DEVELOPMENT OF A PRACTICAL AND HIGH ACCURACY SIMULATION TECHNIQUE BASED ON NUMERICAL MODAL APPROXIMATION FOR FLUID TRANSIENTS IN COMPOUND FLUID-LINE SYSTEMS

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ABSTRACT
A new simulation technique called the “system modal approximation” method for fluid transients in compound fluid-line systems composed of many line elements is developed and presented. This new method is based on numerical modal approximation of the frequency transfer function itself of the output to the input, considering the total system dynamics. This simulation technique also has the feature that the line elements with any kinds of the dynamic characteristics can be applicable because only the numerical data of the frequency response of transfer matrix parameters of individual line element is needed, and that the computation time is very short because the output in time domain can be calculated by the simple algebraic expression in the form of a recurrence formula. Simulation results of this method for the pressure transients in typical three kinds of compound fluid-line are compared with both the solutions from the method of characteristics and experimental results, and the superiority of this technique in easy applicability, flexibility, accuracy, computation time, etc. are demonstrated.

KEYWORDS
Fluid transients, Water hammer, Modal approximation, Compound fluid-line system, Simulation

NOMENCLATURE

\[ A(j\omega) - D(j\omega) \] : Transfer matrix element
\( c \) : Speed of sound
\( f \) : Frequency
\( j \) : Imaginary unit number
\( l \) : Length of line element
\( p \) : Pressure in time domain
\( q \) : Volume flow rate in time domain
\( r \) : Inner radius of line element
\( s \) : Laplace operator
\( t \) : Time
\( \Delta t \) : Step width of time in numerical calculation
\( x \) : Input variable in time domain
\( y \) : Output variable in time domain
\( v \) : Fluid kinematic viscosity
\( \rho \) : Fluid density
\( \omega \) : Angular velocity

INTRODUCTION
Reducing the shock and vibration due to the fluid transients in fluid transmission lines or improving the dynamic characteristics of total system by controlling the fluid transients has been an important technical subject in the field of fluid power engineering. In order to achieve this purpose efficiently, it is most effective to develop a simulation technique capable of predicting the fluid transients fast and accurately and utilize this as a design tool. Since the fluid-line used in the fluid power systems, petroleum transmission lines, etc. are usually not single but compound lines, the desirable simulation techniques have to be easily applied to such compound fluid-line systems. The method of characteristics incorporating both of the frequency dependent viscosity and heat transfer effects has been widely used in many practical applications for numerical simulation of fluid transients [Wylie and
Streeter (1978), Zielke (1968), Brown (1969). However, when the fluid-line becomes complex system with many line elements involving different length and speed of sound, solution procedures become very tedious. To solve the difficulty of numerical analysis in the method of characteristics, modal approximation technique for fluid line modeling was first introduced by Hullender and Healey (1981) and, subsequently, enhanced by many researchers [Hsue and Hullender (1983), Watton (1988), Zhao et al (1989), Yang et al (1991), Yang and Tobler (1991), Muto et al (1993), Piche and Ellman (1995), Seko (1999)]. Approaches are also all basically based on a technique for formulating approximately the flow model in the individual line element by the product series of a finite number of rational polynomial in the Laplace domain, and expressing variables in the form of state space representation in the time domain. However, it was pointed out in our study [1] that when these simulation techniques were applied to compound fluid-line systems the approximation accuracy became substantially worse and they also had several controversial points like the method of characteristics.

The main contribution of this paper is the development of easily applicable, accurate and fast simulation technique for fluid transients in compound fluid-line systems. The distinctive feature of this method is to make use of modal approximations of the frequency transfer function itself of the output to the input considering the total system dynamics unlike other existing approaches based on the modal approximations of the input/output causality of individual line element. In this paper, this proposed simulation technique is called the "system modal approximation" method (abbreviated to SMA method). The new method also has the feature that only the numerical data of the frequency response of transfer matrix parameters of individual line element, which may be given from either theoretical model or experimental measurements, is needed and that the output in the time domain can be calculated selectively by a simple algebraic expression in the form of recurrence formula.

In this paper, as the first stage of development of a general-purpose SMA method, consideration is limited to the cases whose input/output causality relationship can be approximated by a finite number of second order modes alone. These cases correspond to the analysis of pressure output response to pressure input in the case of every boundary but the input points being closed. The fundamental principle and computing procedures of the new simulation technique are explained. Then, simulation results of this method for pressure transients in three compound fluid-line systems consisting of many line elements involving the series, branch, stepped and closed loop junctions are compared with both the experimental results and solutions from the method of characteristics, and superiority of this technique to other methods in accuracy, applicability, flexibility, computation time, etc. are verified.

**FUNDAMENTAL PRINCIPLE AND CALCULATION PROCEDURE OF SMA METHOD**

A new simulation technique, the SMA method, requires the following information of the fluid line system for simulation analysis: the frequency range of interest and the numerical data of frequency response of the transfer matrix parameters of individual line elements. Numerical analysis method of this approach is that the frequency transfer function of output (wanted variable) to input (source) considering the dynamic characteristics of the entire system including every line elements and boundaries is calculated first by using the above-mentioned numerical data of individual line elements and then the modal approximation of the above frequency transfer function is numerically performed. Modal parameters can be determined by a relatively simple numerical calculation without troublesome mathematical modeling of individual line element nor of theoretical derivation of their parameters. Furthermore, only the required output response can be calculated selectively without solving simultaneous equation, and thus considerable reduction of computation time is also expected. The following shows the outline of each step of computing procedures for this proposed SMA method.

1. Establishment of frequency response of transfer matrix parameters of line elements of compound fluid-line system: The relationship between the upstream pressure and volumetric flow rate of ith line element, \( P_{i-1} \) and \( Q_{i-1} \), and the downstream pressure and flow rate, \( P_i \) and \( Q_i \), can be expressed in the following two-port matrix form in the Laplace domain (and in the frequency domain).

\[
\begin{pmatrix}
P_{i-1}(j\omega) \\
Q_{i-1}(j\omega)
\end{pmatrix} =
\begin{pmatrix}
A(j\omega) & B(j\omega) \\
C(j\omega) & D(j\omega)
\end{pmatrix}
\begin{pmatrix}
P_i(j\omega) \\
Q_i(j\omega)
\end{pmatrix}
\]

(1)

Notice that in the SMA method all the line elements do not necessarily have to be lines such as tube, pipe, hose, etc., and that it is all right to obtain the numerical data of these matrix elements by experimental measurements instead of the theoretical model.

2. Construction of frequency response matrix of the entire system: Construct the frequency response matrix of the entire system as follows, considering the input variables, boundary conditions and manner of junctions of each line element.

\[
\begin{pmatrix}
E_{i,j}(j\omega) \\
Y_{i,j}(j\omega)
\end{pmatrix} =
\begin{pmatrix}
F_{i,j}(j\omega) \\
X_{i,j}(j\omega)
\end{pmatrix}
\]

(2)

where \( X_k \) \((k=1-K)\) is input variable vector, \( Y_J(j=1-J)\) output variable (wanted variable) vector, \( E_{i,j} \) an \( J \times J\)
matrix which is constituted by the matrix parameters of individual line element in Eq. (1), 0, 1, and -1, and Fj,k an J x K matrix constituted similarly to Ej,k. This construction procedure can easily be performed by a methodical treatment capable of building an interactive computing system.

[3] Computation of frequency transfer function: Compute the frequency transfer function $G_{j,k}(j\omega)$ of output variables to input variables indicated in the next equation by the Gauss-Jordan method.

$$Y_j(j\omega) = \sum_{k=1}^{K} \{G_{j,k}(j\omega) \cdot X_k(j\omega)\}$$  \hspace{1cm} (3)

[4] Modal approximation of frequency transfer function: Here, only for the purpose of simplifying the description, the equation for the case of single input and single output is shown. In addition, $X_k, Y_j$ and $G_{j,k}$ are simply written as $X, Y$ and $G$, respectively. For the systems of multiple inputs, output variables also can be obtained easily as algebraic sum of the output to each single input. Since the transfer function $G(s)$ in interest in this study is of pressure/pressure, $G(s)$ can be approximated with $G^*(s)$ by the sum of a finite number of second order modes alone as follows.

$$G(s) = \frac{Y(s)}{X(s)} \equiv G^*(s) = b_0 + \sum_{n=1}^{N} \left[ \frac{a_n s + b_n}{s^2 + 2\zeta_n \omega_n s + \omega_n^2} \right]$$  \hspace{1cm} (4)

where $b_0$ is the correction term introduced in order to eliminate the inevitable steady-state error caused by a modal approximation by a finite number of modes.

[5] Determination of modal parameters: The natural frequencies $\omega_n$ and damping ratios $\zeta_n$ in Eq.(4) is numerically determined by an estimation method using the half-power bandwidth commonly used in the modal analysis (i.e., by the method based upon the curve fitting techniques in modal analysis). That is, $\omega_n$ and $\zeta_n$ are estimated from the following equations respectively by searching the frequencies $f_n$, where the imaginary part of $G(j\omega)$ has the extreme value on the coincident quadrature plot, and the frequency width $\Delta f_n$ showing the half-power bandwidth at each of the corresponding natural frequencies.

$$\zeta_n = \frac{\Delta f_n}{2 f_n}$$  \hspace{1cm} (5)

$$\omega_n = 2\pi f_n \sqrt{1 - \zeta_n^2}$$  \hspace{1cm} (5)

The residue coefficients, $a_n$ and $b_n$ (and $b_0$) in Eq.(4) are numerically determined using the least square method under the performance function $H$ defined by Eq.(6).

$$H = \int_0^\infty \left[ \left( \text{Real}[W(j\omega)] \right)^2 + \left( \text{Imag}[W(j\omega)] \right)^2 \right] d\omega \hspace{1cm} (6)$$

where

$$W(s) = \left[ G^*(s) - G(s) \right]/G(s)$$

$$= \frac{G(0) - 1}{G(s)} - \sum_{n=1}^{N} a_n \omega_n e^{-s + 2\zeta_n \omega_n} b_n$$  \hspace{1cm} (7)

[6] Calculation of time response of output variable: Equation (3) can be expressed in the time domain using a numerical convolution integral as follows.

$$y(t + \Delta t) = \int_0^{t + \Delta t} g(t + \Delta t - \tau) \cdot x(\tau) \cdot d\tau$$  \hspace{1cm} (8)

where $g(t)$ is an impulse response (inverse Laplace transform) of $G(s)$. Further, since the impulse response of each element of $G(s)$ has all a form of $e^{at}$ in case of $G(s)$ being able to be approximated by the second modes alone, the above equation can be transformed into the form of recursion formula as follows.

$$y(t + \Delta t) = h(t) + h_0 x(t + \Delta t)$$  \hspace{1cm} (9)

where $h_0$ and $h(t)$ is a constant given by following equation, respectively,

$$h_0 = b_0 + \sum_{n=1}^{N} \left( \beta_{0,n} \gamma_{1,n} + \beta_{1,n} \gamma_{2,n} \right)$$

$$\bar{x} = \left[ x(t - \Delta t) + x(t) \right]/2$$

$$\gamma_{0,n}(t) = \alpha_{0,n} \gamma_{1,n}(t - \Delta t) - \alpha_{0,n} \gamma_{2,n}(t - \Delta t) + \beta_{0,n} \bar{x}$$

$$\gamma_{1,n}(t) = \alpha_{0,n} \gamma_{2,n}(t - \Delta t) + \alpha_{0,n} \gamma_{1,n}(t - \Delta t) + \beta_{1,n} \bar{x}$$

$$h(t) = \sum_{n=1}^{N} \gamma_{0,n}(t - \alpha_{0,n} \gamma_{1,n}(t) + \beta_{1,n} \gamma_{2,n}(t)) \hspace{1cm} (10)$$

where $\alpha_{0,n} \sim \gamma_{n,n}$ are constants decided by the modal parameters in Eq.(4) and the time interval of numerical integral $\Delta t$, and given by following equation.

$$\Omega = \omega_n \sqrt{1 - \zeta_n^2}$$

$$\alpha_{0,n} = \exp(-\zeta_n \omega_n \Delta t) \cos(\Omega \Delta t)$$

$$\beta_{0,n} = \exp(-\zeta_n \omega_n \Delta t/2) \cos(\Omega \Delta t/2) \Delta t/2$$

$$\gamma_{1,n} = \alpha_{0,n}$$

$$\alpha_{n,n} = \exp(-\zeta_n \omega_n \Delta t) \sin(\Omega \Delta t)$$

$$\beta_{1,n} = \exp(-\zeta_n \omega_n \Delta t/2) \sin(\Omega \Delta t/2) \Delta t/2$$

$$\gamma_{2,n} = (b_0 - \alpha_{0,n} \zeta_n \omega_n)/\Omega$$  \hspace{1cm} (11)
As stated above, in the SMA method, when the parameters of modal approximation shown by Eq.(4) are determined the calculation of fluid transients can be performed easily using the recursion formula given by Eq.(9).

**COMPARISON OF SMA SIMULATION RESULTS WITH EXACT SOLUTIONS AND EXPERIMENTAL MEASUREMENTS**

Firstly, the usefulness of the SMA method will be verified by comparing with exact solutions in the application to three kinds of fairly complicated fluid-line systems composed of several line elements (circular rigid tube) with different dimensions including the series, branch, stepped and close loop junctions as shown in Fig.1 (called the System No.1 – No.3). Provided that the ratio of the length of individual line elements was chosen appropriately so that the compatible mesh sizes for the method of characteristics could be determined rationally. In this analysis, the frequency response characteristics of transfer matrix elements of the respective line elements, \( A(j\omega) \approx D(j\omega) \), were determined from the following theoretical equations, although in the SMA method it is also possible to determine them from the numerical data obtained by experiments.

\[
A = \cosh \Gamma, \quad B = Z \sinh \Gamma, \quad C = \sinh \Gamma / Z, \quad D = A
\]

\[
\Gamma = (j\alpha_k / c) \sqrt{\xi(j\omega)}, \quad Z = p c / \pi \sqrt{\xi(j\omega)} \quad \{12\}
\]

\( \xi(j\omega) \) is the transcendental function including Bessel functions and can be approximated fairly accurately over the wide range of frequency by the equation introduced by Kagawa et al (1983).

The parameters and dimensions (length and inner radius) of line elements used for this simulation are as follows:
- \( c = 1350 \text{ m/s}, \quad \nu = 50 \text{ mm/s}, \quad \rho = 867 \text{ kg/m}^3, \quad l_1 = 1.6 \text{ m}, \quad l_2 = 2.1 \text{ m}, \quad l_3 = 0.5 \text{ m}, \quad r_1 = 9.2 \text{ mm}, \quad r_2 = 3.9 \text{ mm}, \quad \text{and} \quad r_3 = 7.5 \text{ mm} \) are common to all the examined systems.
- \( l_4 = 1.1 \text{ m}, \quad l_5 = 2.0 \text{ m}, \quad r_4 = 9.2 \text{ mm}, \quad \text{and} \quad r_5 = 3.9 \text{ mm} \) are for the System No.2.
- \( l_4 = 1.5 \text{ m}, \quad l_5 = 1.2 \text{ m}, \quad r_4 = 3.9 \text{ mm}, \quad r_5 = 9.2 \text{ mm} \) for the System No.3.

The number of modes used in this study is the total number of natural frequencies included in the frequency range up to 2 kHz of interest.

Figure 2 ~ Fig.4 show the comparisons of the SMA simulation results with the exact solutions for the frequency response characteristics of transfer functions, \( P_A / P_0 \), \( P_B / P_0 \) and \( P_C / P_0 \), and the time responses of pressures, \( p_A \), \( p_B \) and \( p_C \) [pressures at the points indicated respectively in Fig.1 (a) ~ (c)], to the step input of \( P_0 \). As can be seen from the comparisons in frequency responses, the approximation of the SMA method agrees well with the exact solutions almost in the entire frequency range of interest except the range nearby the modes of so-called “heavy coupling”, where the coupling between two modes is so severe that the peaks of resonance do not appear clearly. The simulation of time responses of pressures also agrees well with the exact solutions (obtained from the method of characteristics) for their complexity except the minor error due to the
heavy coupling.

Next, comparisons of the SMA simulation results with the experimental measurements will be shown. The compound fluid-line systems tested are three kinds of systems with the same composition as those shown in Fig. 1. Provided that the lengths of individual line elements differ somewhat from those in Fig. 1, because the blocks for installation of pressure transducers and various kinds of couplings are incorporated into the fluid-line systems. Line elements are all steel-made tube capable of being regarded as rigid. The fluid used in the experiments was a commercial hydraulic fluid. A fluid power pump, a relief valve, an accumulator and a directional valve were installed at the upstream end of the test compound fluid-line system. Experimental procedures are as follows. Open the directional valve and adjust the pressure in the test line to about 1.5–2.0 MPa by the relief valve in order to suppress the cavitation occurrence (or column separation occurrence) during transient phenomena and then close the directional valve. Apply the pressure in accumulator to about 4.0–4.5 MPa by adjusting the relief valve. Open the directional valve swiftly by striking the valve spool with a hammer and generate the fluid transients in a test fluid-line system. Measure the pressure fluctuations, $p_0$, $p_A$, $p_B$ and $p_C$, simultaneously by the semi-conductor pressure.

(a) Frequency responses

(b) Time responses

Figure 3 Comparisons of SMA simulation results with exact solutions for compound fluid-line system No. 2

(a) Frequency responses

(b) Time responses

Figure 4 Comparisons of SMA simulation results with exact solutions for compound fluid-line system No. 3
transducers. In this study, the measured inlet pressure, \( p_0 \), was used as the input signal, because \( p_0 \) does not become an exact step input. Provided that such a treatment does not affect the essence of the SMA method.

Figure 5 (a) - (c) show the transient responses of output pressures to the pseudo step change of inlet pressure in the three kinds of fluid-line systems shown in Fig.1, respectively. Measured pressure transients are compared with the SMA simulation results. Very close agreements between measured and simulated values are obtained at least up to time of two or three periods of principal wave in all cases. On closer investigation, since a degree of damping of high-frequency components in the measured values is greater than that in simulation, a little difference is found late in the transient phenomena. This is supposed to be caused mainly by the effects of minor losses at the branch junctions, stepped junctions, etc. which are all neglected in this simulation analysis.

It is noticeable that the computation time required for execution of procedures from step [1] ~ step [6] indicated in the previous section is only 2 or 3 seconds for Fig.5 (a) ~ (c), respectively.

**CONCLUSIONS**

In this paper, a new simulation method called the “system modal approximation” method (SMA method) for fluid transients in compound fluid-line systems, which is able to predict fast and accurately, was proposed. Distinctive feature of this method is that the system information required for analysis is only the numerical data of frequency response of the transfer matrix parameters of individual line element. Another feature is that the required output variable alone can be calculated selectively by the simple algebraic equation in the form of recursive formula. As the first stage of development of the general-purpose SMA method, detailed considerations were limited to the case whose input/output causality relationship could be approximated by the second order modes alone. Fluid transients produced in three kinds of relatively complicated fluid-line systems composed of several line elements were examined. Good agreement between the measured and simulated values as well as between the exact and simulated results was observed. In conclusion, this newly developed SMA method has been found to be far superior to other methods in accuracy, easy applicability, flexibility and computation time in the application to the compound fluid-line systems.

**REFERENCES**