Ductile Forming Limit Stress in Sheet Forming Processes with Nonlinear Strain Path Involving Out-of-plane Forming

Takaaki IGUCHI

Steel Research Laboratory, JFE Steel Corporation, 1 Kawasaki-cho, Chuo-ku, Chiba 260-0835 Japan.

(Received on November 7, 2006; accepted on January 15, 2007)

The ductile forming limit in nonlinear strain paths which include out-of-plane forming such as rolling was investigated for a 11% Cr steel sheet. For strain paths with only a plane stress state, e.g., in-plane strain, the forming limit stress diagram (FLSD) has been proven to be effective, even if the paths are nonlinear. In this research, a method of extending the FLSD theory to strain paths which include out-of-plane strain is investigated experimentally and theoretically. An experimental procedure is used to measure directly the stresses at the onset of diffused and localized necking and at the onset of separation with rolling-stretching loading paths. Theoretically, an assumption is introduced to consider out-of-plane strain in the M-K theory. The experimental and theoretical results of the forming limit stress show good agreement. When large rolling strain is induced in first-stage loading paths, this research demonstrated that there are cases where the FLSD theory is not effective for predicting the ductile forming limit. Those cases are characterized by a stress state of the re-yielding point in the final loading stage. If the point is outside the FLSD curve, the forming limit stress is not on the FLSD curve but located near the subsequent yield surface. Thus, a modified FLSD curve can be defined by the lines connecting the outer line of the conventional FLSD curve and the subsequent yield surface of the loading path under consideration. With this extension, it is possible to demonstrate the applicability of the FLSD method to processes which include out-of-plane loading paths such as rolling.

KEY WORDS: 11% Cr steel; sheet forming; forming limit; yield criterion; rolling; nonlinear path; out-of-plane strain.

1. Introduction

Sheet metal forming usually means a forming process in which the stress condition in the material sheet is assumed to be a plane stress state. The subjects of consideration are in-plane strain or stress and bending strain or stress. In the thickness direction, only thickening or thinning as a result of in-plane deformation is considered.

On the other hand, forming which is positively applied in the thickness direction, such as rolling and ironing, should in principle be classified as a type of bulk forming because the stress and strain condition is 3-dimensional, even if the material being deformed is a thin sheet metal.

Forming limit criteria (only the ductile forming limit is considered in this paper) for sheet metal forming processes such as stamping have been the subject of research by many researchers from an early date. The most popular and primitive method is the strain-based Forming Limit Diagram (FLD). The FLD is known to be useful when the loading path is linear, that is, the ratio between the principal stresses or strains remains constant throughout the forming process.

However, it is also well known that the FLD is completely useless in the case of nonlinear loading. One of the methods on which the authors are focusing to overcome this problem is the stress-based forming limit theory. This theory determines a unique limit line for sheet forming in a principal stress field, not in the strain field, independently of strain path. In this paper, the diagram for the stress field is called a Forming Limit Stress Diagram (FLSD). This theory enables an easy evaluation of the forming limit in all nonlinear loading paths by the same FLSD, which is determined by linear loading paths involving various strain or stress ratios.

Here, it must be mentioned that the theory is not derived from a pure ductile fracture theory of metals, but rather is obtained empirically. Therefore, the applicability of the method should be verified in each individual case of forming and material. It should also be mentioned that the fact that the stresses used as an index cannot be measured directly or easily is a disadvantage of the theory for practical use.

Certain researchers have verified the theory experimentally and theoretically from the viewpoints mentioned above. In this case, the limit was verified mainly by stresses evaluated by transforming the strain limit using plastic deformation theory. Some researchers have also evaluated and verified the stress limit by a numerical method based on the M-K theory. For example, Yoshida et al. directly measured the limit stress in the case of aluminum tube forming. Those researchers demonstrated that the stress-based forming limit theory is quite appropriate in all cases. The authors of this paper have also proven that the theory is
applicable in some cases of nonlinear loading in previous research\textsuperscript{12,13} dealing with 11\% Cr steel sheets, which display large anisotropic behavior. Furthermore, the limit stress was measured directly in some loading cases, and it was then demonstrated that the measured limit stress coincides closely with the theoretically-evaluated one.\textsuperscript{13} From the research mentioned above, it can be concluded that the stress-based forming limit theory is applicable to almost all cases.

However, all of the studies mentioned above are confined to the assumption of a plane stress state, and no research has shown evidence as to whether the theory is applicable to nonlinear loading which involves bulk forming. In the case of ironing, for example, it is apparent that a strain-based FLD is not suitable for evaluating the forming limit. Whether the stress-based theory is applicable to cases of this type or not is a quite interesting issue, but no research has provided an answer to this question.

The most significant difference between sheet forming and bulk forming is the difference in hydrostatic stress, which is usually tensile in sheet forming and compressive in bulk forming. For example, let us consider a sheet which was rolled at a certain elongation and another sheet which was stretched in a plane strain condition to the same elongation. Both have basically the same plastic strain. If the sheets are then stretched in the following process, and the material shows isotropic hardening behavior, the stress which arises in the material should be identical in both sheets. Therefore, the forming limit given by the stress-based theory should also be identical.

However, ductile fracture theory based on void growth\textsuperscript{14-16} says that the limits of these two sheets are different because their stress histories are different. This is because void growth mainly depends on hydrostatic pressure. In the case described above, the sizes of the microroids in the material are different in the two materials before the stretch loading stage, and it is therefore easy to conclude that their limits should be different.

Furthermore, bulk forming can generate much higher strain than the usual forming limit in sheet forming because deformation of the bulk takes place under a compressive stress state. This also makes the problem more complicated.

This paper especially concerns verification of the FLSD theory in the case of a forming process involving such out-of-plane deformation. For this purpose, an approach similar to that utilized in the author’s previous research\textsuperscript{12,13} is employed. Furthermore, extension of the M-K theory to predict the forming limit is also investigated.

2. Rolling-stretching Test

2.1. Rolling-uniaxial Strain

The material investigated throughout this research is cold-rolled sheets of 11\% Cr steel with a 1.2 mm thickness. The true stress–true strain curves obtained by uniaxial tension tests in the longitudinal (L), cross (C), and diagonal (D) directions of the mother sheet are shown in Fig. 1. The $r$-values in the three directions, which are evaluated with a tensile test specimen at 15\% strain, are $r_L = 1.570$, $r_C = 1.963$, $r_D = 1.164$.

The mother sheets were rolled at various reduction ratios, then cut to tensile test specimens. The forming limit strains and stresses were measured in the tensile test.

Combining the direction of rolling and direction of mother material, four types of specimens were prepared, as shown in Fig. 2. In the following denotation, the direction of mother material L and C and the direction of rolling R (rolling) and T (transverse) are combined and denoted as L-R, etc. For convenience, the discussion will begin with a detailed description of the results for only L-R.

The author showed in previous research\textsuperscript{13} that, in the tensile test, the measured stress at the onset of localized necking coincides with the forming limit stress predicted by the mathematical method. Here, we will attempt to verify if this is also true in the present case. This can be done by measuring stresses and strains at the following points, and comparing them with the strain of rolling applied as first-stage loading.

(1) Onset of diffused necking (or maximum nominal stress)
(2) Onset of localized necking
(3) Onset of separation

The above necking points are detected in this method by video-recording the behavior of the specimens. By referring to time, the corresponding tensile load can be determined from the time–load curve. For the diffused necking point (1), theoretically it coincides with the maximum nominal stress point. Hence, the diffused necking point can also be defined as the maximum nominal stress point. In actual cases, the optically-detected necking point usually comes well after the maximum stress points. This is because diffused necking occurs so gradually that time is required to
detect this type of necking optically. The true strains and stresses at the detected point are then defined as the diffused necking strains and stresses. Here, longitudinal strain is evaluated as the logarithmic strain measured by the gauge length \( \frac{GL}{H} \approx 50 \text{ mm} \). The strain is considered to be true strain because the deformation which occurs before diffused necking is uniform. Next, the cross-sectional area of the specimen under loading is evaluated using the constant volume law, and the true stress is evaluated by dividing the tensile load by the cross-sectional area.

The localized necking point (2) can be detected easily by video-recording the specimen. Figure 3 shows a view of the specimens at the onset of localized necking. Figure 3(a) shows that of material with small rolling strain of about 0.1, and Fig. 3(b) shows that with large rolling strain of about 0.8. Throughout this paper, the rolling strain is defined as a logarithm of the thickness reduction ratio. As shown in these figures, localized necking occurs following diffused necking when the rolling strain is small. However, when the rolling strain is large, localized necking occurs immediately without diffused necking.

The problem is how to evaluate true strain at the onset of localized necking. Generally, deformation is no longer uniform at the localized necking point, so strain in local areas cannot be evaluated by longitudinal displacement with a certain gauge length. Therefore, for direct measurement of the thickness and width of the necking point in the specimen, measuring instruments must be installed around the necking point. Practically speaking, this is very difficult because the necking position is not always in the same position. Here, we will utilize a method presented in the author’s previous paper. Figure 4 shows the points to be measured on the specimens after the tensile tests. In this method, it is considered that the portion outside the localized necking band is unloaded and deformation no longer continues after localized necking occurs. Hence, the cross-sectional area of the specimen at the onset of localized necking is evaluated by measuring the thickness and width at the boundary of the necking band and the outside portion. The logarithm of the reduction ratio of the cross-sectional area is defined as true strain, and the stress obtained by dividing the tensile load by the cross-sectional area is defined as true stress at the onset of localized necking.

For the onset of separation (3), strains are evaluated by direct measurement of the cross-sectional area of the fracture surface of the specimen after separation, and stresses are evaluated by dividing the tensile load by the cross-sectional area. The tensile load at the onset of separation is determined from the time detected in the video. Figure 5(a) shows an example of a nominal stress–strain curve with small rolling strain of 0.1. Figure 5(b) shows that with large rolling strain of 0.8. The necking points which are detected by the above-mentioned method are plotted in the figures.
served when rolling strain is small, but the point comes well after the maximum stress point which is observed immediately after the yielding point. That is, the two points do not coincide. In the following discussion, the maximum stress point is considered to be a true diffused necking point for convenience to discuss including the case where the diffused necking is not observed optically. The localized necking point is observed at the point where a rapid decrease of tensile load commences. As shown in Fig. 5(b), when rolling strain is large, almost no diffused necking is observed and localized necking is observed first. The maximum stress point occurs immediately after yielding, then necking occurs after some further deformation.

**Figure 6** shows the relationship between rolling strain and the measured stress at the maximum stress point (as diffused necking stress). The stresses are not constant with respect to rolling strain as pre-strain, and are rather similar to those in the stress–strain curve. Anisotropy is observed in the measured stress, depending on the rolling direction, but is independent of the direction of material.

**Figure 7** shows the relationship between rolling strain and the measured true stress at the onset of localized necking. When the rolling strain is small, stress is larger than that in Fig. 6, but the difference becomes small when rolling strain is large. This is because the localized necking point is closer to the maximum stress point when rolling strain is larger. Furthermore, stress is not constant with respect to rolling strain as pre-strain. This means that the forming limit stress is not applicable. However, stress is almost constant when rolling strain is within the maximum uniform strain of the material, which is approximately 0.3 as shown in Fig. 1. Anisotropy is also controlled by the direction of rolling, as in the case of diffused necking stress.

**Figure 8** shows the relationship between rolling strain and stress at the onset of separation. The stress is far higher than that in Figs. 6 and 7, and the relationship with rolling strain is not clear.

### 2.2. Rolling-plane Strain

As the next loading path, the case of rolling-plane strain is considered. In this case, only the L-R test was conducted, in which the rolling direction is coincident with the longitudinal direction of the mother material and the specimen is cut in the same direction as the rolling direction.

In order to realize a plane strain condition in tensile tests, the slit tensile specimens shown in **Fig. 9** and **Fig. 10** were utilized, as discussed in the author’s previous paper. The same procedure as mentioned above is utilized to measure the necking points, strains, and stresses. In this plane strain case, it is almost impossible to detect the diffused necking points. For this reason, the maximum stress points are always considered to be the diffused necking points. Localized necking can be detected by the above-mentioned method without problem, as shown in **Fig. 11**. Strain at the onset of localized necking is evaluated by measuring width and thickness shown in **Fig. 10**. Separation point will not be discussed anymore because it is already clear that this point has not significance in the forming limit.
The relationship between rolling strain and the stress at the onset of diffused necking as well as the relationship between rolling strain and the stress at the onset of localized necking are shown in Fig. 12. When rolling strain is small, the stress of localized necking is larger than that of diffused necking, but the difference becomes smaller when the rolling strain is larger. Stress is not constant with respect to rolling strain except for localized necking when rolling strain is small. These tendencies are similar to those with the rolling-uniaxial loading path. The stress level is slightly smaller than that with the rolling-uniaxial loading path.

2.3. Rolling-biaxial Strain

The following case is rolling-biaxial strain. The biaxial strain condition is realized by spherical stretching with a punch of 20 mm diameter. A view of the stretched specimen is shown in Fig. 13. In this case, stress under loading cannot be measured directly. Thus, the method of conversion from strain proposed by Stoughton7) is utilized only for this case. Stretching is stopped immediately after localized necking is observed, and strain is measured using a 2 mm square mesh pattern printed on the specimen, as shown in Fig. 13. Diffused necking is not observed in this test. As shown in the figure, necking occurs in a round manner around the top of the punch, but it starts at a certain point on the necking band. The nearest mesh size to this point is measured in order to evaluate strain. It must be noted that this definition of strain is different from that in the case of uniaxial and plane strain, but strain is evaluated with a gauge length of 2 mm. Biaxial stress is then calculated from strain by the method described in detail in a previous paper.12)

The calculated biaxial stresses as the major and minor principal stresses are shown in Fig. 14, where the prestrained material by rolling is prepared in only L-R manner. Stress is constant with respect to changes in rolling strain, which means that the forming limit stress is applicable only in this case.

3. Comparison of Measured Stress with M-K Theory

The M-K theory10) is extended and applied to the case of rolling-stretch loading paths. The procedure mentioned in a previous paper by the author13) is utilized here. In order to apply the theory to nonlinear loading that includes rolling strain which is out-of-plane strain, the following assumption will be introduced. Namely, rolling strain has an effect comparable to that induced by stretching which introduces identical final displacement. This assumption implies that if the material is rolled in one direction with almost no width spreading, the result is comparable to plane strain stretching. However, it should be emphasized that, from the viewpoint of stress, these are essentially different modes of deformation, viz., even though the final displacements are identical, the former is deformation under compressive stress and the latter is deformation under tensile stress. Actually, in the latter case, fracture occurs when strain reaches some level, whereas practically no forming limit exists in the former case. Nevertheless, under the assumption here, both strains are considered to be comparable, considering only final displacement, which is actually true from the viewpoint of plastic theory, in that hydrostatic stress has no influence on yielding. The rolling strains are considered in the numerical model in the same way as stretching without
an initial imperfection, and for this reason, fracture does not occur is the first stage in the model. The initial imperfection, which is 0.995 as the thickness ratio of perfect and imperfect portion of the considered material, is introduced in second stage loading. As an additional assumption, the material is considered to show perfectly isotropic hardening behavior, which means that yield stress is identical under tensile and compressive conditions. In actual case, there is some orthogonal anisotropy in work-hardening behavior as shown in Figs. 6–8, but it will be also neglected in following discussion.

3.1. Rolling-uniaxial Strain

The forming limit stress evaluated by the extended M-K theory in the case of rolling-uniaxial stretching is shown in Fig. 15 as the relationship with rolling strain. In the figure, the stress measured at the onset of diffused necking and onset of localized necking, as well as the uniaxial stress–strain curve, are simultaneously plotted for comparison. Here, all data are for the case of L-R. The equation for the uniaxial stress strain curve is as follows:

\[
\bar{\sigma} = 735.0 (\bar{\varepsilon} + 0.001)^{0.25} \text{ (MPa)} \tag{1}
\]

This is obtained by fitting the power law equation to the results of the uniaxial tension test in the L-direction shown in Fig. 1, which is same as in calculations with the M-K theory. Here, \(\bar{\sigma}\) is the uniaxial equivalent stress, and the value for the equivalent strain \(\bar{\varepsilon}\) is evaluated by the following Eq. (2) from rolling strain \(\varepsilon_R\), assuming that rolling is a plane strain condition and no anisotropic behavior is considered.

\[
\bar{\varepsilon} = \frac{2}{\sqrt{3}} \varepsilon_R \tag{2}
\]

The following results can be mentioned from Fig. 15.

(1) When rolling strain is small (<approx. 0.2), the stress given by the M-K theory is constant and without dependence on rolling strain. It is understood that the limit stress follows the forming limit stress theory in this case.

(2) When rolling strain is large (>approx. 0.2), the stress given by the M-K theory depends on rolling strain and its value increases as rolling strain increases. The increase curve is similar to the uniaxial stress strain curve, but with a certain shift toward a large level.

(3) The experimentally-measured stress at the onset of localized necking is almost coincident with that obtained by the M-K theory throughout the entire range of rolling strain, and the stress is almost constant when the rolling strain is small. Although the absolute value is slightly smaller than the theoretical value when rolling strain is large. On the other hand, the stress at the onset of diffused necking is much smaller than the theoretical stress, especially when rolling strain is small.

3.2. Rolling-plane Strain

For the case of rolling-plane strain, Fig. 16 shows the same comparison of stresses. Here, plane strain yield stress is defined as \(2/\sqrt{3}\) times the uniaxial yield stress shown in Fig. 15. The following comments can be mentioned in this case:

(1) When rolling strain is small (in this case, the range is wider than before, <approx. 0.3), the stress evaluated by the M-K theory is constant and does not depend on rolling strain. Hence, this limit stress follows the forming limit stress theory.

(2) When rolling strain is large (>approx. 0.3), the theoretical limit stress increases depending on the increase of rolling strain. The curve shifts upward from the plane strain yield stress curve. However, the amount of the shift is much smaller than in the rolling-uniaxial case.

(3) The experimentally-measured stress at the onset of localized necking almost coincides with the stress obtained by the M-K theory, and the constant area of measured stress
can be seen when rolling strain is small. On the other hand, the stress at the onset of diffused necking is again much smaller than the theoretical value, especially when rolling strain is small.

3.3. Rolling-biaxial Stretching

In the same way, Fig. 17 shows a comparison of the stresses in the case of rolling-biaxial stretching. Here, stress cannot be measured directly. Hence, the experimental stress is a converted value obtained from strain using Stoughton’s method, as described in Sec. 2.3. The rolling strain is considered as an equivalent plastic strain evaluated in Eq. (2) in the conversion. The plotted experimental stress is the major principal stress. Biaxial yield stress is identical with that of the uniaxial case expressed by Eq. (1).

The following comments can be made based on Fig. 17.

(1) The forming limit stress by the M-K theory is not constant but increases linearly with respect to the increase of rolling strain. However, the increase ratio is small.

(2) The experimentally-obtained limit stress coincides well with that given by the M-K theory, although the definition of stress is basically different from the cases of rolling-uniaxial and rolling-plane strain.

(3) Both the theoretically- and experimentally-obtained stress are always quite large compared to the biaxial yield stress at the same value of rolling strain.

4. Discussion

The results presented in the previous chapter showed that the theoretically-evaluated forming limit stress by the extended M-K theory, in which rolling strain is considered to be comparable with plane strain stretching strain with identical final displacement, without evolution of the initial imperfection and ignoring the difference of the stress state, showed good agreement with the stress measured at the onset of localized necking in the three types of loading paths including rolling.

On the other hand, the author’s previous paper showed that, in loading paths which include only in-plane strain, the stress obtained by the M-K theory is coincident with the stress measured at the onset of localized necking. Based on this, it can be concluded that the extension of the M-K theory applied here is adequate.

Another conclusion is that the forming limit stress theory is not applicable, e.g., the limit stress depends on rolling strain as pre-strain, when the rolling strain is large. The reason is discussed in this chapter.

First, for the case of rolling-uniaxial stretching, the stress paths and fracture point calculated by the M-K theory are plotted on the principal stress field as shown in Fig. 18. The stress paths here consist of rolling, followed by unloading and then uniaxial stretching. The rolling and unloading processes cannot be plotted on this figure, so only the uniaxial stretching stress history is plotted. Rolling strain $\varepsilon_R$ is applied at levels of 0.1, 0.2, 0.3 and 0.5. For further comparison, the FLSD in linear loading paths obtained by the same M-K theory, and also the yield loci in some pre-strain levels, are plotted in the figure. Similar figures for rolling-plane strain and the rolling-biaxial stretching are shown in Fig. 19 and Fig. 20, respectively. From these figures, the re-
sults mentioned above can be understood as follows:

(1) When first-stage strain, e.g. rolling strain in these cases, is small (approx. <0.2) and second-stage loading is uniaxial stretching or plane-strain stretching, the re-yield stress point at second-stage loading is inside the FLSD. During second-stage loading, the stress point travels along the subsequent yield locus from the elastic stress state to the plastic stress state, then plastic deformation progresses further, and finally fracture occurs on the line of the FLSD. Hence, the fracture points are always the same point on the FLSD line, even if the rolling strain level changes. Thus, the forming limit stress theory is valid in this case.

(2) When rolling strain as first-stage strain is large (approx. >0.2) and second-stage loading is uniaxial stretching or plane-strain stretching, the stress point of the re-yield stress point of second-stage loading is outside the FLSD line. The point travels along the subsequent yield locus from the elastic stress state toward the plastic stress state beginning with the start of yielding, and fracture occurs immediately after the stress state reaches the plastic stress state or halfway to it. Therefore, the limit stress is larger than that predicted by the FLSD, and nearly equal to the yield stress after rolling. However, these are not completely coincident, but rather, the limit stress is slightly larger because plastic deformation progresses after stress reaches re-yielding until fracture occurs.

(3) When second-stage loading is biaxial, the limit stress by the FLSD is much larger and well outside the yield locus, until the rolling strain reaches to 0.9 approximately. In this case, the re-yield stress point of second-loading stage is always within the FLSD curve. Thus, the forming limit stress is always constant independent of rolling strain, following the forming limit stress theory. However, in this case, if the rolling strain exceeds 0.9, the same phenomenon as cases (1) and (2) will occur.

The above discussion (1)–(3) describes the changes in the experimental and theoretical limit stress without contradiction. The exceptional case of the forming limit stress is that the re-yielding stress point is outside the FLSD line because the strain in the first stage is sufficiently large. It has also been shown that the limit stress for this exceptional case can be expected to be nearly equal to the subsequent re-yield stress.

5. Conclusions

The ductile forming limit in non-linear strain paths which include out-of-plane strain such as rolling was investigated with a 11% Cr steel. In particular, the applicability of the forming limit stress theory to the above strain paths was verified. Experimentally, the stresses at the onset of diffused and localized necking and at the onset of separation were measured directly for rolling-stretching loading paths and compared with those obtained by the M-K theory. Theoretically, the possibility of extending the M-K theory to cases which include out-of-pane strain such as rolling was investigated. The results of this research are summarized as follows:

(1) In rolling-stretching loading path, the stress measured at the onset of localized necking, which is considered to correspond to the theoretical limit stress, is not constant but increases depending on the increase of rolling strain as pre-strain, especially the rolling strain is large. Hence, the forming limit stress theory is not completely valid for predicting the ductile forming limit.

(2) The M-K theory was extended in such a way that rolling strain as first-stage strain is considered to be comparable with that in-plane strain stretching in which only the final displacement is identical and the difference between the actual stress state is ignored. The limit stress of the second stage evaluated by the extended theory coincides well with the measured results. Thus, this extension of the M-K theory is considered to be adequate.

(3) These experimental and theoretical investigation demonstrated that the forming limit stress theory is applicable when the stress point of the re-yielding in second-stage loading is located inside the FLSD curve of the principal stress field.

(4) When the re-yielding point is located outside the FLSD curve, the limit stress is nearly equal to the re-yielding stress in second-stage loading. Thus, a modified FLSD curve can be defined as the lines connecting the outer line in the conventional FLSD curve and the subsequent yielding surface of the loading path under consideration.

(5) Hence, it can be concluded that the ductile forming limit can be predicted by using the extended forming limit stress theory as described above.

REFERENCES