The Deformation and the Redundant Work Factor in the Axisymmetric Drawing of AISI 420 Stainless Steel Bars—Strain Path Effects Analysis

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(Received on July 9, 2007; accepted on October 1, 2007)

In the present work, experimental techniques for evaluating the deformation and the redundant work factor in the axisymmetric drawing of ferritic AISI 420 stainless steel bars were investigated. Six operation conditions, involving two reductions of area and three die semi-angles, were employed in the study. Regarding the redundant deformation analysis, the visioplasticity technique was considered as the most adequate procedure for estimating the average deformation in drawing. In this case, an increasing relationship between the redundant deformation factor and the parameter $\Delta$ was obtained. On the other hand, the stress–strain curves superposition technique led to redundant deformation factor values almost insensitive to variations of the parameter $\Delta$ and below unity, a phenomenon which was associated with strain path effects. Concerning the redundant work factor study, the experimental results were lower than those obtained through a theoretical approach and, in some conditions, below unity. This was also attributed to strain path effects.

KEY WORDS: drawing; average strain; visioplasticity; redundant deformation factor; redundant work factor.

1. Introduction

One of the most significant aspects of the axisymmetric drawing operation is the occurrence of a non-homogeneous deformation in the cross section of the material. This heterogeneity is related to an internal distortion process that takes place in the metal as it flows through the die, not contributing to the overall dimensional changes but supposedly increasing the drawing force and the average work hardening of the bar.1–4)

Several methods have been employed in the analysis of the deformation in drawing, involving experimental, theoretical and numerical approaches. These studies estimate the average effective strain $\varepsilon_m$ imparted to the material or the redundant deformation factor $\phi$, which represents the relationship between $\varepsilon_m$ and the external deformation $\varepsilon$ in the process (Eq. (1)). The results have usually been associated with the geometrical features of the operation (die semi-angle $\alpha$ and area reduction per pass $r$), eventually described through the so-called parameter $\Delta$ (Eq. (2)).5,6)

\[
\phi = \frac{\varepsilon_m}{\varepsilon} \quad \text{.......................... (1)}
\]

\[
\Delta = \frac{d_i + d_f}{d_i - d_f} \sin \alpha \quad \text{......................... (2)}
\]

where $d_i$ and $d_f$ are the initial and the final diameters of the bar.

Regarding the experimental studies of the deformation in drawing, the existing procedures are the stress–strain curves superposition technique,3,5,7) the hardness profile method4,6) and the visioplasticity.8,9) The first one was introduced by Hill and Tupper for plane strain drawing, arising from an investigation related to the slip line field theory.3,10) The method is based on the assumption that the drawing stress is given by the area under the tensile flow curve of the undeformed metal, up to a certain value of strain corresponding to the average deformation caused by the forming operation.10) It is considered that there is no energy loss in the process and that the results are independent of the structural features of the material.5) Under these circumstances, the tensile yield stress of the drawn bar would be equal to the ordinate of the stress–strain curve of the annealed metal at the equivalent mean strain.10) Figure 1 illustrates the technique, performed for 303 stainless steel bars drawn to two external strain values.5) In this case, the results were obtained through the direct superposition of the drawn metal yield stress on the tensile stress-strain curve of the annealed material.

The hardness profile method was presented by Backofen,4) involving the determination of the hardness distribution in the cross section of the drawn metal and the subsequent transformation of these values into deformation
Table 1. Chemical composition of the AISI 420 stainless steel (wt%).

<table>
<thead>
<tr>
<th>C</th>
<th>Cr</th>
<th>Ni</th>
<th>Mn</th>
<th>Si</th>
</tr>
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<tbody>
<tr>
<td>0.374</td>
<td>12.720</td>
<td>0.234</td>
<td>0.403</td>
<td>0.445</td>
</tr>
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</table>

Fig. 2. Optical micrograph of the annealed AISI 420 stainless steel.

Fig. 1. Example of the stress–strain curves superposition technique for the determination of the average strain in the drawing of 303 stainless steel bars.

2. Material and Methods

2.1. Material

The material used in this investigation was the AISI 420 stainless steel, whose chemical composition (weight percent) is exhibited in Table 1. An isothermal annealing was conducted in the as received bars, involving an initial stage of 3 600 s at 910°C and a cooling rate of 0.058°C/s followed by 7 200 s at 705°C. Prior cold work and final homogeneity were verified through metallography and hardness tests. After the heat treatment, the metal displayed an average Vickers hardness of 186±3HV and a microstructure consisting of chromium carbides dispersed in a ferrite matrix (Fig. 2).
2.2 Specimens

Three different specimen shapes were designed, according to the sequence of experiments. The first one, related to the tension test of the annealed metal, consisted of samples with 10.00 mm diameter and 60.00 mm gauge length. The second group, corresponding to the drawing-tension experiment, comprised specimens with 10.43 mm or 10.85 mm diameter and 33.98 mm or 36.77 mm length, whose dimensions depended on the reduction of area (r) and on the die semi-angle (α) considered in the first forming operation. In this case, thicker sections at the ends of the useful part of the samples were also machined. This special geometry provided, after drawing, 10.00 mm diameter and 40.00 mm gauge length bars with more work hardened extremity sections, avoiding the occurrence of rupture inside the grips during the subsequent tension. Finally, the third set of specimens, associated with the visioplasticity technique, involved sectioned stepped cylindrical bars, whose details are given in Fig. 3. Similarly to the drawing-tension test, the dimensions of the samples were calculated according to the values of r and α. In this case, however, the specimens were designed in order to allow the determination of the strain profiles related to both reductions of area investigated in this work. Further information concerning the drawing operation and the experimental procedure of the visioplasticity technique will follow. After machining, a coordinate grid of 1 mm x 1 mm was electro-chemically marked in the internal flattened surfaces of the samples.

2.3. Tension

Tension was carried out at room temperature in an Instron model 5582 machine. An electronic 25 mm gauge length Instron model 2630-107 extensometer was used in all experiments up to the beginning of necking. The crosshead speed employed in the tests was 6.67 x 10^{-2} mm/s, leading to initial strain rates of 1.10 x 10^{-3} s^{-1} (annealed metal) and 1.17 x 10^{-3} s^{-1} (drawn bars).

2.4. Drawing

Drawing was also conducted in an Instron model 5582 machine, with a device specially designed for the process. The operation was carried out in one pass, at a crosshead machine, with a device specially designed for the process.

2.5. Visioplasticity Technique

In this case, drawing has been carried out in two stages for each specimen (or each die semi-angle). Initially, the operation was conducted up to about half of the original length of the sample related to the area reduction of 8%. The process was interrupted and the specimen was removed from the die. After that, the two parts of the bar were separated and images of the grid distortion (including the deformation zone) were obtained. The halves were then screwed back together and an analogous procedure was performed for the region associated with the area reduction of 15%. Based on the location of the distorted grid lines (in terms of radial and axial directions), values of the flow function of the metal were calculated. A smoothing procedure developed by Shabaik was employed in these results, allowing the determination of an analytical expression for the flow function. Subsequent calculations, involving the velocity, strain rate and strain components and the effective strain, were conducted through the classical method proposed by Thomson. Full details concerning the experiments and the calculation procedure are given in Ref. 20.

3. Results and Discussion

Figures 4–7 display the grid distortion and the complete effective strain distribution of the AISI 420 stainless steel bars during and after drawing obtained through the visioplasticity technique. The deformation zone axis in Figs. 6 and 7 considered the maximum material/die contact length (5.0 mm for α = 3° – Fig. 6, and 9.0 mm for α = 3° – Fig. 7). The final effective strain profiles related to all experiments are presented in Fig. 8. Third-order polynomial fit curves were used to represent the local values of strain in the cross section of the bars.

Despite the occurrence of some irregularities in the
Fig. 4. Grid distortion of the AISI 420 stainless steel bars during drawing $r=8\%$: (a) $\alpha=20^\circ$, (b) $\alpha=8^\circ$ and (c) $\alpha=3^\circ$.

Fig. 5. Grid distortion of the AISI 420 stainless steel bars during drawing $r=15\%$: (a) $\alpha=20^\circ$, (b) $\alpha=8^\circ$ and (c) $\alpha=3^\circ$.

Fig. 6. Effective strain distribution of the AISI 420 stainless steel bars during drawing $r=8\%$: (a) $\alpha=20^\circ$, (b) $\alpha=8^\circ$ and (c) $\alpha=3^\circ$.

Fig. 7. Effective strain distribution of the AISI 420 stainless steel bars during drawing $r=15\%$: (a) $\alpha=20^\circ$, (b) $\alpha=8^\circ$ and (c) $\alpha=3^\circ$. 
curves, probably related to smoothing problems, a general behavior is clearly observed. Considering the experiments performed with $\alpha=3^\circ$, for both values of $r$ the deformation occurs gradually and homogeneously as the metal flows through the die, up to an almost uniform strain profile whose results are close to the external deformation imparted by the drawing operation. On the other hand, for $\alpha=20^\circ$, heterogeneous effective strain distribution curves are observed in the entire deformation zone and at the end of the process. The experiments carried out with $\alpha=8^\circ$ also led to the development of heterogeneous deformation profiles in the metal. The differences between the results of the central region and of the surface of the bars, however, are less pronounced than those observed for $\alpha=20^\circ$.

The development of non-homogeneous deformation distributions in the cross section of drawn bars has been previously brought up in several investigations, involving different strain analysis approaches and also drawing stresses and final mechanical properties prediction theories (e.g. Refs. 2), 4), 8), 21–23)). In these works, the die geometry features, i.e., $\alpha$ and $r$, were reported as the most important variables determining the flow pattern of the metal. Similarly to the results exhibited in Figs. 4–8, strain heterogeneity was found to increase as the die semi-angle value increases, considering the same reduction of area. This phenomenon, as one considers material layers farther from the bar axis, is caused by higher levels of shearing superimposed on the external axisymmetric compression. The strain path followed by the various layers of the bar under processing thus involves an axisymmetric compression, on which various degrees of shearing are superimposed. The magnitude of this shearing depends on the area reduction in the pass, the die semi-angle and the distance of the layer from the bar axis. Finally, even though analogous behaviors were verified in these analyses, such as the development of heterogeneous deformation distribution in the metal drawn with high die semi-angles, quantitatively different responses in terms of local values of effective strain in the cross section of the bars were observed for the various procedures employed in the evaluation of the material deformation.8,9)

Among the studies mentioned before, the investigations concerning the visioplasticity technique did not cover the calculation of average effective strain values $\varepsilon_{\text{av}}$ in the drawing operation, as well as the establishment of relationships between the redundant deformation factor $\phi$ and the parameter $\Delta$. Hence, Table 3 and Fig. 9 display this analysis for the AISI 420 stainless steel bars. In this case, the estimation of $\varepsilon_{\text{av}}$ and $\phi_v$ ($v$ refers to visioplasticity) was performed using the third-order polynomial fit strain curves shown in Fig. 7, the procedure presented in Ref. 4) and Eq. (1).

Despite the data dispersion, an increasing monotonic relationship between $\phi_v$ and $\Delta$ is observed in Fig. 9. These results are similar to those presented in the literature for other metals and various deformation evaluation methods. As usual in the literature3–7) a linear fit was performed for the data points leading to the $\phi_v$ versus $\Delta$ equation ($\phi_v = 1.143 + 0.039\Delta$) displayed in Fig. 9.

In order to allow the comparison between the visioplasticity and other classic experimental method, the average effective strains and redundant deformation factors for the AISI 420 stainless steel obtained through the stress–strain curves superposition technique (denoted as $\varepsilon_{\text{ms}}$ and $\phi_e$) were evaluated and are shown in Table 4. The procedure was conducted as previously described, employing the tensile curves of the annealed and the drawn material and the superposition criteria at 0.2% strain. Figure 10 displays the $\phi_e$ versus $\Delta$ data and the linear expression associated with them ($\phi_e = 0.759 - 0.0001\Delta$), as well as the results already exhibited in Fig. 9. In addition to $\phi_e$ and $\phi_v$, an equation based on the formulation proposed by Caddell and Atkins5) for describing the relationship between $\phi$ and $\Delta$ is also presented ($\phi_C = 0.677 + 0.071\Delta$). This theoretical prediction

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**Table 3.** Values of average effective strain $\varepsilon_{\text{av}}$ and redundant deformation factor $\phi$, obtained through the visioplasticity method.

<table>
<thead>
<tr>
<th>Drawing parameters</th>
<th>$\varepsilon_{\text{av}}$</th>
<th>$\phi_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 20^\circ$, $r = 8^\circ$</td>
<td>0.146</td>
<td>1.754</td>
</tr>
<tr>
<td>$\alpha = 20^\circ$, $r = 15^\circ$</td>
<td>0.266</td>
<td>1.638</td>
</tr>
<tr>
<td>$\alpha = 8^\circ$, $r = 8^\circ$</td>
<td>0.104</td>
<td>1.249</td>
</tr>
<tr>
<td>$\alpha = 8^\circ$, $r = 15^\circ$</td>
<td>0.246</td>
<td>1.154</td>
</tr>
<tr>
<td>$\alpha = 3^\circ$, $r = 8^\circ$</td>
<td>0.091</td>
<td>1.091</td>
</tr>
<tr>
<td>$\alpha = 3^\circ$, $r = 15^\circ$</td>
<td>0.186</td>
<td>1.144</td>
</tr>
</tbody>
</table>

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**Fig. 8.** Final effective strain distribution of the drawn AISI 420 stainless steel bars.

**Fig. 9.** Relationship between $\phi$ and $\Delta$—results obtained through the visioplasticity technique.
derives from an experimental work conducted with the stress–strain curves superposition method, carried out in single pass drawn aluminum, Armco iron and 303 stainless steel bars (some results related to the last material have already been exhibited in Fig. 1). In this study, a general expression for $\phi$ was empirically developed (Eqs. (5)–(7)) employing the Hollomon’s equation terms $\sigma$, $m$ and $e$ ($\sigma = \sigma_0 e^m$), determined with the tensile flow curve of the annealed metal.

$$\phi_{CA} = C_1 + C_2 \Delta \tag{5}$$

$$C_1 = 2.25 \sigma_0^{-0.1} m^{0.28} \tag{6}$$

$$C_2 = 0.404 \sigma_0^{-0.054} m^{0.76} \tag{7}$$

Considering the results related to the Caddell and Atkins’s approach, similarly to the visioplasticity, $\phi_{CA}$ was found to increase as $\Delta$ increases. In this case, however, the results were lower than $\phi$ (except for $\Delta > 14.6$), more sensitive to variations of the geometric parameter and, for $\Delta < 4$, below unity. On the other hand, contrasting with $\phi \times \Delta$ and $\phi_{CA} \times \Delta$ equations and other studies reported in the literature, the redundant deformation factor obtained through the experimental stress strain curves superposition technique $\phi$ was found to be almost insensitive to variations in the $\Delta$ parameter (in fact, a slight decrease in $\phi$ as $\Delta$ increases is observed). Besides this unusual relationship, the values of $\phi$ for all drawing conditions examined in this investigation were not only smaller than those obtained through the visioplasticity method and the Caddell and Atkins’s approach, but also below unity, similarly to $\phi_{CA}$ for $\Delta < 4$. This phenomenon is impossible from the point of view of the geometrical measurement of strain, since the shear deformation imposed on the axisymmetric compression of the metal, as it flows through the die in the drawing process, occurs in addition to the external geometric strain of the bar. This is confirmed by the fact that all $\phi$, values are above unity (see Table 3 and Fig. 9).

Hence, the results presented in Fig. 10 reveal some limitations of the stress–strain curves superposition technique for calculating the average effective deformation in the forming operation. The method is based on the tensile yield stress of prestrained samples, i.e., depends on the mechanical response of the material submitted to a strain path change from drawing to pure tension. Among several investigations and different results (e.g., Refs. 24–31), one of the most commonly observed effects of strain path changes is the occurrence of enhanced or reduced initial flow stresses during the second stage of deformation, revealed through the comparison between the results of the monotonic and the two (or multiple) stage flow curves, considering the same effective strain level. Figure 11 illustrates schematically both of these types of work hardening behaviors. The flow curves of prestrained samples are superposed on the curve associated with the annealed material. In this case, the strain path change is represented by an initial plastic deformation up to the prestrain value indicated in the graph, followed by some deformation operation whose nature or direction differs from that employed in the first stage of the process. The so-called type I is usually related to the development of reloading flow stresses lower than those observed in the material during monotonic straining at the same effective strain level and higher strain hardening rates. The type II behavior is connected to the opposite situation. The structural features of the material, the magnitude of the strain path change and the prestraining value were reported as the most important variables in these studies, leading to the two distinct initial flow stress behaviors after the strain path change previously described. Regarding the results presented in Fig. 10, taking into account the stress–strain curves superposition technique procedure, the occurrence of redundant deformation factors below unity is related to the development of flow stresses at the beginning of the

<table>
<thead>
<tr>
<th>Drawing parameters</th>
<th>$\varepsilon_{ea}$</th>
<th>$\phi_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 20^\circ$, $r = 8%$</td>
<td>0.073</td>
<td>0.880</td>
</tr>
<tr>
<td>$\alpha = 20^\circ$, $r = 15%$</td>
<td>0.088</td>
<td>0.541</td>
</tr>
<tr>
<td>$\alpha = 8^\circ$, $r = 8%$</td>
<td>0.056</td>
<td>0.672</td>
</tr>
<tr>
<td>$\alpha = 8^\circ$, $r = 15%$</td>
<td>0.097</td>
<td>0.599</td>
</tr>
<tr>
<td>$\alpha = 3^\circ$, $r = 8%$</td>
<td>0.076</td>
<td>0.915</td>
</tr>
<tr>
<td>$\alpha = 3^\circ$, $r = 15%$</td>
<td>0.148</td>
<td>0.910</td>
</tr>
</tbody>
</table>

Table 4. Values of average effective strain $\varepsilon_{ea}$ and redundant deformation factor $\phi_s$ obtained through the stress–strain curves superposition technique.
tensile behavior of predrawn AISI 420 and 304 stainless steel bars.

Thus, the results exhibited in this investigation show that the stress–strain curves superposition technique seems to be inappropriate for the analysis of the deformation in drawing, since it involves strain path change effects (from drawing to tension) and, therefore, does not adequately allow the evaluation of the strain heterogeneity of the metal in the forming operation. Visioplasticity seems to be the most adequate procedure, since the results are associated only with the geometrical evolution of each part of the sample during the operation and do not depend on the measurement of the mechanical properties of the material after processing.

Table 5 displays the experimental results of the redundant work factor $\Phi_{\text{exp}}$ for the six drawing conditions evaluated in this study. The experimentally measured drawing forces, converted into average drawing stress values, were used in the calculation, as well as the Sach’s slab method theory for estimating the drawing stress in the process. In this case, $\Phi_{\text{exp}}$ was considered as a multiplying factor (Eq. (3)), as recommend by Wistreich 16) and Johnson and Rowe 2). The friction coefficient $\mu$ used in the formulation was also determined according to the Johnson and Rowe’s study 2) performed with four metals (low carbon steel, oxygen-free copper, commercially pure aluminum and 60/40 brass) and several operation variables. Their results had indicated that under conditions where the parameter $\Delta$ is close to unity (i.e., small die semi-angles and large reductions of area), the redundant work factor would be approximately unity.

Concerning the die geometric features involved in this analysis, the friction coefficient was calculated employing $\alpha=3^\circ$ and $r=15\%$ ($\Delta=1.29$), as well as the drawing stress value related to this situation, which led to $\mu=0.138$.

Figure 13 shows the $\Phi_{\text{exp}}$ versus $\Delta$ data and the linear expression associated with them ($\Phi_{\text{exp}}=0.627+0.098\Delta$). In addition to these results, the equation established by Johnson and Rowe 21) ($\Phi_{\text{exp}}=0.88+0.195\Delta$) is also displayed. Similarly to the redundant deformation factor analysis ($\phi_{\text{r}}$ and $\phi_{\text{CA}}$) (Fig. 10), $\Phi_{\text{exp}}$ was found to increase with $\Delta$. The occurrence of redundant work factor values below unity (for $\Delta<4$) is also observed, analogously to the results obtained for $\phi_{\text{CA}}$ and $\phi_{\text{r}}$ (this last parameter, as previously discussed, was below unity not only for $\Delta<4$, but for all drawing conditions evaluated). Pronounced differences between the $\Phi_{\text{exp}}$ and $\Phi_{\text{exp}}$ are verified. The predicted values were higher than the experimental ones. This may be associated
with the lower work hardening of AISI 420 stainless steel under axisymmetric drawing than under tension, which would lead to experimental drawing stresses lower than those expected for a material whose hardening would be independent of the strain path. It should be remembered that Johnson and Rowe’s approach to the evaluation of the drawing stress through Sach’s equation supposes that no strain path effects are present. It is also noteworthy that $\Phi_{exp}$ below unity are observed (see Fig. 13), which is impossible for strain path free materials. A theoretical equation for the analysis of the redundant work factor in drawing has also been established by Caddell and Atkins. In this case, $\Phi$ was related to the redundant deformation factor $\phi$ and to the Hollomon’s equation hardening exponent $m$, through the equation $\Phi = \phi^{m+1}$. The investigation, however, involves the use of stress–strain curves superposition technique results and, therefore, as formerly discussed, includes strain path change effects.

4. Conclusions

(1) The visioplasticity results confirmed the influence of the die geometry features on the strain profiles and consequently on the average deformation in drawing. An increasing linear relationship between the redundant deformation factor $\phi$ and the parameter $\Delta$ was established.

(2) The stress–strain curves superposition technique seems to be inadequate to estimate the average deformation in drawing because it involves not only the forming operation characteristics but also strain path change effects. For the AISI 420 stainless steel, redundant deformation factor values almost independent of the drawing conditions and below unity were measured.

(3) Similarly to the redundant deformation factor analysis conducted with the stress–strain curves superposition technique, some redundant work factor values estimated, utilizing the experimental drawing stress results and the procedure developed by Johnson and Rowe, were found to be below unity (in this case only for $\Delta < 4$). The phenomenon also seems to be related to strain path change effects.

(4) Considering both deformation and redundant work factor analysis, the use of techniques that involve the work hardening characteristics or the mechanical properties of the metal leads to non-conclusive results. In this case, at least in terms of the evaluation of the deformation in drawing, the visioplasticity is considered as the most adequate experimental procedure, since its results are associated only with the geometrical evolution of the sample during the operation.

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