \phi \cdot f^* u^* &= \sin^2 \phi
\end{align*}
\]  

(12)

Figure 2 shows the load loci on the non-dimensional \( f^*-u^* \) plane. It can be seen that the elliptical major axis is rotated \( \theta = \pi/4 \) from the \( f^* \) axis, and the configuration of the load locus varies from a straight line at \( \phi = 0 \) to a unity circle at \( \phi = \pi/2 \).

Therefore, the non-dimensional force \( f_p^* \) and velocity \( u_p^* \) at the maximum value of the load power \( f^* u^* \) are obtained by Eqs. (9) and (10):

\[
\begin{align*}
    f_p^* &= u_p^* = \cos(\phi / 2)
\end{align*}
\]  

(13)

Next, considering the external force \( f_o \), Eq. (12) becomes:

\[
\begin{align*}
    u^* + (f^* - f_o^*) - 2 \cos \phi \cdot f^* u^* &= \sin^2 \phi
\end{align*}
\]  

(14)

where \( f_o^* = f_o / f_m \), and the load locus within the range of \( u^*>0 \) is moved into the right direction as shown in Fig. 3.

**DRIVE CHARACTERISTIC OF SERVO ACTUATOR**

The steady flow rate \( q_i \) through the critical-lapped servo valve is expressed as follows:

\[
q_i = C_i \sqrt{P_S - P_L}
\]  

(15)

where \( C_i \), \( i \), \( P_S \) and \( P_L \) respectively represent the servo valve constant, driving current, supply pressure and differential pressure across the effective piston area \( A \).

Under the unloaded condition \( P_L = 0 \), the maximum flow rate \( Q \) is given by providing the maximum current \( i \) as follows:

\[
Q = C_i \sqrt{P_S}
\]  

(16)

Neglecting the mechanical and volumetric efficiency, the driving force \( F \) and velocity \( U \) which are generated by the hydraulic cylinder are normalized by Eqs.(7) and (8), and represented by:

\[
\begin{align*}
    F^* &= F / f_m = A P_L / f_m \\
    U^* &= U / u_m = q_i / (A u_m)
\end{align*}
\]  

(17)  
(18)

**Figure 3** Load loci considering external force

**Figure 4** Characteristic curve of actuator drive
Figure 5 Load locus enclosed by drive curves

When the current $i_e$ feeds into the servovalve, the relationship between $F^*$ and $U^*$ is non-dimensionally obtained by using Eq.(15) to (18):

$$U^* = U_m^* \sqrt{1 - (F^*/F_m^*)}$$

$$F_m^* = AP_1/f_m$$

$$U_m^* = Q/(Au_m)$$

where, the maximum drive force $F_m^*$ and velocity $U_m^*$ are also referred to as the stalled force and runaway velocity. Consequently, the drive characteristic for a specification is plotted in a parabolic curve as shown in Fig.4. The driving force $F_p^*$ and velocity $U_p^*$ at the peak output power are obtained by differentiating the output power $F^*U^*$ with respect to $F^*$ and expressed as follows:

$$F_p^* = (2/3)F_m^* = 2AP_1/(3f_m)$$

$$U_p^* = (1/\sqrt{3})U_m^* = Q/(\sqrt{3}Au_m)$$

The maximum overall efficiency $\eta_t$ of the servo mechanism with the load will be obviously defined as the maximum output power divided by the pump power. Using Eq.(20) to Eq.(23) yields the following equation:

$$\eta_t = \frac{F_p^*U_p^*}{P_1Q} \frac{F_p^*U_p^*}{F_m^*U_m^*} = 0.385$$

As obviously known, the maximum efficiency $\eta_t$ is a rather low percentage, 38.5% [1]. In order to attain the high efficiency, it is necessary that the drive curve skillfully enclose the load locus selecting the smallest values for $F_m^*$ and $U_m^*$ as possible. In the next section, based on Eqs. (12), (14), and (19), a discussion will be made how to choose the specification of the servovalve and actuator for the effective design.

EFFECTIVE DESIGN OF HYDRAULIC SERVO DRIVE

Figure 5 illustrates that the characteristic curve of a hydraulic cylinder drive encloses the load locus on the non-dimensional plane considering the external force $f_o$. In this section, firstly, the drive curve is drawn so as to contact internally at a point $(f_p^*+f_o^*, u_p^*)$ having a gradient of -1 on the load locus. Therefore, differentiating Eq.(19) with respect to $F^*$ equals -1, that is:

$$dU^*/dF^* = -1$$

and using Eq. (13), we get:

$$F_m^* = (3/2)\cos(\phi/2) + f_o^*$$

$$U_m^* = \sqrt{3 + 2f_o^*/\cos(\phi/2)}$$

Here, we shall define bias rate $\lambda$ which is the stalled force $F_m^*$ divided by the runaway velocity $U_m^*$ as follows:

$$\lambda = \frac{F_m^*}{U_m^*} = \frac{\sqrt{3 + 2f_o^*/\cos(\phi/2)}}{3\cos(\phi/2)}$$

Secondly, as shown in the thin-line in Fig.5, the drive curve is again plotted so as to take a margin at or near the maximum power point of the load locus, while the runaway velocity $U_m^*$ remains constant. The margin ratio
\( \nu \) will be defined as follows:

\[
\nu = \frac{R - f_0^*}{r} \frac{\cos(\phi/2)}{\cos(\phi/2)} = \frac{R - f_0^*}{r} \frac{\nu^2}{1 - 2f_0^*/\cos(\phi/2)}
\]  

(29)

where, \( r \) is the major radius of the load locus, and \( R \) is the distance between \((f_0^*, 0)\) and the intersection \((F_R^*, U_R^*)\) on the drive curve. In this manner, the new drive curve can enclose the load locus without being biased in view of the effective power transmission, as it gives the margin near the peak point on the load locus. As a result, because the intercept on the \( F^* \) axis shown in Eq.(26) shifts to the right direction, stalled force \( F_m^* \) will be:

\[
F_m^* = \frac{\nu \cos(\phi/2) + f_0^*}{1 - \frac{\nu^2}{3 + 2f_0^*/\cos(\phi/2)}}
\]  

(30)

Here, within the I quadrant in Fig.5, we will discuss the size of the spare areas \( S_a \) and \( S_f \) between both curves, which might be bounded by the elliptical major axis of the load locus. In order to estimate the distribution of the drive curve for the load locus, the area ratio \( J \) is defined by:

\[
J = \frac{S_f}{S_a}
\]  

(31)

\[
S_a = \frac{2\lambda U_m^*}{3} \left(1 - \varepsilon\right) - \frac{\nu^2 \cos^2(\phi/2)}{2} \left( \sin\phi \left( \frac{\pi - \phi}{2} + f_0^* \sqrt{1 - f_0^*} + \sin^{-1} f_0^* \right) - f_0^* \cos\phi \right)
\]  

(32)

\[
S_f = \frac{2\lambda U_m^*}{3} \varepsilon + \frac{\nu^2 \cos^2(\phi/2)}{2} \sin\phi \left( \frac{\pi - \phi}{2} \right) + \frac{\sin\phi}{4} \left( \pi - \phi \right)
\]  

(33)

\[
\varepsilon = \left[ 1 - \frac{\nu \cos(\phi/2) + f_0^*}{\lambda U_m^*} \right]^2
\]

When the drive curve is tangential to the load locus at the peak power without the external force, that is, \( f_0^* = 0 \) and \( \nu = 1 \), the stalled force \( F_m^* \) and runaway velocity \( U_m^* \) become simply generated from Eqs.(26) and (27):

\[
F_m^* = \left( 3/2 \right) \cos(\phi/2)
\]  

(34)

\[
U_m^* = \sqrt{3} \cos(\phi/2)
\]  

(35)

so \( \lambda = \sqrt{3}/2 = 0.866 \) from Eq.(28), and the area ratio \( J \) is obtained by means of Eq.(31) as follows:

\[
J = \frac{0.833 \cos(\phi/2) - (\pi - \phi) \sin(\phi/2)/2}{0.899 \cos(\phi/2) - (\pi - \phi) \sin(\phi/2)/2}
\]  

(36)

Figure 6 Area ratio \( J \), stalled force \( F_m \) and runaway velocity \( U_m \) varied with parameter \( \phi \)

Figure 7 Effect of external force \( f_o^* \) on bias rate \( \lambda \)

Figure 8 Effect of external force \( f_o^* \) on area ratio \( J \)
Figure 6 shows the calculation results of Eqs. (34), (35) and (36). As shown in the thick line, the area ratio \( J \) is less than a unit all over the region of the parameter \( \phi \). Next, the effect on the external force \( f_o \) will be discussed under the condition of a touch on the peak point, that is to say, \( v = 1 \). Figures 7 and 8 indicate that the bias rate \( \lambda \) and the area ratio \( J \) depend on the parameters \( f_o^- \) and \( \phi \). In Fig. 7, note that \( \lambda \) equals 0.866 without the external force \( f_o \). As shown in Fig. 8, the area ratio \( J \) is decreased according to the changes of the external force \( f_o \). Note that the area ratio \( J \) becomes smaller with an increase of the parameter \( \phi \). At \( \phi = \pi/2 \), the tendency is remarkable, although there is only a little wasted space bounded by two curves.

Figure 9 shows the effect of the margin ratio \( v \) on the area ratio \( J \) under the condition of \( f_o^- = 0.2 \) by substituting Eqs. (32) and (33) into Eq. (31). It is found that the area ratio \( J \) remarkably varies with an increase in the parameter \( \phi \). Although the area \( S_a + S_b \) between two curves should be minimized in view of the power consumption, we ideally assumed that the area ratio \( J \) equals a unit. Therefore, if \( S_a = S_b \), the values of margin ratio \( v \) depend on the parameters \( \phi \) and \( f_o^- \) as shown in Fig. 10. In the case of \( f_o^- = 0 \), the margin ratio \( v \) remains constant, namely \( v = 1.07 \), regardless of the change of \( \phi \). Hence both areas are equalized by taking about 7% for the margin at maximum power.

**DETERMINATION OF DESIGN SPECIFICATIONS**

Figure 11 represents the flow chart for selecting the design specification, such as the effective piston area \( A \), the flow rate \( Q \) and supply pressure \( P_s \), which can be determined from the stalled force \( F_m \) in Eq. (30) and runaway velocity \( U_r \) in Eq. (27). Finding the optimum margin ratio \( v \) from Fig. 10 as mentioned in the previous section, is especially crucial for effective power transmission.

**CONCLUSIONS**

An effective design approach of hydraulic servo actuator systems has been proposed by using the non-dimensional force versus velocity diagram, which can visually represent the space area between the load locus and drive curve. To evaluate that the drive curve is symmetrical with respect to the major axis of the load locus, the bias rate \( \lambda \), area ratio \( J \) and margin ratio \( v \) have been introduced. Finally, it has been shown that the design specification can be determined from the given load condition.

**REFERENCES**