DESIGN FOR A NEW FLOW METER UTILIZING FLOW FORCE

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ABSTRACT
This paper presents the development research of a unique flow meter utilizing flow force. This flow meter makes use of flow force by appropriately laying out the inclined plate or tapered shaft in a flow line to generate torque. The feature of this flow meter is as follows: (1) The structure is simple. (2) The pressure loss thorough the flow meter is very small. (3) The flow rate can be instantaneously acquired by measuring torque; thus a good dynamic response can be expected. In this paper, the fundamental principle and the mathematical model of the present flow meter are described. Then as the first step of the development research, the fundamental characteristics for a steady state are investigated theoretically in order to obtain the design criterion. In addition, based on this information, the dimensions of the proposed flow meter are derived as example design cases.

KEY WORDS
Flow meter, Measurement, Flow force, Hydraulic lock

NOMENCLATURE
\(a\): Height of inclined plate
\(b\): Width of inclined plate
\(e\): Offset distance between flow line center and inclined plate (tapered shaft) center
\(\bar{e}\): Non-dimensional offset distance \(e/h_{(0)}\)
\(h_{(0)}\): Height of entrance clearance
\(h_{(2)}\): Height of exit clearance
\(h_{(0)}(0)\): Height of entrance clearance without offset
\(h_{(2)}(0)\): Height of exit clearance without offset
\(\bar{h}_{(2)(0)}\): Non-dimensional height \(h_{(2)(0)}/h_{(1)(0)}\)
\(l\): Length of inclined plate and tapered shaft
\(P_1\): Pressure at entrance of flow meter
\(P_2\): Pressure at exit of flow meter
\(P_c\): Pressure at center of flow meter
\(q\): Flow rate of clearance per width
\(Q\): Total flow rate
\(r\): Radius of tapered shaft
\(R\): Radius of flow meter bore
\(T\): Torque
1. INTRODUCTION

It has been well recognized for many years that measuring a precise and instantaneous flow rate is a priority for engineering research fields such as applications of fluid power systems. Various types of flow meter have been developed [1-5] and many of them, especially for measuring steady flow rate, are widely used in the industrial world. In this research, as a new measuring technique of flow rate, a unique flow meter utilizing flow force is proposed.

In hydraulic components such as spool type valves, flow force will occur if the flow line between the sleeve and spool is tapered. Generally, this often causes a hydraulic lock, which is one of the crucial problems for servo valves [6]. However, the proposed flow meter makes use of flow force by appropriately laying out the inclined plate or tapered shaft in the flow line to generate torque. Since torque around the center of the inclined plate or tapered shaft is proportional to the flow force that depends on the flow rate through the clearance, the flow rate can be instantaneously acquired by measuring this torque.

This paper addresses a design technique for the proposed flow meter. To establish a design criterion, the fundamental characteristics of the present flow meter are examined theoretically. The detail dimensions are also determined based on the obtained information.

2. BASIC STRUCTURE

The basic structure for two types of the proposed flow meter is shown in Fig.1 and Fig.2. These are called inclined plate type (Fig.1) and tapered shaft type (Fig. 2). The rectangular plate, which inclines toward the center of the flow meter, is placed eccentrically in a rectangular flow line. In the upstream line from the vertical centerline of inclined plate, the flow becomes an expanse clearance flow between the two planes, while the flow becomes a narrow clearance flow in the downstream line. Since the centerline of the inclined plate offsets distance e from the horizontal centerline of flow line, the pressure difference occurs between the upper and lower sides of the inclined plate. This pressure difference is symmetrically distributed with respect to the plate center O. As the result, the torque is yielded around the plate center 0. In the proposed flow meter, flow rate can be instantaneously obtained by measuring this torque, which is proportionally related to the flow rate. In another design option, the taper shaft type can be employed. The taper shaft fitted in a circular flow line, instead of the inclined plate and the rectangular flow line combination.

![Schematic diagram of inclined plate type flow meter](Fig.1)
3. MATHEMATICAL MODEL

3.1 Inclined plate type

In a laminar flow region, the theoretical model of the flow in the inclined clearance is well known [7]. The flow rate of the upper and lower side in the upstream channel of the inclined plate center O are described by Eq.(1) and the flow in the downstream channel are also given by Eq.(2).

\[
\begin{align*}
q_u &= \frac{1}{6\mu} \left( \frac{h_u h_2}{h_1 + h_2} \right)^{\frac{3}{2}} \left( P_u - P_i \right) \\
q_l &= \frac{1}{6\mu} \left( \frac{h_l h_2}{h_1 + h_2} \right)^{\frac{3}{2}} \left( P_l - P_d \right)
\end{align*}
\]

(1)

Since \(q_u = q_u'\) and \(q_l = q_l'\), the following relationship can be derived from Eq.(1) and Eq.(2).

\[
\begin{align*}
q_u &= \frac{1}{6\mu} \left( \frac{h_u h_2}{h_1 + h_2} \right)^{\frac{3}{2}} \left( P_u - P_i \right) \\
q_l &= \frac{1}{6\mu} \left( \frac{h_l h_2}{h_1 + h_2} \right)^{\frac{3}{2}} \left( P_l - P_d \right)
\end{align*}
\]

(2)

Therefore, the flow rate through the present flow meter can be obtained as follows.

\[
P_u = P_l = \frac{P_u + P_l}{2}
\]

(3)

Hence, the relationship between the flow rate \(Q\) and the torque \(T_i\) is obtained by introducing the gain factor \(\kappa_i = \frac{T_i}{Q}\) as follows.

\[
\frac{T_i}{Q} = \frac{6\mu \lambda}{h_2} \left( \frac{l}{h_2} \right)^3
\]

(9)

where \(\lambda = 1 - \frac{h_2}{h_2\_0}\).

Let the torque generated in the upstream channel be defined as \(T_u\). The torque around the inclined plate center O appears as

\[
T_u = 2T_u = \left[ 2 \int \Delta P (1 - \bar{x}) d\bar{x} \right] b l (P_i - P_f)
\]

(7)

and the non-dimensional torque is

\[
\tilde{T}_u = \frac{2T_u}{b l (P_i - P_f)} = 2\int_0^1 \Delta \bar{P} (1 - \bar{x}) d\bar{x}
\]

(8)

Hence, the relationship between the flow rate \(Q\) and the torque \(T_i\) is obtained by introducing the gain factor \(\kappa_i = \frac{T_i}{Q}\) as follows.

\[
\frac{T_i}{Q} = \frac{6\mu \lambda}{h_2} \left( \frac{l}{h_2} \right)^3
\]

(9)

The gain factor \(\kappa_i\) is numerically calculated using Eq.(5) through Eq.(8) provided that the non-dimensional parameters \(\tilde{h}_2\) and \(\tilde{e}\) depending on the flow channel configurations are given. By choosing these parameters to maximize the gain factor \(\kappa_i\), the adequate non-dimensional parameters \(l/h_{20}\) can be determined for a desired dynamic and static characteristics in the torque detection.

3.2 Tapered shaft type

For the tapered shaft type flow meter shown in Fig.2, the upstream channel is described by

\[
\Delta \bar{P} = \frac{1}{2} \left[ \left( \frac{1 - \tilde{e}}{1 + \tilde{e} - \lambda \tilde{e}} \right)^{\gamma} - 1 \right] - \left[ \left( \frac{1 + \tilde{e}}{1 + \tilde{e} + \lambda \tilde{e}} \right)^{\gamma} - 1 \right]
\]

(6)

where \(\lambda = 1 - \tilde{h}_2\).

Let the torque generated in the upstream channel be defined as \(T_u\). The torque around the inclined plate center O appears as

\[
T_u = 2T_u = \left[ 2 \int \Delta \bar{P} (1 - \bar{x}) d\bar{x} \right] b l (P_i - P_f)
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![Fig.2 Schematic diagram of tapered shaft type flow meter](image-url)
theoretical model of flow in the tapered clearance is also well known for a laminar flow. The local flow rate \( dq \) of the local circumferential width \( dw \) in \( x \)-direction is given by Eq.(10).

\[
dq = \frac{1}{12 \mu} \frac{(h_{m} h_{p})}{h_{m} + h_{p}} (P_{1} - P_{2}) dw
\]

(10)

The total flow rate can be obtained by integrating Eq.(10) with respect to the circumferential-direction.

\[
Q = e_{s} \frac{\pi R (P_{1} - P_{2})}{6 \mu} h_{1}^{3}
\]

(11)

Where, the flow coefficient \( e_{s} \) is expressed by

\[
e_{s} = \frac{1}{2} \left[ \frac{3}{2} \varepsilon (1 + \tilde{h}_{1}) - \left( \frac{1 + \tilde{h}_{2}}{2} \right)^{3} + 2 \tilde{h}_{1} (1 + \tilde{h}_{2}) \right]
\]

(12)

The non-dimensional pressure difference \( \Delta P \) between \( \theta \) and \( \theta' \) is given by

\[
\Delta P = \frac{1}{2} \left\{ \frac{1 - \bar{\epsilon} \cos \theta}{1 - \bar{\epsilon} \cos \theta - \alpha \tilde{\epsilon}} \right\}^{2} - 1
\]

\[
- \frac{1 + \bar{\epsilon} \cos \theta}{1 + \bar{\epsilon} \cos \theta - \alpha \tilde{\epsilon}} \left( \frac{1 + \bar{\epsilon} \cos \theta}{\tilde{h}_{1} + \bar{\epsilon} \cos \theta} \right)^{2} - 1
\]

(13)

where \( \alpha = 1 - \tilde{h}_{1} \).

The non-dimensional torque \( T_{n} \) around the tapered shaft, which is originated by the pressure difference, is described by

\[
T_{n} = \frac{2 T_{n}}{R} = \frac{2}{\pi} \int_{0}^{\pi/2} \frac{\Delta P (1 - \dot{x}) \sin \alpha d \theta}{1 - \bar{\epsilon}} d \theta
\]

(14)

Thus, the relationship between flow \( Q \) and torque \( T_{n} \) can be obtained by the following equation.

\[
\frac{T_{n}}{Q} = 6 \mu \chi \left( \frac{1}{h_{1}} \right)^{3}
\]

(15)

As with the inclined plate type, the gain factor for the tapered shaft is defined as \( \chi = T_{n} / (\pi \varepsilon_{s}) \). The comparison of both gain factors will be discussed in the latter session.

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4. FUNDAMENTAL CHARACTERISTICS

4.1 Inclined plate type

Figures 3 (a) and (b) show the relationship between the pressure difference in the upstream channel and the non-dimensional distance from the plate edge (\( \bar{x} = x/l = 0 \)) to the center (\( \bar{x} = 1 \)), plotted with parameters \( \bar{\epsilon} \) at \( \tilde{h}_{m} = 2 \) and \( \tilde{h}_{m} = 5 \) respectively. It can be seen that the
peak of the pressure difference is amplified and shifts toward the plate edge with an increase of the non-dimensional height $h_2$. Note that the pressure difference increases over all range of $\tilde{x}$ concerning the increase of the non-dimensional offset distance $\tilde{e}$. Figure 4 shows the relationship between the non-dimensional height $h_2$ and the gain factor $\kappa_i = \frac{\theta}{\epsilon}$, in Eq.(9) plotted with $\tilde{e}$ as the parameter. For the design process of this flow meter, it is necessary to carefully select the non-dimensional parameters $h_2$ and $\tilde{e}$ so as to maximize the torque-flow gain factor $\kappa_i$. The suitable design would be possible to acquire the sufficient torque, even though the measuring flow rate is extremely small. It can be also seen from Fig.4 that the torque-flow gain factor $\kappa_i$ is maximized where the parameter $h_2 = 1.5 \sim 1.7$.

4.2 Tapered shaft type
The fundamental characteristics for the parameter changes of the tapered shaft type are almost similar to those of the inclined plate type. However, the crucial distinction is the pressure distribution on the upper and lower side of the flow channel due to the surface configuration. In the tapered shaft type, the difference between the pressures at the angle $\theta$ and the angle $\theta'$ decreases as the angle increases. Figure 5 shows the pressure difference distribution with the angle variation at $h_2 = 2$ and $\tilde{e} = 0.9$. It can be seen that the pressure difference $\Delta P$ decreases accordingly with increases in the angle $\theta$.

Figure 6 shows the relationship between the torque-flow gain factor $\kappa_i = \frac{T}{irss}$ and $h_2$, plotted with parameter $\tilde{e}$. A similar distribution to the inclined plate type is shown in this figure. However, from the quantitative point of view, the torque-flow gain factor $\kappa_i$ is only about 10 percent of the inclined plate type.

5. DESIGN EXAMPLES

5.1 Determinations of Appropriate Dimensions
Based on the theoretical model and fundamental characteristics of the present flow meter discussed in the previous section, the design parameter and dimensions are determined as example design cases. The required design specification and design parameters are listed in Table 1.

These parameters are chosen based on the information in

![Fig.5 Pressure difference distribution of upstream channel in tapered shaft type flow meter ($h_2=2\ ', \tilde{e}=0.9$)](image)

![Fig.6 Torque-flow gain factor $\kappa_i$ of tapered shaft type flow meter](image)

Table 1 Specification and parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. flow rate</td>
<td>$Q = 30 \text{ [L/min]} \ (0.5 \times 10^{-3} \text{ [m}^3/\text{s]})$</td>
</tr>
<tr>
<td>Max. torque</td>
<td>$T = 0.49 \text{ [Nm]}$</td>
</tr>
<tr>
<td>Viscosity of oil</td>
<td>$\mu = 1.634 \times 10^{-5} \text{ [Pa} \cdot \text{s]}$</td>
</tr>
<tr>
<td>Parameters</td>
<td>$h_2 = 1.7$</td>
</tr>
<tr>
<td>$\tilde{e} = 0.65$</td>
<td></td>
</tr>
<tr>
<td>$\kappa_i = 0.05$</td>
<td></td>
</tr>
<tr>
<td>$\tilde{e} = 0.0053$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Example dimensions and resulting calculated values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Inclined plate type</th>
<th>Tapered shaft type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{10} \ (h_1)$</td>
<td>2.0</td>
<td>1.5</td>
</tr>
<tr>
<td>$h_{20} \ (h_2)$</td>
<td>3.5</td>
<td>2.63</td>
</tr>
<tr>
<td>$e$</td>
<td>1.3</td>
<td>0.975</td>
</tr>
<tr>
<td>$b \ (R)$</td>
<td>12.7</td>
<td>13.4</td>
</tr>
<tr>
<td>$l$</td>
<td>117.0</td>
<td>87.8</td>
</tr>
<tr>
<td>$a$</td>
<td>22.0</td>
<td>16.5</td>
</tr>
<tr>
<td>$P_1 - P_2$</td>
<td>0.030</td>
<td>0.051</td>
</tr>
<tr>
<td>$\omega_k$</td>
<td>475</td>
<td>822</td>
</tr>
</tbody>
</table>
Fig.4 and Fig.6. The value of the non-dimensional height \( h_{20} \) is chosen as 1.7, because the maximum torque-flow gain factor can be obtained. It has been clarified that torque-flow gain factor becomes larger with an increase of non-dimensional offset distance \( \tilde{e} \) until its maximum value \( \tilde{e} = 1.0 \). However, the value of \( \tilde{e} \) is adapted as 0.65 by taking into account a strain effect of torque meter and manufacturing error. In order to obtain the required torque flow gain \( T/Q = 980\text{[Nm/(m}^3\text{s)]} \), the ratio \( b/h_1 \) is determined to be 58.5 using Eq.(9).

Regarding to the two cases that select \( h_1 = 2 \text{ mm} \) or \( h_1 = 1.5 \text{ mm} \) as the reference length \( h_1 \), the example dimensions of the flow meter parameter are calculated as shown in Table 2. These are derived from the above parameters assuming the flow passing through the flow meter channel is a laminar (Reynolds number: \( Re = 1700 \)). For the height of inclined plate, \( Aa \) of 22 mm and 16.5 mm are chosen. It should be noted that this parameter only affects on the dynamic characteristics.

Once the dimensions \( h_{10}, h_{20}, l, b \) and so on are obtained, the pressure loss \( P_1, P_2 \) can be calculated by Eq.(4) or (11). In the example designs, it is 0.03 MPa in the case of \( h_1 = 2 \text{ mm} \), and 0.051 MPa in the case of \( h_1 = 1.5 \text{ mm} \). It can be said that the pressure loss of the present flow meter is negligible and it is one of the advantages of this flow meter.

5.2 Consideration on Time Delay

It should be generally considered that the time delay is caused by the torque detection, the differential pressure changes between both sides of the plates, and the fluid compressibility in the flow meter chambers. However, the time delay due to the torque detection has the greatest effect on the dynamic characteristics of the flow meter. In the torque detecting system, the torque sensor connected with the shaft detects the torsion angle \( \gamma \) of the sensor as the torque \( T \) using the strain gauge. Then, the motion equation detecting the torque \( T \) is described as follows.

\[
T = J\ddot{\gamma} + D\dot{\gamma} + k\gamma
\]

(16)

Where, \( J \) is the moment of inertia about center \( O \) of the inclined plate, \( D \) is the damping coefficient and \( k \) is the torsion stiffness. In the case of the designed inclined plates made of Aluminum alloy, the natural frequencies of the torque detecting system \( w_0 = (k/J)^{1/2} \) are shown in Table 2.

To design the flow meter with the excellent dynamic characteristics, the length of the inclined plate and the reference length \( h_1 \) must be as small as possible. Therefore, the pressure loss \( P_1, P_2 \) becomes larger by adopting these dimensions that have the better dynamic characteristics of the torque detecting system.

6. CONCLUSIONS

A new flow meter that makes use of flow force is proposed in this paper. As the first step of development work, the design criterion has been established by examining the fundamental characteristics theoretically. Based on the criterion, the dimensions of the present flow meter are determined. Further work for the development of the present flow meter is being undertaken. The experimental work on the inclined plate type flow meter has been carried out and will be reported in the next issue.

REFERENCES