A New Linearized Equation for Modelling a Servovalve in Hydraulic Control Systems

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ABSTRACT

In the procedure of the hydraulic control system design, a linearized approximate equation described by the first order terms of Taylor's series has been widely used. Such a linearized equation is effective just near the operating point. However, pressure and flowrate in actual hydraulic systems are usually not confined near an operating point. This study suggests a new linearized equation for servovalve modelling as a modified form of the conventional linearized equation. And also, methods to determine effective operating points for the new linearized equation and the conventional linearized equation are proposed. From the evaluations of time responses and frequency responses obtained from simulations for a hydraulic control system, the effectiveness of the new linearized equation and the methods to determine effective operating points is confirmed.

KEY WORDS

Servovalve, Linearization, Operating Point, Hydraulic Control System

INTRODUCTION

This study suggests a new linearized equation for precisely approximating the characteristics of a servovalve installed in a hydraulic control system. The linearized equation is devised to be applicable to a hydraulic control system with excellent computational accuracy, even though pressure and flowrate in the system vary in a large scale during the system operation. The computational error of the suggested linearized flowrate equation is evaluated by denoting the characteristics curves of the servovalve on a load

pressure (p_l) versus load flowrate (Q_l) diagram. Subsequently, the authors propose a procedure for determining an effective or representative operating point, for the new linearized equation, to ensure excellent computational accuracy. Also, an other way for determining an effective operating point guaranteeing excellent computational accuracy for the conventional linearized equation is proposed. However, the usage of the last approach for the conventional linearized equation is restricted to just the case when the load pressure value in the effective operating point stays near zero.

For evaluating the effectiveness of the proposed new linearized equation and the procedures for determining

effective operating points, dynamic behaviors of a hydraulic control system are investigated through numerical simulations of time responses and frequency responses with various linearized equations and a nonlinear equation.

EQUATIONS FOR DESCRIBING THE SERVOVALVE CHARACTERISTICS

Basic Equations for the Valve Characteristics

General hydraulic control system consists of a servovalve, a servo actuator and load elements. Fig. 1 shows an example of a hydraulic control system using a symmetrical cylinder as a servo actuator. In the figure, p_s is the supply pressure, p_1 , p_2 are pressures inside the actuator, Q_1 , Q_2 are flowrates in the servovalve, and i_v is current in the torque motor of the servovalve.

The object servovalve in this study is an ideal critical center type valve with matched and symmetrical throttle passages. The load flowrate Q_l in the servovalve can be written as the following equation [1].

$$Q_l = C_d w x_v \sqrt{\frac{1}{\rho} (p_s - \frac{x_v}{|x_v|} p_l)} \quad (1)$$

where, C_d : discharge coefficient of orifice, w : area gradient of rectangular port, x_v : displacement of spool valve, p_l : load pressure, and ρ : density of oil.

Equation (1) is described as follows under the assumption that x_v varies in proportion to current i_v [2],[3].

$$Q_l = K_{sv} i_v \sqrt{p_s - \frac{i_v}{|i_v|} p_l} \quad (2)$$

where, $K_{sv} = Q_{ro} / (i_{vr} \sqrt{p_s})$, Q_{ro} : flowrate under

Figure 1 Schematic diagram of a hydraulic control system

$i_v = i_{vr}$ and $p_l = 0$, i_{vr} : rated value of i_v . Equation (2) is valid under steady state.

Conventional Linearized Flowrate Equation in a Servovalve

Conventional and widely practiced expression of the linearized flowrate equation in a servovalve is given by the following equation[1]~[5].

$$\Delta Q_l = k_i \Delta i_v - k_p \Delta p_l \quad (3)$$

where, k_i : flow-current coefficient and k_p : flow-pressure coefficient.

k_i and k_p are described as following equations through the differentiation process of equation (2) and some transformation process.

$$k_i = K_{sv} \sqrt{p_s - \frac{i_v^*}{|i_v^*|} p_l^*} \quad (4)$$

$$k_p = \frac{|Q_l^*|}{2} \left(p_s - \frac{i_v^*}{|i_v^*|} p_l^* \right) \quad (5)$$

where, i_v^* , p_l^* and Q_l^* are current, load pressure and load flowrate in an operating point.

We obtain another form of equation (3) near an operating point as follows.

$$\begin{aligned} Q_l &= Q_l^* + k_i (i_v - i_v^*) - k_p (p_l - p_l^*) \\ &= k_i i_v - k_p p_l + k_p p_l^* \end{aligned} \quad (6)$$

As an example, non-dimensional $p_l \sim Q_l$ relationships computed from equation (2) and (6) are plotted in Fig. 2, in which $i_v^* = i_{vr}$, $Q_l^* = 0.65 Q_{ro}$,

Figure 2 Comparison of $p_l \sim Q_l$ diagram computed by the nonlinear equation (2) and the linearized equation (6)

$p_l^* = 0.57 p_s$. From this figure, it is known that Q_l computed from equation (6) is not zero even at the origin of the diagram with $p_l = 0$ and $i_v = 0$, and significant computational errors may occur in the computed results with the equation (6) in the valve operation range apart from the predetermined operating point.

Suggestion of a New Linearized Equation for Modelling a Servo Valve

The object servovalve in this study is an ideal critical center type valve with matched and symmetrical throttle passages. Therefore, the straight line group obtained from equation (6) must appear in a symmetric style about the origin of a $p_l \sim Q_l$ diagram. Based on such a physical characteristic for the linearized flowrate equation, the authors devise the following equation as a modified form of equation (6).

$$\begin{aligned} Q_l &= k_i i_v - k_p p_l + k_p p_l^* \left(\frac{i_v}{i_v^*} \right) \\ &= \left(k_i + \frac{k_p p_l^*}{i_v^*} \right) i_v - k_p p_l \end{aligned} \quad (7)$$

k_i and k_p in equation (7) can be obtained by substituting the relevant physical values into equation (4) and (5).

Fig. 3 shows an example for applying equation (7) to a servovalve with the same physical values and the same operating point as Fig. 2. In the figure, the nonlinear equation (2) appeared as the chain lines and the linearized equation (7) as the solid lines. From the figure, it was ascertained that the symmetry was recovered completely and computational accuracy was improved considerably with the linearized equation (7).

As of now, there is no general rule on how to determine an effective operating point of a servovalve in the process of applying the linearized equations to analyzing a hydraulic control system.

Figure 3 Comparison of $p_l \sim Q_l$ diagram computed by the nonlinear equation (2) and the linearized equation (7)

Figure 4 Illustration of a $p_l \sim Q_l$ diagram for explaining the method for determining an effective operating point

Here, the authors suggest a procedure for determining an effective operating point that enables to get excellent computational accuracy in computation with equation (7). The procedure is described as follows.

(i) Dynamic behavior of the object hydraulic system is computed while a reference input signal is taken. To compute flowrate in the servovalve, the nonlinear equation (2) is used.

(ii) k_i and k_p values at every specified points on a locus denoting a system response are calculated. On the locus in Fig. 4, as an example, there are 20 points, which are generated based on the concept of constant time division for one cycle time of a response.

(iii) k_{mi} and k_{mp} , which are the arithmetic mean values of k_i and k_p respectively, are calculated with the following equations.

$$k_{mi} = \frac{1}{n} \sum_{j=1}^n k_i(j), \quad k_{mp} = \frac{1}{n} \sum_{j=1}^n k_p(j) \quad (8)$$

where, n is number of data used for the calculation.

(iv) An effective operating point is determined by obtaining p_l^* and Q_l^* values through substitution of k_{mi} and k_{mp} values to equation (4) and (5). Also, i_v^* is calculated by applying p_l^* , Q_l^* values obtained already to equation (2). By substituting the values of k_{mi} , k_{mp} , p_l^* , Q_l^* obtained in the present step to the second row of equation (7), we get the following equation.

$$\begin{aligned} Q_l &= \left(k_{mi} + \frac{k_{mp} p_l^*}{i_v^*} \right) i_v - k_{mp} p_l \\ &= k_i^{**} i_v - k_{mp} p_l \end{aligned} \quad (9)$$

How to Determine an Effective Operating Point of a Conventional Linearized Equation for Modelling a Servovalve

It was ascertained that comparatively good computation accuracy was obtained by the conventional linearized equation when an operating point of a servovalve was on the Q_l axis of the $p_l \sim Q_l$ diagram, i.e. $p_l^* = 0$ [2]. Equation (3) can be rewritten when $p_l^* = 0$ as follows.

$$Q_l = k_i i_v - k_p p_l \quad (10)$$

Under the assumption that the computational error of equation (10) is zero, the following equation is obtained from equations evaluating traveled distance of the piston in Fig. 5.

$$\int_0^t (k_i i_v - k_p p_l) dt = \int_0^t Q_l dt \quad (11)$$

Q_l in the right hand side of equation (11) must be computed by the nonlinear flowrate equation. Equation (11) is rearranged to get

$$k_p = \frac{1}{\int_0^t p_l dt} \left(k_i \int_0^t i_v dt - \int_0^t Q_l dt \right) \quad (12)$$

where, the constant k_i can be denoted as Q_{ro}/i_{vr} .

p_l and Q_l in the right side of equation (12) can be obtained from a numerical simulation using the nonlinear flowrate equation. Q_l^* , flowrate in an operating point of a servovalve, is obtained by the substituting k_p value calculated from equation (12) into the equation (5).

In the calculation of equation (12), a special attention must be paid on determining the integration time not to permit the results of integrations to be near zero.

Figure 5 Schematic diagram of the object system for simulation

The linearized equation described by equation (10) and (12) is easy to handle compared to equation (9). However, it is a drawback of the linearized equation given by equations (10) and (12) that the usage of the linearized equation is restricted to just the case when the effective operating point stay on Q_l axis of the $p_l \sim Q_l$ diagram.

NUMERICAL SIMULATION

Basic Equations for a Hydraulic Control System

Numerical simulation will be done for a hydraulic control system shown in Fig. 5.

By applying the principle of continuity to each of the cylinder chambers, we have following equation.

$$Q_l = A_p \dot{x}_p + C_{tp} p_l + \frac{V_l}{4\beta_e} \dot{p}_l \quad (13)$$

The equation of motion of the combined body of the load and the piston is given as

$$A_p p_l = M_t \ddot{x}_p + D_l \dot{x}_p + F_l \quad (14)$$

Dynamic characteristics of torque motor and spool in a servovalve is described in Laplace domain as

$$G_{vx}(s) = \frac{x_v(s)}{i_v(s)} = \frac{k_v \omega_v^2}{s^2 + 2\zeta_v \omega_v s + \omega_v^2} \quad (15)$$

The linearized flowrate equation (3) can be rewritten by considering the dynamic characteristics of a servovalve described by equation (15) as follows.

$$\begin{aligned} \Delta Q_l &= k_a \frac{G_{vx}(s)}{k_v} k_x \Delta V_a(s) - k_p \Delta p_l(s) \\ &= k_i \frac{G_{vx}(s)}{k_v} \Delta i_v(s) - k_p \Delta p_l(s) \end{aligned} \quad (16)$$

where, V_a : input voltage of servo amplifier, k_a : gain of servo amplifier, k_x : flow-displacement coefficient, $k_i = k_v k_x$.

The Object System for Numerical Simulations

The hydraulic control system for simulation is shown in Fig. 5 and physical values in the object system is listed in Table 1. Reference input of 1 V to the system is equivalent to position signal of 0.005 m.

In numerical simulation, equation (13), (14) and flowrate equation in the servovalve were used. For the nonlinear flowrate equation, equation (1) and a transformed equation of (15) to time domain were used

Table 1 Physical values of the object system for numerical simulations

parameters
$A_p (\text{m}^2) : 0.0094, C_{tp} (\frac{\text{N}}{\text{m/s}}) : 0, D_l (\frac{\text{m}^3/\text{s}}{\text{N/m}^2}) : 18000,$
$F_l (\text{N}) : 0, K_a (\frac{\text{mA}}{\text{V}}) : 3, K_{LT} (\frac{\text{V}}{\text{m}}) : 200,$
$V_l (\text{m}^3) : 6.0 \times 10^{-4}, \beta_e (\text{N/m}^2) : 7 \times 10^8,$
$M_l (\text{kg}) : 118, p_s (\text{MPa}) : 7, Q_{ro} (\text{l/min}) : 20,$
$\omega_v (\text{rad/s}) : 760, \zeta_v : 0.84,$
$K_{sv} (\frac{\text{m}^3/\text{s}}{\text{mA} \sqrt{\text{N/m}^2}}) : 7.6 \times 10^{-4}$

as a coupled form.

For the linearized flowrate equation, equation (9) or (10) was used. To incorporate the dynamic characteristics of the servovalve into the linearized flowrate equations, k_i^{**} in equation (9) and k_i in equation (10) were replaced with $k_i^{**} G_{vx}(s)/k_v$ and $k_i G_{vx}(s)/k_v$, and these equations were transformed to time domain equations, respectively.

The numerical simulations were carried out by numerical integration programs with Runge-Kutta method.

Simulation Results of Time Responses and Discussions

Fig. 6 shows a $p_l \sim Q_l$ diagram and time responses of the system, which are obtained from numerical simulation with input signal of $a \sin(2\pi ft)$ ($a = 2.76 \text{ V}$, $f = 5 \text{ Hz}$) under open loop control scheme. Here, the open loop scheme was introduced deliberately to see clearly the relative error between responses.

In Fig. 6 (b), the 'linear - 1(1)' shows a time response by applying linearized equation (10) with the operating point '1' in Fig. 6 (a). The 'linear - 1(2)' is obtained by equation (10) with the operating point '2'. The 'linear - new' is obtained by using the linearized equation (9) with the operating point 'op'. The 'linear - 2' is obtained by linearized equation (10) with the operating point 'op-2'. The 'nonlinear' is obtained by nonlinear flowrate equation.

In the figure, the time response 'linear - 1(1)' and 'linear - 1(2)' did not show good correspondence with the 'nonlinear' response. However, the time response 'linear - new' and 'linear - 2' show good agreement with the 'nonlinear' response. From these results, the effectiveness of flowrate equation (9) was ascertained, and also the usefulness of equation (10) and (12) was clarified.

Simulation Results of Frequency Responses and Discussions

(a) $p_l \sim Q_l$ diagram

(b) time response

Figure 6 $p_l \sim Q_l$ diagram and time response of the object system, actuated by input signal with $a \sin(2\pi ft)$ ($a = 2.76 \text{ V}$, $f = 5 \text{ Hz}$) under open loop control

To demonstrate the effectiveness of the linearized flowrate equations for a broad frequency range of input signal, frequency responses for the hydraulic control system shown in Fig. 5 were investigated and denoted in Fig. 7 and Fig. 8. In these frequency responses, input and output signals are reference input voltage, v_i and cylinder displacement signal denoted by voltage value, v_p .

The amplitude of input signal in the simulation to obtain Fig. 7 and Fig. 8 is described as $ab^f \text{ V}$, where $a = 3.05$, $b = 0.98$ and f is frequency (Hz) of input signal. The meaning of legends in Fig. 7 and Fig. 8 is the same as the ones in Fig. 6.

To get the responses indicated with the 'linear - new' and the 'linear - 2' in Fig. 7, linearized flowrate equations were obtained in advance with effective operating points based on the system input signal of amplitude 2.79 V and frequency 5 Hz.

In the same way, for the 'linear - new' and the 'linear - 2' in Fig. 8, linearized flowrate equations were obtained based on the input signal of 1.24 V and 50 Hz.

From Fig. 7 and Fig. 8, it is confirmed that the

(a) gain diagram

(b) phase diagram

Figure 7 Frequency responses of the object system by numerical simulations under open loop control (operating point tuned at 5 Hz)

computed results by the linearized flowrate equations based on equation (9) or equation (10) and the procedures to get effective operating points suggested in this study show very good correspondence with the results by the nonlinear flowrate equation near the tuned frequencies for the effective operating points of 5 Hz or 50 Hz.

CONCLUSIONS

In this study, a new linearized flowrate equation for modeling servovalve and a procedure to obtain an effective operating point were suggested.

For evaluating the effectiveness of the new linearized equation and the procedure to obtain an effective operating point, dynamic behaviors of a hydraulic control system were investigated through numerical simulations of time responses and frequency responses with various linearized equations suggested in this study and a nonlinear equation.

From the results of the simulations, the usefulness of the new linearized equation and the procedure to obtain an effective operating point were fully confirmed.

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(a) gain diagram

(b) phase diagram

Figure 8 Frequency responses of the object system by numerical simulations under open loop control (operating point tuned at 50 Hz)

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