Numerical Simulation of Effect of Tuyere Angle and Wall Scaffolding on Unsteady Gas and Particle Flows Including Raceway in Blast Furnace

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We have performed the numerical simulation for the particle and gas flows in the raceway region in a blast furnace of which dimension is almost the same as that of the commercial blast furnace using Distinct Element Method for the computation of the multi-body interaction among coke particles, Hard Sphere Model for two body interaction of powder particles based on Direct Simulation of Monte-Carlo Method, and Finite Difference Method for the numerical analysis of Navier–Stokes equations with the interaction terms between gas and particles for the gas flows. In the present simulation we have calculated the particle and gas flows in the raceway regions in which tuyere angles are 0, 3, 7 and 11 degree downward. The downward inclination of tuyere means that the air injects to the higher pressure side. This would stabilize the air flow and the raceway would become stable. However if the inclination angle is too high, the flow becomes unstable by various conditions near the bottom of blast furnace. The coke particle flow rate from the center region of blast furnace and its flow width increase with increasing the tuyere downward angle from the horizontal and attains the maximum value at near 7 degree. It means that the coke particle flow becomes widely uniform at about 7 degree tuyere angle except the region near the furnace wall.

We have also calculated the effect of scaffolding on the furnace wall on the particle and gas flows. The coke particle flow distributions with scaffolding on the wall become narrower. The scaffolding is nearer to the raceway, the effect of that becomes stronger. The raceway is not spherical and becomes unstable in cases with scaffolding on the wall. The coke particle velocity becomes higher by the narrow coke particle flow distribution caused by the existence of the scaffolding on the wall and it concentrates coke particles on the upper part of raceway near the furnace wall. The coke particle flow is dammed by the scaffolding and the wide area in which the coke particle velocity is very low is formed on the scaffolding. The gas flow distribution with scaffolding becomes non-uniform, particularly in the area between the softening melting cohesive zones and the scaffolding due to their interaction. The gas flow is also dammed up by the scaffolding and softening melting cohesive zones. The existence of scaffolding near softening melting cohesive zones strongly affects the gas flow.

KEY WORDS: numerical simulation; distinct element method; finite difference method; Navier–Stokes equation; blast furnace; raceway; coke particle; tuyere angle; wall scaffolding.

1. Introduction

Numerical simulation of particle and gas flows in a blast furnace is one of the most important subjects in the steel manufacturing technology and also in the particulate matter mechanics. The DEM (Distinct Element Method) simulation by Yamaoka and Nakano¹) in the cold model blast furnace of which width is 0.5 m and height is 1.0 m showed the effect of the tuyere diameter, the tuyere length and the bosh angle on the gas and particle flows. The continuum model calculation by Xu et al.²) in the two-dimensional bed indicated the dependency of raceway and fluidization phenomena on gas velocity by the size and shape of the mobile zone, flow patterns and particle forces. Zhang et al.³) numerically showed that the mass loss strongly affected the solid flow pattern and deadman profile in a two dimensional cold model blast furnace with 0.43 m width and 0.8 m height using the continuum model for both gas and solid phases. Increasing the solid consumption rate increased the solid velocity and decreased the deadman size. The calculated result of the stress distribution on the deadman surface by Takahashi et al.⁴) using their extending Walters theory⁵,⁶) assuming deadman to be a conical body showed that the floating mode depended on the horizontal profile of vertical load. Nouchi et al.⁷) numerically showed that the flow pattern and stagnant zone profile are affected by the level of liquid and the position of discharging hole in a water model using DEM. DEM calculation results of Nogami et al.⁸) who considered the heat exchange and the chemical reactions in a small scale model of blast furnace indicate that the blast temperature and the blast compositions are able to control the raceway shape and size, and the gas temperature.
in it. All of these studies are for a small scale model blast furnace. Results and findings for an actual blast furnace are necessary.

In this study we have calculated the effects of tuyere angle and scaffolding on the blast furnace wall on particle and gas flows in the raceway region in a blast furnace of which dimension is almost the same as that of the commercial blast furnace for the better understanding and the correct controlling the various phenomena in an actual blast furnace, and for the proposing the most effective operation method to minimize CO₂ discharge. DEM[9] has been used for the computation of the multi-body interaction forces among coke particles. We have used Hard Sphere Model for two body interaction forces of powder particles based on Direct Simulation of Monte-Carlo Method (DSMC)[10] and Finite Difference Method for the numerical computation of Navier–Stokes equations with the interaction terms between gas and particles for the gas flows.[11]

2. Computational Procedure

The simulation and calculation procedures and methods in the present study are almost the same as those of Yuu et al.[12] and Umekage et al.[13] Then these are omitted in this paper except the governing equations.

The governing equations for the gas phase are the three-dimensional Navier–Stokes equations for the incompressible fluid with interaction terms between the gas and particles, and the fluid continuity equation.

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla \cdot \mathbf{e} \mathbf{e} - \mathbf{St} \tag{1}
\]

\[
\frac{\partial e}{\partial t} + \mathbf{V} \cdot \mathbf{e} = 0 \tag{2}
\]

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla \cdot \mathbf{e} \mathbf{e} - \mathbf{St} \tag{1}
\]

\[
\frac{\partial e}{\partial t} + \nabla \cdot \mathbf{e} = 0 \tag{2}
\]

\[
\text{St} \text{ and St}_l \text{ in Eq. (1) are the interaction terms which are the drag and the lift forces between the gas and particles. Motions of the gas and particles linked through these interaction terms.}
\]

We used the fourth order central difference scheme for the convection terms and the second order central difference scheme for other spatial derivative terms. The second order Runge–Kutta method was used for the time derivative terms.

The governing equations for the particle motion are as follows:

\[
m_p \frac{d(U_p)}{dT} = \sum_j (F_{Dj} + F_{Lj}) \tag{3}
\]

\[
I_p \frac{d(\Omega_p)}{dT} = \sum_j (M_{Mj} + M_{Lj}) \tag{4}
\]

Eqs. (3) and (4) are three-dimensional. The fluid drag force, \(F_{Dj}\), and the fluid lift force, \(F_{Lj}\), are similar to \(\text{St}\) and \(\text{St}_l\) in Eq. (1). Subscripts \(i\) and \(j\) indicate the reference and the contacting particle numbers.

Figure 1(a) shows the computational domain and the boundary conditions. The computational domain is a region from which is a cylindrical part on a bosh to near the bottom of a blast furnace of which dimension is almost the same as that of the commercial blast furnace including the raceway flows. The cross-sectional area in \(x-y\) plane of computational domain in which there is one tuyere is semi-circular and is approximately represented by small rectangular cells as shown in Fig. 1(b). In this simulation tuyere angles shown in Fig. 1(c) were four cases for the horizontal, 3, 7 and 11 degree downward. The particle number calculated and other computational conditions are shown in Table 1. As shown in Fig. 1(a), the loads which were calculated using the elastic-plastic model by Katayama et al.[14]
and are correspond to the granular pressures are distributed in the top layer of the granular matter in the present computational domain.

The model in which coke particles were removed to represent the burned cokes in a raceway and the calculation procedure for powder particles and softening melting cohesive zones, which were also taken into account in this paper, were the same as those presented by Yuu et al.\textsuperscript{12} and Umekage et al.\textsuperscript{13}. The size distribution and the initial size inclination of coke particles used for the present calculation were also the same as those used in above mentioned literatures. Then these were omitted in this paper.

The void fraction was calculated by the following equation.

\[ \epsilon = \frac{V_0 - V}{V_0} \tag{5} \]

We used the personal computer, which is Intel, Itanium 2 (1.6 GHz) processor \( \times 2 \), for the computation. It took 2,500 h for the calculation of 25 s phenomena.
3. Results and Discussion

The calculated particle and gas velocities in figures in this paper are the values averaged in a computational cell. The figures in this paper show the calculated results in the cross-sectional area of nozzle center. The position of an arrow in the figures is a center of computational cell. Figures 2(a), 2(b), 2(c) and 2(d) show calculated instantaneous coke particle locations for various tuyere directions. In these figures all of coke particles existing in one computational cell depth were plotted. Figures 2(a), 2(b), 2(c) and 2(d) clearly indicate the raceway shape for each tuyere direction. The raceway for the horizontal nozzle direction is small and slender, and its shape is elliptic. When the nozzle direction is 3 degree downwards, the raceway becomes larger and particularly its height grows larger. For 7 degree downward nozzle direction the raceway is nearly spherical, since its height becomes still larger and for 11 degree downward the raceway reforms to be slightly non-spherical. The raceway height depends upon the nozzle direction but the raceway depth is almost unchanged in this tuyere direction range.

We calculated the depth and the height of raceway according to the definition shown in Fig. 2(a). Figures 3(a), 3(b), 3(c) and 3(d) show calculated results of raceway depth and height as a function of time. The results indicate that both of the depth and the height of raceway periodically change and they have roughly two different periods. One is a short period and another is long one as shown examples in Fig. 3(a). The short period would indicate the usual unsteady fluctuation of the raceway and the long period would do the large scale change of the unstable flow due to, for examples, particle blockade, yield of particle bed and etc. In the case of horizontal tuyere direction, the calculated raceway depth shown in Fig. 3(a) increases with the time and reaches the maximum value after about 12 s from the start. After that the raceway depth decreases and starts to increase again at about 14 s. As a whole the raceway depth changes periodically with long period as shown in Fig. 3(a). The raceway depth also fluctuates with short period. However in this case the difference between the maximum and the minimum values of raceway depth for a short period is small. For 3 degree downward tuyere direction the large scale fluctuation of raceway depth becomes smaller than that for the horizontal nozzle direction as shown in Fig. 3(b). For 7 degree downward nozzle direction the large scale fluctuation shown in Fig. 3(c) still becomes smaller than that for 3 degree direction. However the large scale fluctuation for 11 degree nozzle direction slightly increases as shown in Fig. 3(d). The small scale fluctuation of raceway depth does not depend upon the tuyere direction eminently, but the size of small scale fluctuation slightly increases with increasing the nozzle direction angle in these conditions.

For the raceway height the large scale fluctuation exists in each tuyere direction but the sizes of these fluctuations are smaller than those of raceway depth and the minimum value exists at about 7 degree downward direction. How-
ever the small scale raceway height for 7 degree nozzle direction is largest in these angles. These mean that the raceway is stable for the long scale period but there are small scale unstable raceway flows. The difference between the raceway depth and height decreases with increasing the downward angle from the horizontal. However it becomes the minimum value at 7 degree and it turns for the difference to increase with increasing the angle. Figure 3(c) shows that the raceway depths in 7 degree are nearly equal to the raceway heights. It means that the shape of the raceway almost spherical as shown in Fig. 2(c). The downward inclination of tuyere means that the air injects to the higher pressure side. This would stabilize the air flow and the raceway would become stable. However if the inclination angle is too large, the unstable flow recurs by the existence of blast furnace bottom and various conditions near the bottom.

As shown in our previous paper,13) the calculated mean values of the raceway depth for 7 degree nozzle direction are fairly in good agreement with data which were measured in Kobe steel co. Kakogawa third blast furnace using micro wave reflectiongunned through tuyere by Matsui et al.13) The precise remark could not be gained from the comparison because there are differences between the experimental conditions and our computational ones. However the fairly good agreement between them indicates that our computational procedure and results presented in this paper are reasonable. Our calculated values show the rough but the proper estimation of raceway characteristics particularly the effect of tuyere angle.

Figures 4(a), 4(b), 4(c) and 4(d) show the calculated iso-contours of typical instantaneous coke particle velocity scalar for each tuyere direction. The elliptic circles in figures of the present study as appeared in Fig. 4(a) are softening melting cohesive zones setting in the present calculation. The coke particle flow rate from the center region of blast furnace and the flow width increase with increasing the tuyere downward angle and attains the maximum value at 11 degree in these direction angles. This would be because that the downward component of air velocity from a tuyere with a large downward direction becomes larger and drags down a number of coke particles. The coke downward velocity near the furnace wall for 11 degree tuyere direction is also the largest. Figures 4(a), 4(b), 4(c) and 4(d) show that the region with the uniform coke velocity is the largest for 7 degree tuyere angle. It means that the coke particle flow becomes widely uniform at about 7 degree tuyere angle except the region near the furnace wall. As shown in Fig. 2(c), the raceway is spherical and large at 7 degree downward angle. This would make uniform coke flow. On the other hand the coke velocity for 11 degree tuyere direction has the distribution which has the higher velocity at the central region of the distribution and near the furnace wall. The high coke velocity area near the furnace wall for 11 degree tuyere direction is the largest. The coke velocity difference in the region except near the furnace wall also is the largest. These indicate that 11 degree tuyere generates the unstable flow.

Figures 5(a), 5(b), 5(c) and 5(d) show the instantaneous air velocity vector for each tuyere direction. The downward air flow from the bottom of raceway increases with increasing the tuyere downward angle and the upward air flow from the top of raceway also increases with increasing the tuyere downward angle and attains the maximum value at near 7 degree. These are some of the reasons why the raceway becomes spherical and largest at 7 degree downward angle. The horizontal air velocity is the largest at 0 degree (horizontal) downward angle, so the raceway depth and height at 0 degree are the largest and the smallest, respectively. The detail comparison of these air flow in Figs. 5(a), 5(b), 5(c) and 5(d) show that the air flow to the center region of blast furnace and its flow width slightly increase with increasing the tuyere downward angle and attains the maximum value at near 7 degree. The stable raceway at 7 degree would make the uniform air flow.

Figures 6(a), 6(b) and 6(c) show calculated the typical instantaneous coke particle velocity vector diagrams in blast furnaces without and with scaffoldings. In the calculation with scaffoldings the tuyere angle was 7 degree downward. As shown in Fig. 6(a), coke particles in the blast furnace without scaffolding flow down from the wider region. On the other hand, the coke particle flow distributions with scaffolding on the wall in the blast furnace become narrower as shown in Figs. 6(b) and 6(c). This is because the coke particle flow is dammed up by the scaffolding. Then the wide area in which the coke particle velocity is very low is formed on the scaffolding. The coke particle flow width with scaffolding on the bosh part of the wall is narrower as shown in Fig. 6(c). This means that the scaffolding which is nearer to the raceway affects more strongly the coke particle flow as shown in Figs. 6(b) and 6(c). The coke particle flow changes the flow direction due to the existence of scaffolding and attaches to the furnace wall as shown in Figs. 6(b) and 6(c). Then the flow rate of coke particle near the furnace wall becomes larger.

Figures 7(a), 7(b) and 7(c) show calculated time-averaged coke particle velocity vector diagrams with scaffoldings in blast furnaces. The comparison of Figs. 6 and 7 indicates that the typical instantaneous coke particle velocity vector distributions are almost the same as time-averaged ones. Figure 7(c) shows the coke particle flow with two scaffoldings on the furnace wall. The flow interaction between two scaffoldings forms the complicated coke flow distribution which might be an origin that causes the large scale unstable phenomenon.

Figure 8 shows the magnified flow distribution of coke particles on the scaffolding on the lowest part of cylindrical wall just on the bosh as shown in Fig. 6(b). The coke particle velocity in the region on the scaffolding is very small but not zero. If the adhesion force acts on the coke particle, the quiescent coke piles would be formed on the wide region on the scaffolding. These coke piles also might be an origin which causes the large scale unnecessary flow in a blast furnace.

Figures 9(a), 9(b) and 9(c) show calculated instantaneous coke particle locations. In the case of coke flow without scaffolding on the furnace wall, the raceway of which shape is spherical is formed. On the other hand the raceway is not spherical and is unstable in cases with scaffolding on the wall. As shown in Fig.6, the coke particle velocity becomes higher by the narrow coke particle flow distribution caused by the existence of the scaffolding on the wall and it
Fig. 10(a). Calculated color iso-contour of instantaneous void fraction of coke particle without scaffolding for 7 degree downward nozzle direction in the actual blast furnace (T=6.0 s).

Fig. 10(b). Calculated color iso-contour of instantaneous void fraction of coke particle with a scaffolding on the cylindrical part of the actual blast furnace wall for 7 degree downward nozzle direction (T=6.0 s).

Fig. 10(c). Calculated color iso-contour of instantaneous void fraction of coke particle with a scaffolding on the part of bosh of the actual blast furnace wall for 7 degree downward nozzle direction (T=6.0 s).

Fig. 4(a). Calculated color iso-contour of coke particle velocity scalar for horizontal nozzle direction in the actual blast furnace (T=10.0 s).

Fig. 4(b). Calculated color iso-contour of coke particle velocity scalar for 3 degree downward nozzle direction in the actual blast furnace (T=10.0 s).

Fig. 4(c). Calculated color iso-contour of coke particle velocity scalar for 7 degree downward nozzle direction in the actual blast furnace (T=10.0 s).

Fig. 4(d). Calculated color iso-contour of coke particle velocity scalar for 11 degree downward nozzle direction in the actual blast furnace (T=10.0 s).
concentrates coke particles on the upper part of raceway near the furnace wall. The large amount of coke particles which flow into the narrow part of raceway would produce the unstable raceway shape and state shown in Figs. 9(b) and 9(c).

Fig. 5(a). Calculated instantaneous air velocity vector for horizontal nozzle direction in the actual blast furnace ($T=10.0 \text{ s}$).

Fig. 5(b). Calculated instantaneous air velocity vector for 3 degree downward nozzle direction in the actual blast furnace ($T=10.0 \text{ s}$).

Fig. 5(c). Calculated instantaneous air velocity vector for 7 degree downward nozzle direction in the actual blast furnace ($T=10.0 \text{ s}$).

Fig. 5(d). Calculated instantaneous air velocity vector for 11 degree downward nozzle direction in the actual blast furnace ($T=10.0 \text{ s}$).

Fig. 6(a). Calculated instantaneous coke particle velocity vector without scaffolding on the actual blast furnace for 7 degree downward nozzle direction ($T=4.6 \text{ s}$).

Fig. 6(b). Calculated instantaneous coke particle velocity vector with scaffolding on the cylindrical part of the actual blast furnace wall for 7 degree downward nozzle direction ($T=4.6 \text{ s}$).

Fig. 6(c). Calculated instantaneous coke particle velocity vector with scaffolding on the part of bosh of the actual blast furnace wall for 7 degree downward nozzle direction ($T=4.6 \text{ s}$).

**Figures 10(a), 10(b) and 10(c)** show calculated instantaneous iso-contours of void fraction in the blast furnace. The high void fraction region is formed under the scaffolding. Particularly the void fractions in the wide area between the raceway and the scaffolding on the furnace wall are higher.
since the coke particle velocity distributions become narrower and their velocities become higher due to the existence of scaffolding. It would be easy that an unstable flow happens in the high void fraction area.

Figures 11(a), 11(b), 11(c) and 11(d) show calculated results of gas velocity vector distribution. The gas flow in the blast furnace without scaffolding flows up uniformly from the wider region of both side of softening melting cohesive zones as shown in Fig. 11(a). On the contrary the gas flow distribution with scaffolding becomes non-uniform, particularly in the area between the softening melting cohesive zones and the scaffolding due to their interaction as shown.
in Figs. 11(b) and 11(c). The gas flow is dammed up by the scaffolding and softening melting cohesive zones. The interaction between them complicates the gas flow. The existence of scaffolding near softening melting cohesive zones strongly affects the gas flow as shown in Fig. 11(c). It might happen to blockade the gas flow by the interaction of softening melting cohesive zones and scaffolding on the wall. Figure 11(d) shows the gas flow with two scaffoldings on the furnace wall. The flow interaction among two scaffoldings and softening melting cohesive zones forms the complicated gas flow distribution near the furnace wall which would cause the large scale unusual and unstable phenomena. The comparison of Figs. 11(a), 11(b), 11(c) and 11(d) clearly indicates that softening melting cohesive zones and the scaffolding near the tuyere strongly affect the flows. The effect would be caused mainly by the scaffolding on the tuyere.

4. Conclusion

We have calculated the effects of tuyere angle and scaffolding on the furnace wall on particle and gas flows in the raceway region in an actual blast furnace. Then the following concluding remarks are obtained.

(1) The downward inclination of tuyere stabilizes the air flow and then the raceway becomes stable. However if the inclination angle is too high, the flow becomes unstable by various conditions near the bottom of blast furnace.

(2) The difference between the raceway depth and height decreases with increasing the downward angle from the horizontal. It becomes the minimum value at 7 degree.

(3) The coke particle flow rate from the center region of blast furnace and its flow width increases with increasing the tuyere downward angle and attains the maximum value at near 11 degree.

(4) The raceway is spherical and large at 7 degree downward angle. This would make uniform coke flow.

(5) The air flow to the center region of blast furnace and its flow width also increase with increasing the tuyere downward angle and attains the maximum value at near 7 degree. The stable raceway at 7 degree would make the uniform air flow.

(6) The scaffolding affects both coke and gas flows. The coke particle flow distributions with scaffolding on the wall become narrower. The scaffolding is nearer to the raceway, the effect of that becomes stronger.
(7) The raceway is not spherical and is unstable in cases with scaffolding on the wall. The coke particle velocity becomes higher by the narrow coke particle flow distribution caused by the existence of the scaffolding on the wall and it concentrates coke particles on the upper part of raceway near the furnace wall.

(8) The coke particle flow is dammed by the scaffolding and the wide area in which the coke particle velocity is very low is formed on the scaffolding.

(9) The gas flow distribution with scaffolding becomes non-uniform, particularly in the area between the softening melting cohesive zones and the scaffolding due to their interaction. The gas flow is dammed up by the scaffolding and softening melting cohesive zones. The existence of scaffolding near softening melting cohesive zones strongly affects the gas flow.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$D$</td>
<td>Radius of blast furnace at tuyere section (m)</td>
</tr>
<tr>
<td>$D_0$</td>
<td>Damping force vector at contact point $j$ (N)</td>
</tr>
<tr>
<td>$D_m$</td>
<td>Powder mean diameter (m)</td>
</tr>
<tr>
<td>$E$</td>
<td>Modulus of elasticity (Pa)</td>
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<td>$F_i$</td>
<td>Contact force vector at contact point (N)</td>
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<td>Fluid drag force vector acting on particle $i$ (N)</td>
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<tr>
<td>$F_{L_i}$</td>
<td>Fluid lift force vector acting on particle $i$ (N)</td>
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<tr>
<td>$F_g$</td>
<td>Gravitational force vector acting on particle $i$ (N)</td>
</tr>
<tr>
<td>$I_p$</td>
<td>Inertia moment of particle (kg m$^2$)</td>
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<td>$M_j$</td>
<td>Moment due to contact force at contact point $j$ (Nm)</td>
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<tr>
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<tr>
<td>$Re$</td>
<td>Reynolds number $=DU_p\rho/\mu$</td>
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<td>$St$</td>
<td>Nondimensional interaction term vector due to fluid drag force $i$</td>
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<td>$St_f$</td>
<td>Nondimensional interaction term vector due to fluid lift force $i$</td>
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**Greek letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$\varepsilon$</td>
<td>Void fraction $(-)$</td>
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<tr>
<td>$\mu$</td>
<td>Air viscosity (Pa·s)</td>
</tr>
<tr>
<td>$\mu_p$</td>
<td>Friction coefficient (particle-particle) $(-)$</td>
</tr>
<tr>
<td>$\mu_w$</td>
<td>Friction coefficient (particle-wall) $(-)$</td>
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</tr>
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<tr>
<td>$\rho_p$</td>
<td>Particle density (kg m$^{-3}$)</td>
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<tr>
<td>$\Omega$</td>
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<tr>
<td>$\nabla$</td>
<td>Nondimensional nabla operator $(-)$</td>
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**REFERENCES**