Approximate Model for Predicting Roll Force and Torque in Plate Rolling with Peening Effect Considered

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An approximate model for predicting roll force and torque in plate rolling is proposed. In this model, peening effect which unavoidably occurs in plate rolling due to a small ratio of work roll radius over mean thickness of slab (material) being deformed is considered. Besides, the proposed model does not need any iterative scheme to compute maximum reduction ratio per pass.

The proposed roll force model consists of three parts: i) an ideal roll force under homogeneous deformation condition and ii) geometrical factor depending on inhomogeneous deformation state in roll gap and roll radius, material width and thickness and finally iii) the ratio of work roll radius increased by flattening over original roll radius. Multiplication of these three parts gives roll force in a given pass. A similar procedure was taken for roll torque model. A lever arm ratio which plays a vital role in computing roll torque was modified so that inhomogeneous deformation state of material in roll gap is fully covered.

The validity of the proposed model is verified by applying it to No. 2 plate mill in POSCO. Results reveal that the predicted roll force and torque are in a good agreement with measured data, in comparison with conventional model being used. Accuracy of roll force is remarkably improved from 16.7 to 5.4 % and that of roll torque is outstandingly enhanced from 26.0 to 6.3 %.

KEY WORDS: peening effect; roll force; roll torque; approximate model; plate rolling.

1. Introduction

In plate rolling, setting of draft schedule is one of the most important tasks for a process designer to optimize rolling schedule. The draft schedule is based on maximizing the reduction ratio at each pass without exceeding the admissible limits of mill capabilities such as roll force and roll torque. There have been many research activities to develop a model which predicts the roll force and torque in plate rolling. They are divided into two groups: one-dimensional analytic model based on the slab method and numerical model based on finite element method.

The 1-D analytic model$^{1-4}$ calculates the roll force and torque with reasonable accuracy in case of strip rolling which has small non-homogeneous deformation in hot working, compared with plate rolling. However, prediction of roll force and torque in plate rolling with the 1-D analytic model gives rather poor accuracy due to large ratio of thickness to roll radius and thick material which leads to severe non-homogeneous deformation in the roll gap. The 1-D analytic model was modified with adaptive learning coefficients and has then been used as a standard model for predicting roll force and torque in plate rolling. But its accuracy is still not good enough because of its over-simplification of deformation phenomenon of material in roll gap.

Finite element method has been widely used in the analysis of rolling process.$^{5-9}$ FEM is very effective in calculating plastic deformation state, temperature distribution, roll force and torque, but requires at least half hour to run a program for a single pass. Hence, considering computational time and complicated mechanical/thermal boundary conditions (friction condition at the roll/material interface and heat transfer coefficients dependent on the roll pressure) necessary in FE formulation, an approximate model with reliable accuracy is still necessary.

Kwak and Hwang$^{10}$ developed an approximate model predicting roll force and power applicable to finishing stand of tandem hot strip mill using rigid visco-plastic FEM and concept of hypothetical deformation mode of material in roll gap. They reported that their model gives high prediction accuracy as far as thin strip rolling is concerned. However, the approximate model is problematic to be applied to plate rolling since the ratio of work roll radius over thickness of material in strip rolling is very large compared with that in plate rolling. In addition, it requires an iterative calculation route for one to determine the reduction ratio within an allowable maximum roll force and torque in a given pass.

In this paper, we propose a model which eliminates this problem and predicts roll force and torque in plate rolling. The basic idea considered is that the roll force can be computed by multiplying three parts: i) an ideal roll force under homogeneous deformation condition and ii) geometrical factor depending on inhomogeneous deformation state in roll gap, roll radius and material width and thickness and iii) the ratio of work roll radius increased by flattening over original roll radius.

To obtain the geometrical factor, we carry out rigid visco-plastic FE analysis of plate rolling and determined critical parameters affecting the geometrical factor. In order to remove the interrelation between the critical parameters, geometrical factor is expressed using the process variables at each pass known in advance such as friction coefficient, radius of work roll and entrance thickness of material. Following the analysis of roll forces measured in plate mill, the ratio of work roll radius flattened over original roll radius is approximated by combination of the width and thickness of material at each pass, and the function of mixture resist-
Advantages of the proposed model are followsings. First, it can compute allowable maximum reduction ratio per pass without an iteration scheme if the values of roll force and torque allowable in a plate mill are provided. Second, the deformation resistance equation is formulated based on roll force data measured from load cell in plate mill and is then included implicitly in the mixture resistance. Hence we can save the time required for taking extensive laboratory scale compressive tests to gain the equation for deformation resistance whenever material is changed. Third, ‘peening’ typically occurring in the width rolling stage of plate mill is considered in the proposed model. When the ratio of contact length between work roll/material to mean thickness of the material in roll gap is less than about 1.0, the peening arises and its effect on roll force and torque is considerable.

Roll torque is computed on the basis of roll force model proposed. A similar procedure taken in the derivation of geometrical factor is adapted. The lever arm ratio which plays a crucial role in computing torque is formulated so that inhomogeneous deformation state of material in roll gap is fully covered. To verify the proposed model, it was applied to No. 2 Plate Mill in POSCO. About 65,000 actual data sets (roll forces and torques) were obtained and compared with the computed ones.

2. Conventional Roll Force Model

Sims’s roll force model\(^1\) is composed of multiplication of three parts, i.e., deformation resistance of material, \(K_m\), contact area, \(W_l\), and geometrical factor, \(Q_p\), determined by rolling conditions (reduction ratio, entry thickness of material, radius of work roll) and thus given as a following form

\[
F = K_m W_l Q_p \quad \text{...............}(1)
\]

\(W\) denotes the plate width and \(l_f^r\) is a projected contact length at the plate/roll interface and can be approximated as \(\sqrt{R(l_1-l_2)}\). The geometrical factor is expressed in terms of reduction ratio, \(r\) in a given pass, radius of curvature of work roll flatten in the region where roll contact material being deformed, \(R_l\) and outgoing thickness of plate, \(H_2\) as follows

\[
Q_p(r,H_2,R_l) = \frac{\pi}{2\delta} \tan^{-1}\delta - \frac{\pi}{4} - \frac{1}{\delta H_1/R_l} \ln\left(\frac{H_n\sqrt{1-r}}{H_2}\right) \quad \text{...............}(2)
\]

where

\[
\delta = \sqrt{r(1-r)} \\
H_n = H_2 + 2R_l(1-\cos\phi_n) \\
\phi_n = \frac{H_2}{R_l} \tan\left(\frac{1}{2} \tan^{-1}\delta + \frac{\pi}{8} \frac{H_n}{R_l} \ln(1-r)\right)
\]

\(H_1\) and \(H_2\) represent the incoming (entry) and outgoing (exit) thickness of material in a given pass. \(H_2\) is the material thickness in the roll gap at the neutral point.

The background for the geometrical factor, \(Q_p\) is as follows; Slab method gives approximate solution for the roll pressure of Orowan’s 1-D differential equation.\(^3\) However, the slab method assumes homogeneous and isothermal deformation state in roll gap. In practice, as shown in Fig. 1, deformation and thermal state of material in roll gap is inhomogeneous and non-isothermal. \(Q_p\) in Fig. 1 indicates the lever arm used in the calculation of roll torque. Explanation of this will be given later.

Slab method always results in lower roll force than the measured one owing to a difference between homogeneous deformation assumption and inhomogeneous deformation in reality.\(^1\) Thus, the geometrical factor, \(Q_p\) must be introduced to compensate this difference.

In Sims model, the sticking friction was assumed in the entire contact region. As a result, the variation of roll force in the deformation zone due to friction was neglected. In reality, however, the friction coefficient decreases almost linearly from entrance to the neutral plane and then increases in the same pattern to the exit.\(^1\) For this reason, prediction accuracy of Eq. (1) is not good.

To overcome complexity of Eq. (2) and enhance its computational speed, Denton and Crane\(^4\) suggested \(Q_p\) with a quite shortened form using the concept of geometric mean aspect ratio in the deformation zone, \(Z_g\)

\[
Q_p(r,H_2,R_l) = 0.655 + 0.265Z_g \quad \text{...............}(3)
\]

where

\[
Z_g = \frac{l_f}{\sqrt{H_1H_2}} = \sqrt{\frac{R_l}{H_2}}
\]

Denton and Crane model was also based on assumption that material was compressed between two inclined planes with sticking friction. In the Denton and Crane model, however, the effect of work hardening of material during rolling was ignored at the expense of over simplification of \(Q_p\) and subsequently its roll force prediction accuracy is not good either.

Both Sims model and Denton and Crane model have many drawbacks to be used in plate mill: The radius of curvature of work roll flatten in the region where work roll contacts material being deformed should be computed in advance at each pass. Since the effect of friction on roll force was ignored in Eqs. (2) and (3) by assumption of sticking condition, the predicted roll forces is always deviated from measured roll forces. In addition, the equation for deformation resistance in roll force model (Eq. (1)) has to be established through hot compression test whenever material is changed. A complex iterative scheme is also neces-
sary to calculate tolerable maximum reduction ratio at a pass. Consequently, Eqs. (2) and (3) are rarely used in an actual plate mill.

Hence, in an actual plate mill, a roll force model that the flow stress equation and friction effect is implicitly imbedded and no iterative scheme for computing reduction ratio is needed has been highly desirable. But, developing this type of model is very difficult. Therefore, in plate mill, empirical-based roll force and torque model has been developed and used. For example, an empirical-based roll force model used in plate mill of POSCO is shown in Eq. (4).

\[ F = B_0 r^s \frac{R}{R_0} \] .................................(4)

where

\[ B_0 = K_m W C_\gamma C_f C_\phi C_R \]
\[ C_f = a_f \exp[C_{a_f}(1 - (T_1 / T_{ref})^2)] \]
\[ C_\gamma = a_\gamma + a_\gamma^2 H_T + a_\gamma^3 H_T^2 \]
\[ C_R = a_R + a_R^2 V_R \]

\( r \) and \( V_R \) denote reduction ratio and roll speed, respectively. \( C_{a_f} \) denotes equivalent carbon content which takes into account the effect of the chemical composition of material. \( R' \) and \( R \) denote the radius of work roll with and without flattening influence. \( K_m, T_{ref} \) and \( T_1 \) are the reference strength, reference temperature and entry mean temperature of material being deformed. \( a_\gamma, a_R \) are the empirical constants determined by mill operators based on mill log data. Even though actual roll force data is reflected into Eq. (4), there are too many constants to be determined. Significant cost and time for trial-and-error are required for determining the constants. Hence, usefulness of Eq. (4) is limited.

3. Proposed Model

We propose a roll force model which does not need to calculate the radius of curvature of work roll flattened, \( R' \) before rolling and can set up constitutive equation of material from the measured roll force data in plate mill whenever material is changed. The effect of friction on roll force is reflected as a proportionality factor between roll force in non-homogeneous deformation condition, \( F_r \), and roll force under assumption of homogeneous deformation condition, \( F_t \).

\[ F_r = W \int \sigma_y dF = \frac{2RW}{\sqrt{3}} \int_0^{\theta_2} \cos \phi \, d\phi \]

\[ \equiv K_m W \] .................................(8)

where \( K_m = (2/\sqrt{3}) \sigma_0 (\varepsilon_m, \theta_2, T) \). Substituting Eq. (8) into Eq. (1) gives

\[ F = Q_r \frac{R}{R_0} \] .................................(9)

It shows that if the work roll is not flattened during rolling, the geometrical factor, \( Q_r \), plays as a proportionality factor between roll force in non-homogeneous deformation condition, \( F \) and roll force under assumption of homogeneous deformation condition, \( F_t \).

3.1. Geometrical Factor, \( Q_r \)

The process variables which might affect the geometrical factor, \( Q_r \), are as follows; Roll speed, diameter of work roll, entry mean temperature of material, carbon contents, reduction ratio, arithmetic average aspect ratio (ratio of contact length to mean thickness of plate in roll gap) and contact heat transfer coefficient between work roll and plate. The ranges of these process variables are summarized in Table 1.

Eulerian finite element model (9) was adopted for the analysis of the steady-state rigid-viscoplastic deformation occurring in the roll gap. The governing equations for rigid-viscoplastic deformation of the workpiece, namely the equilibrium equation, constitutive relationship and the incompressibility condition are stated. The boundary conditions at the workpiece surface are prescribed to fulfill the contact condition by penalizing the normal velocity of the workpiece relative to the tool at the contact surface. Variational equation is derived for the above boundary value problem that results in a set of non-linear algebraic equations that may be solved either by a direct iteration method or by the Newton-Raphson method. Due to work roll-plate contact and interaction between the thermal and mechanical behavior of the rolled material, the transport phenomena in roll gap is strongly interdependent. In order to resolve the
coupled aspects of the problem, an iterative technique is adopted as shown in Fig. 2. The boundary conditions selected for FE model are summarized in Table 2 and Fig. 3.

Figure 4 reveals that $Q_p$ does not vary with rolling speed, diameter of work roll, mean temperature of incoming material, carbon contents and contact heat transfer coefficient changed. However, $Q_p$ fluctuates when arithmetic average aspect ratio, reduction ratio and friction coefficient vary. With these three terms, the geometrical factor is expressed as follows

$$Q_p(r, H_1, R, \mu) = a(\ln s)^2 + b \ln s + c \quad \text{.................(10)}$$

where

$$a = 0.552198 + 0.34999$$

$$b = 0.018377 s^2 - 0.26279 r - (2.32529\mu - 2.41723)$$

$$c = -0.56628 s - 0.191051 \mu + 1.63922$$

$$s = \frac{2 R r (1 - r)}{2 - r} \sqrt{\frac{H_2}{H_1}}$$

But Eq. (10) is not appropriate to compute allowable maximum reduction ratio per pass without an iteration scheme since $Q_p$ is an implicit function of reduction ratio, friction coefficient and outgoing thickness of material (slab). Hence an alternative form for $Q_p$ is needed. Therefore, we suggest Eq. (11) in which $Q_p$ is expressed explicitly in terms of reduction ratio, friction coefficient, $\mu$ and incoming thickness of material, $H_1$

$$Q_p(r, H_1, R, \mu) = \exp(P_1)^{R^1} \quad \text{.................(11)}$$

where

$$P_1 = 0.0003207 H_1 + 0.27071\mu - 1.09315 + 0.382162 x_2$$

$$+ x_2 x_3 (0.004686 x_3 - 0.050602)$$

$$Q_I = -0.565149 + 0.207347 x_2 - 0.00113008 x_3$$

$$+ 0.0100622 x_3 x_4 - 1.3542 x_4^2$$

when $x_3 = 10/H_1$ and $x_4 = \sqrt{R/H_1}$.

In Fig. 5(a), $Q_p$ calculated from the proposed model (Eq. (11)), Denton model and Sims model is compared with that from FE analysis. $Q_p$ computed by the proposed model is very close to no error line. Sims model looks working well but noteworthy deviations are observed at middle region. Denton model is definitely out of the no error line. Figure 5(b) illustrates that $Q_p$ calculated from those models is examined in terms of arithmetic average aspect ratio. It also shows that $Q_p$ obtained from the proposed model catches up

### Table 2. Boundary conditions for FE analysis of plate rolling process.

<table>
<thead>
<tr>
<th>Condition Type</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Thermal boundary conditions for heat transfer analysis of work roll</td>
<td>$q_r = 0.5(s - v^r) \sigma_{sl}^r$</td>
</tr>
<tr>
<td>(b) On $\Gamma_m$, $\Gamma_2$ and $\Gamma_3$, surfaces ($\sigma_{sl}^r$)</td>
<td>$h_{in} = 0.00875, h_{in} = 0.035 \text{MW/m}^2\text{C}$</td>
</tr>
<tr>
<td>(c) Thermal properties of work roll; $\gamma$-Grained chilled steel</td>
<td>$k_{sl} = 0.0235 \text{MW/m°C}, \rho C_{pl} = 0.00405 \text{MJ/g°C}$</td>
</tr>
</tbody>
</table>

| (2) Thermal boundary conditions for heat transfer analysis of material (plate) | $h_{in} = 2.95 \text{MW/m}^2\text{C}, \varepsilon_r = 0.8, T_0 = 20°C$ |
| (b) Thermal properties of material | $k(\text{W/mK}) = 21.9 + 14.33^t$ where $t = T - 11000$ |
| (c) Flow stress of plate proposed by Shida | $\sigma_r = 0.56(\mu u_h u_r)$, $\sigma_r = -\mu_2\sigma r u_h$ |

**Fig. 2.** Computational procedure for coupled FE analysis.

**Fig. 3.** Boundary conditions for FE models: (a) mechanical boundary conditions for plate (where $t_r$ is traction vector along $x$), (b) thermal boundary conditions for plate (where $q_r$ is surface heat flux), (c) thermal boundary conditions for work roll.

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the behavior of \( Q_p \) well while the reduction ratio and arithmetic average aspect ratio are changed. But the Denton model and Sims model do not follow up at all the variation of \( Q_p \) in case where arithmetic average aspect ratio, \( s \) is less than 1.0.

A sharp increase of \( Q_p \), as the arithmetic average aspect ratio is less than 1.0, is due to the increased amount of the inhomogeneous deformation in roll gap. Under this condition, predicting roll force using Denton model and Sims model inevitably causes error. It should be noted that the thickness of slab and diameter of work roll in plate rolling process are relatively larger than that in hot strip rolling process. A rolling condition where thicker materials is rolled with smaller drafts (reduction) is called peening. This peening phenomenon usually occurs in plate rolling process. Width rolling sequence for manufacturing products with wide width in plate mill has occasionally a short contact length caused by restricting the amount of reduction to prevent the up or down bending of top or bottom parts of plate along its rolling direction. Therefore, the peening effect on the prediction of roll force in plate rolling process should be considered.

3.2. Deformation Resistance

An equation for deformation resistance of a material is usually determined on the basis of tensile, torsion and compressing tests for the various temperature and strain rates. In addition, each test is usually repeated at least twice for a given test condition to confirm reproducibility. Consequently obtaining the equation for deformation resistance requires a lot of cost and time on every occasion material is changed. Therefore, a method which determines the equation for deformation resistance based on the actual data set measured in plate mill is used in this proposed model.

The deformation resistance in hot rolling can be described as a function of average strain, average strain rate, rolling temperature and chemical compositions of steel such as \%C, \%Mn, \cdots and \%Ti.
Thus, equation for roll force, 

\[ F = W_1 \sqrt[3]{R/H_1} \exp(P_r) r^{0.3} \times \frac{2}{\sqrt[3]{3}} \exp(C_{ch}) \left( \frac{2}{\sqrt[3]{3}} \sqrt{R H_1} \right) \exp \left( \frac{T_0}{T_1} \right) \]  

\tag{17}

Taking logarithm for Eq. (17) and replacing \( F \) with \( F_{\text{actual}} \) gives 

\[ \ln \left( \frac{2 W_1 \sqrt[3]{R/H_1} \exp(P_r) r^{0.3}}{C_{ch} + n_r \ln r + n \ln \left( \frac{2}{\sqrt[3]{3}} \sqrt{R H_1} \right)} + \frac{T_0}{T_1} \right) \]  

\tag{18}

Note that \( F_{\text{actual}} \) is the roll force measured at a plate mill.

Finally, the coefficients on the right hand side of Eq. (18) are determined by nonlinear regression using actual load data set such as roll force, reduction ratio, rolling speed and entrance temperature for each pass. As an example, the coefficients for carbon–manganese steel are as following.

\[ C_{ch} = -0.0408123 + 0.8162[\% C] + 0.21411[\% Si] \]
\[ + 0.06235[\% Mn] + 0.6801[\% P] - 2.1513[\% V] \]
\[ - 0.4554[\% Ni] - 0.23888[\% Mo] + 0.16039[\% Cr] + 3.5199[\% Ti] \]

\[ n_r = 0.228308 + 0.42157[\% C] \]
\[ n = 0.054011 \]
\[ T_0 = 6311.14 \]

### 3.3. Roll Force Model–Final Form

Rewriting Eq. (9) gives a final form of roll force model which is composed of three parts; mixture resistance, \( H_m \), reduction ratio, \( r^p \) per pass and ratio of roll flattening, \( k_R \) in the followings

\[ F = H_m r^p k_R \]  

\tag{19}

where

\[ n_r = Q_r + n_r + 0.5, \quad k_R = \sqrt{R/H_1} = k_0 + \sqrt{1 + k_0^2} \]

\[ k_0 = \frac{c_r H_m}{2 W_1 H_1} \]

The geometric factor, \( Q_r \) and roll force under condition of homogeneous deformation, \( F_h \) in Eq. (9) are imbedded in the mixture resistance, \( H_m \) and reduction ratio in Eq. (19). The mixture resistance is expressed as a reference strength which takes into account the chemical composition of material, slab thickness at each pass, rolling speed and mean entry temperature of material at each pass in a form

\[ H_m = \sigma_{\text{ref}} W_k k_{H} k_{r} k_{r} \]  

\tag{20}

where

\[ \sigma_{\text{ref}} = \frac{2}{\sqrt[3]{3}} \exp(C_{ch}), \quad k_{H} = \sqrt{R H_1} \exp(P_r), \]

\[ k_r = \frac{2}{\sqrt[3]{3}} \sqrt{R H_1} \exp \left( \frac{T_0}{T_1} \right) \]

It should be reminded that the reference strength can be calculated through regression of roll force data measured in an actual plate mill. Other values such as \( k_{H} \), \( k_{r} \) and \( k_{r} \) are
computed explicitly from material constants and process variables known beforehand. Hence we need not to take extensive laboratory scale tests to know the mixture resistance, \(H_m\), whenever material is changed.

To express \(k_g\) in terms of known variable, \(k_0\), we use Hitchcock’s equation\(^{11}\)

\[
R' = R \left(1 + \frac{c_1 F}{W(H_1 - H_2)} \right) = R \left(1 + \frac{c_1 H_2 \rho^{-1} - 1}{W H_1} \right) \quad \text{(21)}
\]

where \(c_1 = 16(1 - \nu_0^2)(\pi E_0)\). \(E_0\) and \(\nu_0\) denote Young’s modulus and Poisson’s ratio for the work roll. Putting \(c_1 H_2 \rho^{-1} = k_f \rho^{-1}\) and substituting it into Eq. (21) yields

\[
k_g = 2(k_f \rho^{-1} - 1) = 0 \quad \text{(22)}
\]

Solution of Eq. (22) is \(k_g = k_f \rho^{-1} + \sqrt{1 + (k_f \rho^{-1})^2}\). This is approximated as followings

\[
k_g \approx k_0 + \sqrt{1 + k_0^2} \quad \text{(23)}
\]

Hence we have expression for approximated roll radius flatten during rolling as follows

\[
R' = R(k_0 + \sqrt{1 + k_0^2})^2 \quad \text{(24)}
\]

To check usefulness of this approximation, we examine the difference of roll flattening ratio computed by Hitchcock’s Eq. (21) and that by the Eq. (23) using actual data set of POSCO No. 2 plate mill. Figure 6 shows the error is less than 1.4% as the roll flattening ratio computed by Hitchcock’s equation goes up to 1.018. This indicates that Eq. (23) is valid enough to be used.

4. Proposed Roll Torque Model

The roll torque due to the moments about the roll axis is calculated using the shear stresses for both work rolls\(^{11}\)

\[
G = 2W \int_0^{l_f} \frac{\tau}{\cos \phi} \, R dx \equiv WRR' k_m(2\phi - \theta) \quad \text{...(25)}
\]

\(\phi\), \(\theta\) and \(\phi_f\) denotes contact angle at the arbitrary position, total contact angle and neutral angle in roll gap. \(l_f\) is a projected contact length at the plate/roll interface. \(f'\) denotes the case where the flattening of work roll is considered. Equation (25) does not give good accuracy if the angle of neutral point is not determined precisely beforehand. For this reason, equation for roll torque can be expressed without the term ‘neutral angle’ in the followings

\[
G = 2W \int_0^{l_f} p \sin \phi \times R dx \equiv 2W \frac{R}{R'} \int_0^{l_f} px' dx' \quad \text{...(26)}
\]

\(x' = R' \sin \phi\). Here we define average pressure in the roll gap and it is approximated as follows

\[
K_G = \frac{\int_0^{l_f} px' dx' \equiv \int_0^{l_f} px dx}{\int_0^{l_f} x' dx' \equiv \int_0^{l_f} x dx} \quad \text{(27)}
\]

Here we introduce a ratio of the average pressure for torque over deformation resistance, \(H_g\), in the following way

\[
H_g = K_G / K_m \quad \text{...(28)}
\]

Substituting Eq. (27) into Eq. (26) and rearranging it using Eq. (28) gives

\[
G = 2W \frac{R}{R'} K_m H_g \frac{(l_f')^2}{2} = F_h I_d H_g \quad \text{...(29)}
\]

Equation (29) indicates that ratio, \(H_g\) plays a role as a lever arm ratio under the condition that homogeneous deformation of material in the roll gap is assumed. Therefore we need a modified lever arm ratio which can reflect inhomogeneous deformation of material in roll gap. Here we have an equation for roll torque, \(G\) which reflects inhomogeneous deformation condition in roll gap by plugging Eq. (29) into Eq. (9)

\[
G = 2W \frac{R}{R'} (l_f')^2 Q_m \quad \text{...(30)}
\]

\(Q_m = H_g/(2Q_s)\) indicates a lever arm ratio. Note that the lever arm ratio can be used when material (plate) in roll gap is subject to either inhomogeneous or homogeneous deformation state.

At this point it should be mentioned that the empirical-based torque model currently used at No. 2 plate mill in POSCO is of the following form

\[
G = 2W \frac{R}{R'} (l_f')^2 Q_m = 2W \frac{R}{R'} B_m r \sqrt{R' F / H \rho \phi} Q_m
\]

\[
= B_m \sqrt{RH} \rho \phi Q_m \quad \text{...(31)}
\]

where

\[
Q_m = \frac{1}{2} (1 + f_s) (1 - \nu) \rho \phi = \frac{1}{2}
\]

\(Q_m\) depends on reduction ratio and forward slip, \(f_s\). But forward slip is also an implicit function of reduction ratio, arithmetic average aspect ratio and friction coefficient. For convenience, therefore, \(Q_m\) was simplified as a constant, 0.5. Thus, some error in torque calculation in the stage of width rolling sequence is always observed due to the constant ‘0.5’. If torque is regarded as the results of the roll force times a lever arm (the roll force is not directed through the roll center), the peening effect will elongate the lever arm as shown in Fig. 1 and thus increase the roll torque.\(^{19}\) Therefore, a new lever arm ratio, \(Q_g\) should be built up to predict roll torque correctly.

4.1. Modified Lever Arm Ratio, \(Q_m\)

In the following, a procedure which derives the lever arm ratio in terms of known variables is explained in detail. A series of FE simulation is carried out to examine the effect of different rolling parameters on the lever arm ratio. Figure 7 illustrates \(Q_g\) is not affected at all by the change of rolling speed, diameter of work roll, entry mean tempera-
ture, friction coefficient, carbon contents and contact heat transfer coefficient. However, $Q_M$ is strongly dependent on arithmetic average aspect ratio (ratio of contact length to mean thickness of plate in roll gap) and reduction ratio. Thus, $Q_M$ can be expressed as followings

$$Q_M = 0.367392 + 0.146794r + 0.074667y
- (0.0222y + 0.06768)xy + (0.38739r - 0.55009)x
+ (0.050985y - 0.0375431)s^2 + (0.158534 - 0.084654y + 0.028012rs - 0.20474r^2)s$$  \( \ldots (32) \)

where $x = \ln(s)$, $y = \ln(s)$.

Structure of Eq. (32) is still difficult to be used in the calculation of roll torque since reduction ratio should be given in advance. As a result, different type of expression for $Q_M$ which requires only known rolling parameters such as roll radius and entry thickness of plate at each pass is proposed

$$Q_M = \exp(P_H)r^{0.5}$$  \( \ldots (33) \)

where

$$P_H = -(0.0029912x_1^4 - 1.0098x_1^2 + 7.194x_1 + 0.04386)x_1
+ 0.0040186x_2 - 0.100738x_2^3 + 0.18067x_2^2
- 0.030157x_3^4 + 0.0013161x_3^5
+ 1.5686 - 0.003947R + 0.00076331H_t$$

$$Q_H = 0.3485 - 0.53439x_1 + 0.05468x_1^2 - 0.005491x_1^4$$

$$+ [13.6934 - 5.9894x_1 + 0.59747x_1^5 - 0.0107038x_1^7]x_1$$

where $x_1 = 10/H_t$, $x_2 = \sqrt{R/H_s}$.

Figure 8(a) shows the comparison of $Q_M$ predicted by proposed model and that by FE simulation of plate rolling. The empirical-based torque model definitely shows much large difference. Denton and Crane \( ^4 \) suggested a model for lever arm ratio as followings

$$Q_M = \frac{0.795 + 0.22Z_g}{1.31 + 0.53Z_g}$$

where $Z_g = \frac{I_f^{1/3}}{H_1H_2} = \sqrt{\frac{R^3}{H_2^2}}$  \( \ldots (34) \)
Denton and Crane model starts deviating as $Q_M$ predicted from FE simulation goes up more than 0.55. But a good agreement is noted for the proposed model. Figure 8(b) illustrates usefulness of the proposed model when the arithmetic average aspect ratio (ratio of contact length to mean thickness of plate in the roll gap) is changed. When the arithmetic average aspect ratio is less than 2.0, $Q_M$ predicted from the proposed model start increasing and goes beyond 0.5. When arithmetic average aspect ratio approaches 0.5, $Q_M$ has a range of 0.64 and 0.66 with different reduction ratios. These factors may cause considerable error in roll torque computation when the constant lever arm ratio, 0.5.

4.2. Torque Model—Final Form
Substituting Eqs. (19) and (33) into Eq. (30) yields a final form of torque model

$$G = H_{wF}^{-n_G} k_{H}^{G} \exp(P_{H}) \text{ (35)}$$

where $k_{H}^{G} = 2\sqrt{R_{H}} \ exp(P_{H})$, $n_G = Q_{M}^{H}/n_F + 0.5$.

5. Roll Force and Torque—Predictions vs. Measurements
Figure 9(a) shows the schematic of POSCO No. 2 plate mill. After a slab with 250–300 mm thickness, 1570–2200 mm width and 2500–4200 mm length is discharged from reheating furnace, it is rolled to a plate with specified target dimensions (thickness, width) through two four-high stands. Figure 9(b) illustrates the schematic of plate rolling process divided into two rolling stages. The first stage is 'width rolling' which rolls the slab until the width of deformed slab, i.e., plate, reaches a specified target width. The second stage is 'longitudinal rolling' that rolls the plate until the thickness of the plate reaches to an aimed thickness. In Fig. 10, the roll forces and torques calculated by the proposed model and the empirical-based model currently used in POSCO No. 2 plate mill in terms of pass number are compared with those measured. The rolling condition that thickness of slab was reduced from 302 to 96.88 mm and its width was increased from 2200 to 3588 mm requires 13 passes (eight-width rolling passes and five-longitudinal rolling passes). The roll forces and torques calculated by the proposed model was in overall a good agreement with measured ones, compared with those by the empirical-based torque model. It should be mentioned that the difference in the width rolling stage is much larger than that in the longitudinal rolling stage. This is because the peening effect was not considered in the empirical-based model. Especially, if the arithmetic average aspect ratio (ratio of contact length to mean thickness of plate in the roll gap) is under 0.5, the peening effect is enlarged and subsequently the difference is noteworthy. Note that the peening effect is important in the width rolling stage, but not much in the longitudinal rolling stage.

The measured 65000-actual data sets have some deviations in its magnitude since rolling conditions are not always the same for every pass and slab owing to the differences between extracting temperature of slab in reheating furnace and its temperature measured just before rolling, various slab size and irregular wear profile of work roll. Hence we inevitably have the scattered values of roll forces and torques if we plot them. In Fig. 11, the calculated roll forces and torques are compared with the measured ones. To verify the effectiveness of the model proposed in this study we first define the accuracy of model as follows.

For roll force, $\chi_F \% = |F_{\text{actual}} - F_{\text{actual}}|/F_{\text{actual}}$ and For roll torque, $\chi_G = |G_{\text{actual}} - G_{\text{actual}}|/G_{\text{actual}}$ where $F_{\text{actual}}$ and $G_{\text{actual}}$ indicate the actual roll force and torque measured during rolling. $F$ and $G$ represent the calculated roll force and torque.

We can observe that there are large deviations and moreover the deviations increases significantly as roll force and torque do. This is attributable to the peening effect, which has low arithmetic average aspect ratio (ratio of con-
tact length to mean thickness of plate in roll gap) in the width rolling stage, was not reflected at all on the empirical-based torque model.

However, we could observe when the proposed model was applied, the accuracy of roll force was remarkably improved from 16.7 to 5.4% and the standard deviation was reduced from 12.3 to 4.7%. We have similar results for roll torque calculation. The accuracy of roll torque was outstandingly improved from 26.0 to 6.3%. The standard deviation also decreased from 15.2 to 5.2%. When the proposed model is used, the deviation does not propagate and it converges somehow as the roll force and torque increase. This is because the peening effect has been taken into account in the proposed model already. This is one of the promising things in the proposed model.

6. Concluding Remarks

This paper describes a procedure to make an approximate model for predicting roll force and torque in plate rolling. Forte of the proposed model is that the peening effect, which arises due to low arithmetic average aspect ratio in the width rolling stage of plate rolling, was considered accordingly. In the proposed model, the geometrical factor, $Q_p$ and lever arm ratio $Q_L = H_0/(2Q_p)$ which plays a crucial role in computing roll torque has been formulated so that inhomogeneous deformation state of material in roll gap is fully covered.

In addition, the roll force and torque data measured from load cell in plate mill is included implicitly in the proposed model. Hence we can save the time required for performing extensive laboratory scale tests to get the equation for deformation resistance whenever material is changed. To prove the usefulness of the proposed model, it was applied to POSCO No. 2 plate mill. The conclusions are summarized as follows:

When the proposed model is used, the accuracy of roll force was remarkably improved from 16.7 to 5.4% and its standard deviation reduced from 12.3 to 4.7%. Using the proposed model also gives that the accuracy of roll torque was outstandingly enhanced from 26.0 to 6.3% and its standard deviation decreased from 15.2 to 5.2%.

A draft schedule for plate rolling can be directly calculated using the proposed model since the proposed model can compute allowable maximum reduction ratio per pass without an iteration scheme if the allowable maximum values of roll force and torque in any plate mill are provided. Hence, the proposed model can be used effectively to design a draft rolling schedule if one needs a guide line for re-vamping the rolling line of a plate mill.

REFERENCES