Infiltration Phenomena of Molten Powder in Continuous Casting Derived from Analysis Using Reynolds Equation

Part 2: Unsteady Analysis

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A new type of model for the powder infiltration in continuous casting of steel is developed. Succeeding the previous report on a steady state analysis, this paper presents an unsteady model that uses Reynolds equation and the equation of the motion of the solidified shell as fundamental equations.

The analysis clarifies in details the cyclic movement of a solidified shell during mold oscillation. Mold powder can be infiltrated because the cyclic movement of the shell lags behind the mold oscillation. This provides a period when the mold/cast metal gap opens, during the downward motion of the mold. In this period, molten powder is infiltrated down into the channel. The model also clarifies that the powder infiltration is greatly influenced by two factors; the peak distance between solidified shell and mold wall during the oscillation, and the phase shift of the movement of solidified shell from the mold oscillation. Furthermore, the tendency of the powder consumption to decrease with increasing the powder viscosity can be easily understood through the mechanism of the powder infiltration derived from the analysis.

Those results indicate that the model essentially reveals the unsteady behavior of powder infiltration during the mold oscillation.

KEY WORDS: slag; lubrication; continuous casting; powder infiltration; mold oscillation model.

1. Introduction

In the preceding paper,¹⁾ we reported the analysis results of the infiltration behavior of molten casting powder into the gap between a mold and a cast metal in the continuous casting process of steel. The analysis assumed that the distance between the mold and the cast metal can be changed, and used a stationary-state model that incorporated the Reynolds equation not considering the time term as one of the fundamental equations. The analysis clarified some characteristics of the movement of the solidified shell caused by the vertical movement of the mold. The clarified characteristics included the following:

- (1) The shape of the mold/cast metal gap filled with the molten powder determines the behavior of powder infiltration into the gap, and for the molten powder to flow into the gap it is important that the gap is wider downward.
- (2) The cast metal moves away from the mold when the mold moves upward.
- (3) The cast metal moves closer to the mold when the mold moves downward.

However, these results were obtained assuming a stationary state, and therefore the analysis could not clarify the non-stationary infiltration behavior of the powder under the influence of mold oscillation. Nor could it clarify the influ-

ence of casting conditions such as the viscosity of the molten powder on the infiltration behavior in detail.

This paper introduces a new powder infiltration model that uses the Reynolds equation, used also for the model of the preceding paper,¹⁾ and an equation of motion of the solidified shell as the fundamental equations and applies these equations to a non-stationary state. The paper also discusses the analysis results obtained by using the model, namely the mechanism of the molten powder infiltration and the influence of the viscosity of the molten powder on its consumption.

2. Outline of Analysis

2.1. Method of Analysis

The subject region of the analysis is the two-dimensional area of the gap between the mold and the cast metal from the level of the upper surface of the molten steel down to the lower end of the region filled with the molten powder (see Fig. 1).

The analysis employed the following fundamental equations:

$$\frac{\partial}{\partial x} \left[\frac{\rho h^3}{12\mu} \left(\frac{\partial P}{\partial x} - \rho g \right) \right] = \frac{V_c + V_o}{2} \frac{\partial \rho h}{\partial x} + \rho \frac{\partial h}{\partial t} \quad \dots (1)$$

Fig. 1. Analysis region and notations.

$$\frac{d^2h}{dt^2} + B \frac{dh}{dt} + Kh = \frac{1}{M} \int_0^L P dx \quad \dots\dots\dots(2)$$

where, ρ indicates the density of the molten powder, h the distance between the mold and the cast metal, μ the viscosity coefficient of the molten powder, P the pressure, g the gravitational constant, V_c the casting speed, V_o the velocity of mold oscillation, x the distance in the casting direction, t time, and L the depth of the region filled with the molten powder. These parameters are shown also in Fig. 1. With respect to the mold/cast metal gap viewed two-dimensionally, Eq. (1), the Reynolds equation, satisfies the law of conservation of mass, which is expressed as

$$\int_0^h \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) dz + \frac{\partial h}{\partial t} = 0 \quad \dots\dots\dots(3)$$

where, u indicates the velocity in the casting direction, w the velocity in the direction normal to the mold surface, and z the distance in the same direction. Equation (1) is obtained by replacing Eq. (3) using Eqs. (4) and (5), which are obtained from the Navier–Stokes equations. (How to obtain Eqs. (4) and (5) was explained in the preceding paper.¹⁾)

$$u = \frac{1}{2\mu} \left(\frac{\partial P}{\partial x} - \rho g \right) (z^2 - zh) + \frac{z}{h} V_c + \frac{h-z}{h} V_o \quad \dots\dots(4)$$

$$w = \frac{V_{cz}}{h} z \quad \dots\dots\dots(5)$$

Since $\partial h / \partial x$ in Eq. (3) represents the powder inflow and outflow at the upper and lower ends of the subject region of analysis, $\partial h / \partial x$ in Eq. (1) represents the same. The analysis region is very elongated: the mold/cast metal distance is in the order of several hundred micrometers, and the depth of the region filled with the molten powder is roughly 10 cm. Considering another dimension of the two-dimensional analysis region in the horizontal direction along the mold surface, when the mold/cast metal distance fluctuates, most of the powder that flows into or from the elongated analysis region is presumed to occur not vertically at the upper and lower ends but horizontally along the mold surface (see Fig. 2). Based on this presumption, we assumed that the value of $\partial h / \partial x$ was very close to zero, and omitted it from Eq. (1).

Equation (2) is the equation of motion of a solidified shell. The first term on the left side of the equation represents the inertial force, and the third term the restoring force (stiffness) of the solidified shell. The second term on

Fig. 2. Inflow and outflow of casting powder around analysis region.

the left side is an attenuation term representing the resistance force from outside against the movement of the solidified shell. The above means that B , K and M represent the resistance coefficient, modulus of elasticity and mass of the solidified shell, respectively.

Equation (1) can be solved without $\partial h / \partial x$ and yields

$$\begin{aligned} \int_0^L P dx = & 6\mu(V_o + V_c) \left(F_1(L) - F_2(L) \frac{f_1(L)}{f_2(L)} \right) \\ & + P_0 \left(L - \frac{F_2(L)}{f_2(L)} \right) + \frac{F_2(L)}{f_2(L)} P_1 \\ & + \left(\frac{\rho g L^2}{2} - \frac{F_2(L)}{f_2(L)} \rho g L \right) \quad \dots\dots\dots(6) \end{aligned}$$

It is provided here that

$$f_1(x) = \int_0^x \frac{dx}{h^2} \quad \dots\dots\dots(7)$$

$$f_2(x) = \int_0^x \frac{dx}{h^3} \quad \dots\dots\dots(8)$$

$$F_1(x) = \int_0^x f_1(x) dx \quad \dots\dots\dots(9)$$

$$F_2(x) = \int_0^x f_2(x) dx \quad \dots\dots\dots(10)$$

(How to obtain Eq. (6) was explained also in the preceding paper.¹⁾)

The consumption of the powder Q is expressed by the following equation:

$$Q = - \frac{\rho}{12\mu f_2(L)} [P_1 - P_0 + \rho g L - 6\mu(V_o + V_c) f_1(L)] \quad \dots\dots\dots(11)$$

where, P_1 is the pressure at the lower end of the region filled with the molten powder, and P_0 is the pressure at the level of the upper surface of the molten steel. How to obtain Eq. (11) was explained in the preceding paper.¹⁾ Since the present analysis is a non-stationary-state analysis, the value of Q changes over time.

Furthermore, when the mold/cast metal distance h is zero, the pressure diverges, becoming equal to negative infinity. In reality, however, when h is zero, the cast metal deforms or the molten powder turns into a compressible fluid, and it is very difficult to mathematically deal with such a condition. For this reason, for the present analysis we set forth a condition that, when the solidified shell moved closer to the mold, the distance between them would not become less than a certain minimum. This minimum is hereinafter called the closest distance.

In an iterative procedure, the value of $\int_0^L P dx$ was obtained by solving Eq. (6). The mold/cast metal distance h was calculated from $\int_0^L P dx$ thus obtained and Eq. (2). The value of Q was calculated from h thus obtained and Eq. (11). Then, the value of $\int_0^L P dx$ was calculated again using Eq. (6) after renewing time. Then, the above procedures were repeated.

2.2. Analysis Conditions

The system used for the present analysis was a simplified one in which the contour of a cast metal was approximated using straight lines (see Fig. 3). In addition, since the surface of a real cast metal is considered to lean such that the mold/cast metal distance increases downward,¹⁾ we assumed that the mold/cast metal distance would increase by 0.023 cm every 10 cm in the downward direction. The powder viscosity was set from 1 to 5 poise at an interval of 1 poise, the casting speed at 1.4 m/min, and the depth L of the region filled with the molten powder at 10 cm. The boundary conditions of pressure were set as follows:

$$P_0 = \rho g d \quad \text{and} \quad \dots \dots \dots (12)$$

$$P_1 = 0 \quad \dots \dots \dots (13)$$

where, d is the thickness of the powder pool above the cast metal, and was set at 1 cm. The closest distance between the mold and cast metal defined earlier was set at 1 μm .

Fig. 3. Analysis region.

As explained before, B , K and M in Eq. (2) represent the resistance coefficient, modulus of elasticity and mass of the solidified shell, respectively. It must be noted here that correct analysis is impossible using the values of the modulus of elasticity and mass only in the analysis region. This is because a solidified shell extends outside the analysis region downwards as well as horizontally along the mold surface, and the portions outside the analysis region inevitably influence the values of the modulus of elasticity and mass in the region. For this reason, it is difficult to determine the values of K and M . In consideration of the above, their values were determined through fitting so that the analysis results fell within respective reasonable ranges. With respect to B , an analogy of the resistance acting on a sphere in a fluid²⁾ indicates that the resistance coefficient B depends on the viscosity or the Reynolds number of the fluid. The Reynolds number is a dimensionless number that expresses degree of the turbulence of molten powder flow, and it is affected by the casting speed, the cycle and stroke of mold oscillation, and so forth. All these mean that the value of B depends on the viscosity coefficient of the molten powder, the casting speed, the cycle and stroke of mold oscillation, and other factors. For the present analysis, we assumed that

$$B = \mu B' \quad \dots \dots \dots (14)$$

Because the objective of the present paper is to clarify the dependency of the powder infiltration solely on the powder viscosity, we assumed that B depended only on the viscosity coefficient of the powder, and disregarded the influences of casting speed and the cycle and stroke of mold oscillation. Note that the value of B' was determined also by fitting. The appropriateness of the fitted values of M , K and B' used for the analysis are discussed later.

Table 1 shows specific values of the above analysis parameters and other related items.

3. Powder Infiltration Mechanism

3.1. Analysis Results

Figure 4 shows the changes of the mold velocity, mold/cast metal distance and powder consumption during a cycle time of mold oscillation obtained through the analysis. The lower part of the figure schematically shows the behavior of the casting powder in the mold/cast metal gap presumed from different phases of the curves.

3.2 Discussion

3.2.1. Relation between Mold Oscillation and Powder Infiltration

The results of the stationary-state model¹⁾ showed that, when the mold moved upward, force acted in the direction to separate the solidified shell from the mold and the

Table 1 Analysis conditions.

Casting speed V_c	$2.33 \times 10^{-2} (\text{m/s})$	Mold oscillation cycle	$2.17 (\text{c/s})$
Viscosity μ	$0.1, 0.2, \dots, 0.5 (\text{Pa}\cdot\text{s})$	Mold oscillation stroke	$0.0068 (\text{m})$
Closest distance	$1 \times 10^{-6} (\text{m})$	Thickness of molten powder layer d	$0.01 (\text{m})$
M	$64000 (\text{Kg/m})$	Powder-filled depth L	$0.1 (\text{m})$
Fitting parameters K	$20/M (\text{s}^{-2})$		
B'	$2.63/\mu M (\text{m/Kg})$

Fig. 4. Behavior of solidified shell and casting powder.

Fig. 5. Behavior of casting powder in stationary analysis.

mold/cast metal gap would open, and when the mold moved downward, on the contrary, force acted in the direction to attract the solidified shell to the mold and the mold/cast metal gap would decrease (see Fig. 5). The model also showed that the mold/cast metal gap was open during the upward movement of the mold, but since the molten powder also moved upward together with the mold, it would not flow into the gap. This means that the stationary-state model explained the opening of the mold/cast metal gap, but failed to explain the infiltration of the molten powder into the gap. The reason for the above is that the mold/cast metal gap is not open during the period of the downward move of the mold, when the powder can move downward.

On the other hand, according to the results of the non-stationary-state model, it is clear from Fig. 4 that the solidified shell begins to move away from the mold shortly after the mold begins to move upward (Period a); the powder consumption at this time is negative because the mold is also moving upward. The figure also shows that the solidified shell remains away from the mold even when the mold begins to move downward and the force acts to attract the solidified shell toward the mold (Period b), presumably because of the inertia of the shell. At this phase, the powder consumption becomes positive because the mold moves

downward and so does the molten powder; thus the molten powder flows into the gap. Thereafter, immediately after the mold begins to move upward, the cast metal comes closest to the mold (Period c).

The above shows that the periodical shifting of the solidified shell has a certain time lag in relation to the oscillation of the mold. Since there is a time period when the mold moves downward and the cast metal is away from the mold, the molten powder can flow into the gap.

3.2.2. Factors Determining Powder Infiltration Amount

The powder infiltration mechanism figured out through the model analysis indicates that two factors significantly influence the molten powder infiltration into the mold/cast metal gap.

According to the model, the gap widens and narrows as a result of the mold oscillation. The larger the gap width, the better for the powder infiltration. In addition, the larger the downward velocity of the mold when the mold/cast metal gap is at the largest, the larger the amount of powder infiltration becomes.

The schematic diagram of the mold velocity and the mold/cast metal distance shown in Fig. 6 expresses these two factors. The larger the peak value of the mold/cast metal distance (hereinafter referred to as the peak distance), and the larger the time lag between the time of the peak distance and the time of a peak of the mold velocity curve (hereinafter referred to as the phase shift), the larger the powder consumption becomes.

Unless the wave form of the mold/cast metal distance changes significantly, only these two factors determine the powder consumption. On the other hand, Fig. 7 compares the changes of the mold/cast metal distance and the mold velocity in two cases of different values of the molten powder viscosity. The graph shows that, while the magnitude of

the peaks of the mold/cast metal distance are not significantly different between the two cases, the phase shift is smaller in the larger viscosity case than in the smaller viscosity case. From the influence of the powder viscosity on the phase shift, one can deduce the generally known tendency of the powder consumption to decrease with increasing powder viscosity.³⁾ In other words, of the two factors that influence the powder consumption, the phase shift is the cause of the tendency of the powder consumption to decrease with increasing powder viscosity. This is discussed in more detail in the following section.

The above two factors are a totally new concept obtained from a viewpoint different from those of past studies.

Fig. 6. Two factors influencing powder infiltration.

Fig. 7. Wave forms of mold velocity and mold/cast metal distance in two viscosities.

(a) Peak distance

4. Dependency of Molten Powder Infiltration on Viscosity

4.1. Analysis Results

Figure 8 shows the influence of the molten powder viscosity on the powder consumption obtained through the analysis. The powder consumption value is the average of Q of Eq. (11) for a period of 6 s. The graph also shows the observed values in the operation of a real continuous caster (marked with x).

4.2. Discussion

The analysis results and actual measurements shown in Fig. 8 agree well with each other in terms of magnitude and tendency. This seems to corroborate the appropriateness of the fitted values of B' , K , M and other parameters used for the analysis. Figure 8 shows, in addition, the well-known tendency of powder consumption to decrease with increasing powder viscosity.³⁾

Figure 9 shows the influence of the molten powder viscosity on (a) the 6-s average of the peak distance and (b) that of the phase shift. As seen in the figure, the peak distance seems neither to steadily increase nor to steadily decrease as the powder viscosity increases. On the other hand, the phase shift tends to decrease. It is inferred that the decrease in the powder consumption with increasing powder viscosity results from this decreasing tendency of the phase shift.

Fig. 8. Influence of viscosity on powder consumption.

(b) Phase shift

Fig. 9. Influence of viscosity on peak distance and phase shift.

In this connection, a larger viscosity causes that, when a wall begins to move, the flow of a viscous fluid contacting the wall grows within a shorter time and the flow velocity is larger. This is clear also from the solutions of the fundamental equations of some basic phenomena such as the Couette flow in which a wall contacting a fluid suddenly begins to move.⁴⁾ On the other hand, a movement of the mold, namely a wall, causes the molten powder, namely a viscous fluid, to flow and the flow of the molten powder causes the solidified shell to vibrate. The above relations explain why the phase shift between the curves of the mold velocity and the mold/cast metal distance decreases with increasing powder viscosity. It also explains why both the force to separate the solidified shell from the mold as the mold moves upward and that to attract the shell to the mold as it moves downward increase with increasing powder viscosity. Since the value of the peak distance is influenced by these two mutually opposing factors, it is impossible to predict simply whether it increases or decreases with increasing powder viscosity. The average peak distance in Fig. 9(a) has a tendency to be roughly constant in relation to the powder viscosity, presumably because the opposing influences of the two factors are nearly balanced.

Thus, with using the model, the tendency of the powder consumption to decrease with increasing of the powder viscosity can be easily understood.

5. Characteristics of Analysis Model

The model used for the present analysis is a novel model that simultaneously solves the Reynolds equation and the equation of motion of the solidified shell and considers the change of the mold/cast metal distance over time. Thanks to these characteristics, the model yielded unique results unprecedented in past studies. One of the results of the model is the relation between mold oscillation and powder infiltration. All conventional models concluded that casting powder infiltrated into the mold/cast metal gap stably regardless of mold oscillation. According to the model of the present analysis, on the other hand, the powder flows into the gap only when the mold is oscillated. This means that the present model is the only model that maintains that continuous casting operation is impossible without mold oscillation.

Another unique finding of the new model quite different from those of conventional models is that the motion of a solidified shell is significantly different depending on the shape of the gap into which the molten powder flows: the model shows that the infiltration behavior of the powder is significantly different depending on whether the mold/cast metal gap becomes narrower downward or wider (see Fig. 9 of the preceding report¹⁾).

What is more, the model of the present analysis showed that the gap between a mold wall and a cast metal opened when the upward velocity of the mold was larger than the casting speed.¹⁾ The above indicates that the new model points to the well-known fact that continuous casting operation is viable only when there is a negative strip time in mold oscillation. As far as we know, the new model is the only one that is capable of mathematically deriving this fact.

According to the new model having the above character-

istics, the period during which the molten powder flows into the mold/cast metal gap is from the end of a positive strip time of mold oscillation to the beginning of the next positive strip time, including all the negative strip time in between (see Fig. 4). In the past, some maintained that the molten powder flowed into the gap during a positive strip time,^{5,6)} and others during a negative strip time.^{7,8)} As far as the timing of the molten powder infiltration is concerned, the new model may be viewed as being close to the negative-strip time theory, but the infiltration mechanism that the model describes is significantly different from those that have been proposed so far: the new model states that the powder infiltration occurs because the opening and closing of the gap due to the movement of the solidified shell lags behind the vertical movement of the mold (see Fig. 4). The positive-period theory, on the other hand, bases itself on the fact that the amount of powder infiltration increases with longer positive periods. However, according to the powder infiltration mechanism of the present analysis, the powder infiltrates into the gap during a negative period, because cast metal moves away from the mold owing to lift during a positive period and the mold/cast metal gap thus formed is maintained until the succeeding negative period. This means that the gap does not open without a positive period, and that the longer the positive period, the larger the gap becomes and the better it is for the powder infiltration. As described above, the powder infiltration mechanism according to the present analysis can explain, albeit qualitatively, the relation between the length of a positive period and the amount of powder infiltration.

6. Summary

To clarify the non-stationary behavior of the infiltration of molten casting powder into the gap between a mold and a cast metal, we formulated a model using the Reynolds equation and the equation of motion of a solidified shell as the fundamental equations, and analyzed the non-stationary behavior of the powder infiltration using the model.

The analysis clarified that the mold/cast metal gap opened shortly after the mold began to move upward, the gap remained open even when the mold began to move downward and thereafter, and it closed shortly after the mold began to move upward again. The analysis also clarified that the powder flowed into the gap during the period from the end of a positive strip time of mold oscillation to the beginning of the next positive strip time, including all the negative strip time in between.

The analysis clarified further that the powder infiltration was determined by two factors, namely (1) the peak value of the vibration of the solidified shell and (2) the phase shift between the curves of the mold oscillation velocity and the vibration of the solidified shell. Thus, it became possible to explain the dependency of powder consumption on the viscosity of the molten powder using the two factors.

Thus, the model clarified the essentials of the non-stationary behavior of the powder infiltration due to mold oscillation.

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