Evaluation of Plastic Deformation Limit by Circumferentially Notched Tension Test

Keitaro ENAMI and Kotobu NAGAI

Steel Research Center, National Institute for Materials Science, 1-2-1 Sengen, Tsukuba, Ibaraki 305-0047 Japan.

(Received on January 18, 2005; accepted on March 14, 2005; originally published in *Tetsu-to-Hagané*, Vol. 91, 2005, No. 2, pp. 285–291)

Circumferentially notched and smooth round tension tests were carried out at room temperature with and without compressive prestrain in a hot-rolled ferrite–pearlite SM490B steel, and the effects of the compressive prestrain and the notch on the plastic deformation limit (PDL) were investigated. The PDL was defined as a termination point in a true stress–true strain curve which was caused by a ductile or cleavage fracture. The notch caused variations in the average stress–true strain curve but did not affect the true stress–true strain curve. A true strain at the plastic instability was determined by a work hardening exponent, regardless of the notch. On the other hand, the true strain at the PDL differed according to the notch. An axial stress at the PDL was found to be about 1 300 MPa, regardless of the ductile and cleavage fracture.

KEY WORDS: plastic deformation limit; circumferentially notched tension test; true stress; true strain; axial stress.

1. Introduction

The Japanese automobile industry recently approved of hydroforming in manufacturing to achieve a light weight automobile body.^{1,2)} However, an evaluation method for a plastic deformation limit (PDL) in hydroforming has not been established yet.³⁾ A burst that is usually caused by a ductile fracture is the limit in hydroforming.⁴⁾ Therefore, it is important to know the PDL accurately through a quantitative measure.

PDL is defined as the onset of ductile (or cleavage) fracture in this study. The PDL is said to be closely related to an axial stress.⁵⁾ For example, in a smooth round tension test, a tension specimen does not fracture before local necking, and a hydrostatic stress develops and the axial stress becomes the largest at the center of the neck. The ductile fracture occurs at the center of the neck after local necking. On the other hand, a true strain is uniform in the neck. If the PDL is determined by the true strain, the fracture can occur anywhere in the neck. Therefore, the PDL must be determined by the axial stress. The PDL depends on the history of plastic working, working temperature, strain rate, hydrostatic stress state, and some other factors.

In this study, a hot-rolled JIS SM490B steel was pre-strained 30% in compression to investigate the effect of compressive prestrain on the PDL. Tension tests using circumferentially notched specimens with various notch root radii were carried out at room temperature to investigate how the PDL varies with the hydrostatic stress state. A smooth round specimen was also used for comparison.

2. Experimental Method

A hot-rolled JIS-SM490B steel with a chemical composition of 0.16C–0.44Si–1.46Mn–0.013P–0.004S (mass%) and a plate thickness of 16 mm was used.

2.1. Circumferentially Notched Tension Test

A test piece (length: 50 mm, width: 40 mm, thickness: 16 mm) was machined from the material and compressed in the rolling direction of the material by die-pressing at room temperature. Compressive prestrains of 0%, 10% and 30% were used. The bulging of the test piece due to die-pressing was significant for the 30% prestrain. Then, the circumferentially notched tension specimen, as shown in Fig. 1, was machined from the compressed test piece. The longitudinal direction of the specimen was parallel to the direction of the prestrain. The neck of the specimen was chosen at the center of the bulging. The axial stress state in the neck can be changed by changing the radius of the neck curvature ($R=1, 2, 5, 15$ mm). The depth of the initial neck was 2 mm in every specimen. Tension tests were carried out at room temperature, and the stroke rate of the test machine was

Fig. 1. The circumferentially notched tension specimen.

Fig. 2. The state of the stress and the strain in the neck section after general yielding.

1.0 mm/min. The tensile load and the diameter of the neck were recorded. More details of the experimental method are reported elsewhere.⁶⁻⁹⁾

2.2. Smooth Round Tension Test

A JIS 14A specimen (initial diameter of the cross section: 8 mm, gage length: 40 mm) was used in the smooth round tension test with a 0% prestrain. The longitudinal direction of the specimen was parallel to the rolling direction of the material. A tensile test was carried out at room temperature, and the stroke of the test machine was 1.2 mm/min. The tensile load and the gage elongation were recorded.

2.3. The States of Stress and Strain in the Neck

Figure 2 illustrates the states of stress and strain in the neck after general yielding. The center axis is denoted as the Z-axis, and the radial axis as the X-axis, and the origin is chosen at the center of the neck. The circumferential axis is denoted as the Y-axis and is normal to the page space.

In this figure, R is the radius of the neck curvature, a is the radius of the neck section, σ_z is an axial stress, σ_x is a radial stress, σ_y is a circumferential stress, σ is an average stress, F is the true stress (or an equivalent stress), and ε is the true strain (or an equivalent strain). The true stress and the true strain are assumed to be uniform in the neck. The axial stress is the largest at the center of the neck.

The average stress, the true stress and the true strain are expressed as follows¹⁰⁻¹²⁾:

$$\sigma = \frac{P}{\pi a^2}, \quad P = \int_{X=0}^a 2\pi X \sigma_z dX \quad \dots\dots\dots(1)$$

$$F = \frac{1}{\sqrt{2}} [(\sigma_z - \sigma_x)^2 + (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2]^{1/2} \quad \dots\dots\dots(2)$$

$$\varepsilon = 2 \cdot \ln \frac{a_0}{a} \quad \dots\dots\dots(3)$$

where P is the tensile load, and a_0 is the initial radius of the neck section.

Considering the axisymmetry, σ_y is equal to σ_x . The axial stress is expressed as follows by substituting $\sigma_y = \sigma_x$ into Eq. (2):

$$\sigma_z = F + \sigma_x \quad \dots\dots\dots(4)$$

The maximum axial stress at the center of the neck is ex-

Fig. 3. The relationship between σ_z/F , σ/F and a/R .

pressed as follows¹²⁾:

$$\sigma_z = F \left(1 + \ln \left(1 + \frac{a}{2R} \right) \right) \quad \dots\dots\dots(5)$$

In the smooth round tension, a uniaxial stress state exists until local necking. In this case, $\sigma_x = \sigma_y = 0$, $\sigma_z = F$. By substituting $\sigma_z = F$ into Eq. (1), the average stress becomes equal to the true stress. That is,

$$\sigma_z = \sigma = F \quad \dots\dots\dots(6)$$

In the circumferentially notched tension, $\sigma_x > 0$. Substituting Eq. (4) into Eq. (1),

$$\sigma = F + \frac{1}{\pi a^2} \int_{X=0}^a 2\pi X \sigma_x dX \quad \dots\dots\dots(7)$$

Since $\sigma_x > 0$, the average stress is always larger than the true stress. The relationship between the average stress and the true stress is expressed as follows¹²⁾:

$$\sigma = F \left(1 + 2 \frac{R}{a} \right) \ln \left(1 + \frac{a}{2R} \right) \quad \dots\dots\dots(8)$$

Figure 3 shows Eqs. (5) and (8) as functions of a/R . The data were plotted on the condition that $a = a_0 = 3$ mm and $R = 1, 2, 5, 15$ mm. The average stress and the axial stress increase with a/R .

3. Experimental Results

3.1. Circumferentially Notched Tension Test

Figures 4(a)–4(c) show the tensile load—the diameter of neck section curves in the circumferentially notched tension test. The maximum load point, $P_{\max}$ (the point of plastic instability) is indicated by Δ and the sudden load decrease point, P_f by $\circ$. The maximum load point increased as the radius of the neck curvature decreased. On the other hand, the diameter of the neck section at the maximum load point was not influenced by the initial radius of the neck curvature.

Regarding fractures, a ductile fracture was observed in the 1R, 2R, 5R, and 15R specimens with a 0% and 10% prestrain, and in the 2R, 5R, and 15R specimens with a 30% prestrain. On the other hand, a cleavage fracture was observed in the 1R specimen with a 30% prestrain, and also locally in the 1R specimen with a 10% prestrain and the 2R

Fig. 4. The load-diameter of the neck section curves in the notched tension test; (a) 0% prestrain; (b) 10% compressive prestrain; (c) 30% compressive prestrain.

specimen with a 30% prestrain.

Figures 5(a)–5(c) show the average stress–true strain curves that were converted from the data in Figs. 4(a)–4(c) through Eqs. (1) and (3). The approximated average stress–true strain curves are also shown. The approximated curves were determined by Eq. (15) (which is discussed later), by replacing the true stress with the average stress. The symbols of Δ and $\circ$ in Fig. 5 correspond to those in Fig. 4. As shown in Fig. 5, the sudden load decrease point in Fig. 4 corresponds to the average stress drop point from the approximated curve. Since the average stress drop point in Fig. 5 was determined clearly, this point was defined as the point of the PDL (Hancock *et al.* regarded this point as the ductile failure point.^{10,11}). According to this definition, the PDL is clearly different from the plastic instability point, which is defined as the maximum tensile load point in the load-diameter curve in Fig. 4.

Figures 6(a)–6(c) show the true stress–true strain curves that were converted from the data in Figs. 5(a)–5(c) through Eq. (8), together with the approximated curves that are discussed later. Furthermore, Fig. 6(a) includes the true stress–true strain curve in the smooth round tension that is

Fig. 5. The average stress–true strain curves; (a) 0% prestrain; (b) 10% prestrain; (c) 30% prestrain.

also discussed later. As shown in the three figures, the true stress–true strain curve is unique regardless of the initial radius of the neck curvature. However, the true stress–true strain curve of the 1R specimen with a 0% prestrain is slightly lower than that of the 2R, 5R, and 15R specimens in Fig. 6(a). There is a possibility that the change of neck geometry by plastic deformation is significant for the 1R specimen. The true stress–true strain curve of the circumferentially notched tension also agrees with that of the smooth round tension in this figure.

Figure 7 shows the true stress–accumulated equivalent plastic strain curves in the 15R notched specimen. The accumulated equivalent plastic strain contains the prestrain. The true strain equivalent to the prestrain is expressed as follows¹³:

$$\varepsilon = \ln \left(\frac{1}{1 - \varepsilon_{\text{pre}}} \right) \dots \dots \dots (9)$$

where ε is the true strain, and ε_{pre} is the compressive prestrain.

A 55% true strain (equivalent to a 40% prestrain) was included in the 30% prestrain by the accumulated equivalent

Fig. 6. The true stress–true strain curves; (a) 0% prestrain; (b) 10% prestrain; (c) 30% prestrain.

lent plastic strain to make the stress–strain curve unique. Since the bulging due to die-pressing is significant for the 30% prestrain, there is a possibility that the 40% prestrain was introduced at the center of the bulge.

3.2. Smooth Round Tension Test

Figure 8 shows the nominal stress–nominal strain curve in the smooth round tension test. The nominal stress and the nominal strain are expressed as follows¹⁴⁾:

$$\sigma_n = \frac{P}{\pi a_0^2} \quad (10)$$

$$\varepsilon_n = \frac{\Delta l}{l_0} \quad (11)$$

where σ_n is the nominal stress, P is the tensile load, a_0 is the initial radius of the cross section, ε_n is the nominal strain, Δl is the gage elongation, and l_0 is the gage length.

The plastic instability point is indicated by Δ , the onset of local necking by $\square$, and the break point by $\circ$.

In Fig. 6(a), the true stress–true strain curve that was converted from the data in Fig. 8 was included. The true

Fig. 7. The true stress–accumulated equivalent plastic strain curves in the 15R notched tension.

Fig. 8. The nominal stress–nominal strain curve in the smooth round tension test at room temperature.

stress and the true strain are converted as follows¹⁴⁾:

$$F = \sigma_n(1 + \varepsilon_n) \quad (12)$$

$$\varepsilon = \ln(1 + \varepsilon_n) \quad (13)$$

The symbols of Δ , $\square$, $\circ$ in Fig. 6(a) correspond to those in Fig. 8. The onset of local necking is defined as the true stress decrease point in Fig. 6(a). The onset of local necking in the smooth round tension is different from the PDL in the circumferentially notched tension. Although the true stress seems to decrease after the local necking, the true stress in the neck still increases even after the local necking.

4. Discussions

4.1. The Onset of Plastic Instability

The onset of plastic instability is expressed as follows¹³⁾:

$$\frac{dF}{d\varepsilon} = F \quad (14)$$

The true stress–true strain curve is approximated as follows¹⁴⁾:

$$F = c \cdot \varepsilon^n \quad (15)$$

where c is the true stress when the true strain is unity, and n is a work hardening exponent. As shown in Figs. 6(a)–6(c), the true stress–true strain curve can be approximated quite well by Eq. (15). The approximations were made from the result of the smooth round tension for the 0% prestrain and from the result of the 15R notched tension for the 10% and

Fig. 9. The relationship between the true strain at the PDL and the post failure true strain in the notched tension test.

30% prestrain. $c=865$, $n=0.19$ were obtained for the 0% prestrain, $c=926$, $n=0.15$ for the 10% prestrain, $c=926$, $n=0.04$ for the 30% prestrain.

Substituting Eq. (15) into Eq. (14),

$$\varepsilon = n \dots\dots\dots (16)$$

The true strain at the onset of plastic instability is equal to the work hardening exponent, and Figs. 5(a)–5(c) show that they are almost equivalent regardless of the initial radius of the neck curvature. In other words, the true strain at the onset of plastic instability is uniquely defined by the true stress–true strain curve and is independent of the initial neck shape.

4.2. The Evaluation of True Strain at the PDL in the Smooth Round Tension Test

Figure 9 shows the relationship between the true strain at the PDL and the post failure true strain in the circumferentially notched tension. The post failure true strain was evaluated from Eq. (3) by measuring the radius of the cross section after failure. The true strain at the PDL was measured directly from Figs. 5(a)–5(c) (or Figs. 6(a)–6(c)). The true strain at the PDL was smaller than the post failure true strain. In the ductile fracture, the relationship between the post failure true strain and the true strain at the PDL was approximated as follows, regardless of the prestrain:

$$\varepsilon_{\text{post}} = \varepsilon_{\text{limit}} + 0.35 \dots\dots\dots (17)$$

where $\varepsilon_{\text{post}}$ is the post failure true strain, and $\varepsilon_{\text{limit}}$ is the true strain at the PDL. In the cleavage fracture, the figure showed that the post failure true strain was close to the true strain at the PDL.

On the other hand, the true strain at the PDL in the smooth round tension cannot be measured directly from Fig. 6(a). In the smooth round tension, the relationship between the true stress and the nominal stress before local necking is expressed as follows:

$$F = \sigma_n \left(\frac{a_0}{a} \right)^2 \dots\dots\dots (18)$$

Substituting Eqs. (3) and (15) into Eq. (18),

$$\frac{c \cdot \varepsilon^n}{\exp(\varepsilon)} = \sigma_n \dots\dots\dots (19)$$

Fig. 10. The evaluation method for obtaining the radius of the neck curvature.

That is, the nominal stress is a function of the true strain. Strictly speaking, Eq. (19) becomes invalid after local necking, but Eq. (19) is still used to evaluate the true strain at the PDL as a first approximation. By assuming that the breaking point, B in Fig. 8 is equivalent to the PDL, P_f in Fig. 5 in the notched tension, $\sigma_n=307$ MPa was substituted into Eq. (19). The true strain at the PDL was estimated as 1.047. On the other hand, the post failure true strain was measured as 1.417. This data for the smooth round tension was plotted in Fig. 9 by an open circle. Since the data agreed with Eq. (17), we can assume that the true strain at the PDL is 1.047 in the smooth round tension.

4.3. The Evaluation of the Axial Stress at the PDL

Figure 5 shows that the true strain at the PDL decreases with the radius of the initial neck curvature. Therefore, the PDL cannot be determined by the true strain. The true strain at the PDL was also decreased significantly by the 30% compressive prestrain.

The axial stress is the highest at the center of the neck in the notched tension after general yielding as shown in Fig. 2. Earlier experiments have revealed that the fracture occurs at the center of the neck.^{10,11,15} Therefore, we can assume that the PDL is determined by the axial stress.

To evaluate the axial stress at the PDL by Eq. (5), the following procedure was used to obtain the radius of the neck curvature from the fractured specimen in the smooth round tension. An optical microscope (magnification: 50) was used to observe the neck bottom of a pair of the fractured specimens. The origin was chosen at the center of the neck bottom as shown in Fig. 10 with the X axis parallel to the longitudinal direction of the specimen and the Y axis parallel to the radial direction of the specimen. The coordinates X_0 and Y_0 on the neck profile were measured from the photograph. Then, the radius of the neck curvature, R_f was determined by solving the equations in Fig. 10. Y_0 was fixed at 0.2 mm to diminish the scatter of measurement. The radius of the neck curvature in the 1R, 2R, 5R and 10R notched specimens which were used in the other tensile tests¹⁶ was determined by the same method. Then, a/R (which is denoted as a neck ratio) was evaluated by measuring a_f , the radius of the neck section after failure. Figure 11 shows the relationship between the evaluated neck ratio, a_f/R_f and the initial one, a_0/R_0 (a_0 : the initial radius of the neck section, R_0 : the initial radius of the neck curvature).

Based on the evaluated neck ratio, a_f/R_f in Fig. 11, the axial stress at the PDL was evaluated by Eq. (5) for the 0% prestrain. In the notched tension, the true stress at the PDL was determined from Fig. 6(a). In the smooth round tension, the true stress at the PDL was estimated by substituting $\varepsilon=1.047$ into Eq. (15). a_f/R_f in the 15R notched specimen was estimated by the approximation in Fig. 11. Strictly

Fig. 11. The relationship between the initial a/R and the evaluated a/R after fracture (a : the radius of the neck section, R : the radius of the neck curvature).

Fig. 12. The axial stress-true strain relationship at the PDL.

speaking, a_f/R_f is the neck ratio after failure. To evaluate the axial stress correctly, it is necessary to use the neck ratio at the PDL. But as the first approximation, the axial stress at the PDL was evaluated using a_f/R_f .

Figure 12 shows the axial stress-true strain relationship at the PDL with a 0% prestrain. Since a_f/R_f was not evaluated for the 10% and 30% prestrain, the corresponding data were not plotted in the figure.

Figure 12 also includes the data of 1R circumferentially notched tension tests at -80 and -100°C that were carried out in other studies.¹⁶⁾ The 1R notched specimen was fractured in cleavage at -80 and -100°C . Since the true strain at the PDL is close to the post failure true strain in the cleavage fracture as shown in Fig. 9, the true strain was evaluated from Eq. (3), using a_f . On the other hand, the average stress was calculated from Eq. (1) using the breaking load P_f and a_f . Then, the true stress was evaluated from Eq. (8), and the axial stress was evaluated from Eq. (5), using a_f/R_f , which was determined in an optical microscope photograph.

The axial stresses at the PDL for the specimens in ductile fracture (solid circles in Fig. 12) were 1274 MPa for the smooth round tension, 1285 MPa for the 15R notched tension, 1259 MPa for the 5R notched tension, 1250 MPa for the 2R notched tension, and 1194 MPa for the 1R notched tension. Although the axial stress in the 1R notched tension was slightly smaller than others, the axial stress at the PDL was almost constant regardless of the initial radius of the neck curvature. Figure 6(a) shows that the true stress estimated by Eq. (8) at the PDL in the 1R specimen was slightly smaller than that in the other specimens. If the true stress-true strain curve of the 1R specimen agreed with the

approximation from Eq. (15), the true stress at the PDL in the 1R specimen would be 748 MPa. If the 748 MPa-true stress was substituted into Eq. (5), the axial stress at the PDL in the 1R specimen would be 1332 MPa.

In the 1R notched specimens in cleavage fracture (an open circle and open triangles in Fig. 12), the axial stresses at the PDL were 1309 MPa at -100°C , and 1294–1426 MPa at -80°C . Therefore, despite the scattering of data in Fig. 12, the axial stress at the PDL was about 1300 MPa regardless of ductile and cleavage fracture, and the ductile-cleavage transition range was from 0.3 to 0.45 in the true strain. This agrees with the result which was obtained by Otsuka *et al.*¹⁵⁾ that the true strain at the ductile-brittle transition was about 0.3.

4.4. The Significance of Circumferentially Notched Tension Test

The PDL is clearly different from the plastic instability. The plastic instability is defined by Eq. (14) thus, it can be determined in the true stress-true strain curve. On the other hand, the PDL can be determined from the axial stress. The work hardening exponent, n is now believed to be the most important factor in determining the PDL in hydroforming²⁾; however, the work hardening exponent is not related to the PDL, but to the plastic instability.

In the smooth round tension test, the true stress-true strain curve can be obtained up to the true strain at the local necking, which is about 0.2 in Fig. 6(a). On the other hand, in the notched tension test, the true stress-true strain curve can be obtained even after local necking up to the PDL as shown in Fig. 6(a). In the 15R notched tension, the true stress-true strain curve can be obtained up to the 0.8 true strain. Therefore, the evaluated range of the true stress-true strain curve in the 15R notched tension is 4 times wider than that of the smooth round tension. Moreover, the approximation of Eq. (15) is valid up to the 0.8 true strain.

The most important factor is for the axial stress at the PDL in the tested material to be about 1300 MPa. Then it will become possible to evaluate the PDL of a stress concentrated region in a steel-frame building (such as a joint between the beam and the column) or of hydroforming by using this value. Numerical simulation is necessary to carry out these evaluations.

5. Conclusions

The plastic deformation limit in a hot rolled JIS SM490B steel was evaluated by a circumferentially notched tension test and a smooth round tension test. The main conclusions are summarized as follows:

(1) There are two stages in the circumferentially notched tension: the plastic instability and the plastic deformation limit (PDL). The PDL is defined as a stress drop point in an average stress (or true stress)-true strain curve. There are three stages in the smooth round tension: the plastic instability, the local necking, and the PDL.

(2) A true stress-true strain curve is unique regardless of the circumferentially notched tension and the smooth round tension. A true strain at the plastic instability is determined uniquely by a work hardening exponent, n , regardless of the initial radius of the neck curvature.

(3) A true strain at the PDL varies with the compressive prestrain and the initial radius of the neck curvature. The PDL is not determined by the true strain. The true strain at the PDL is different from a post failure true strain.

(4) According to the evaluation method in this paper, an axial stress at the PDL was about 1 300 MPa, regardless of ductile and cleavage fracture in the tested hot-rolled JIS SM490B steel.

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