Note

An Extended Model for Slag Eye Size in Ladle Metallurgy

Krishnakumar KRISHNAPISHARODY
and Gordon A. IRONS

Steel Research Centre, McMaster University, 1280 Main Street
West, Hamilton, ON, Canada L8S 4L7.

(Received on July 7, 2008; accepted on August 29, 2008)

1. Introduction

The slag eye is the term for the region where slag is pushed to the periphery of a steel ladle by the action of inert gas bubbling. The formation of the eye has been the subject of some investigations in the recent past.^{1–5)} In modern ladle metallurgy practice, the eye and its central spout portion raised above the bath level are some of the few regions where molten steel is directly exposed to the ambient environment. These are sites for reactions like Oxygen and Nitrogen pick up. Furthermore, the slag metal interface at the periphery of the eye region may break up into slag droplets, often referred to as ‘slag emulsification’. Such droplet generation may increase the interfacial area for slag–metal reactions, but also may result in undesirable effects like slag entrainment. Thus, the overall influence of eye formation on the end quality of steels is important.

Reliable predictive models for the eye size could form an integral component of any control strategy for on-line process control. Such models of the eye sizes may also be useful for process design considerations. With this aim, a previous mechanistic model of slag eye formation⁵⁾ is extended in a new mathematical model to quantify the eye size directly from the plant operating variables. The technique of dimensional analysis as well as analysis of published eye size data from several liquid–liquid systems have been employed in the derivation of the model.

2. Dimensional Analysis of Slag Eye Size

The slag eye arises out of interactions among multiple phases (gas–liquid–liquid) in the gas-stirred ladle. Thus, in principle, the size of the eye is expected to depend on a large number of variables in the multi-phase system. These include the gas flow rate (Q), the bath height (H), the height of the second phase (h), the diameter of the ladle (D), gravitational acceleration (g), the physical properties (*i.e.*, the densities and viscosities) of the fluids involved, the interfacial tension between the upper and lower phase liquids and the characteristics of the gas injection device. Of these, the parameters with a demonstrable secondary influence on the eye size are eliminated from the analysis, as explained below.

The effect of molecular viscosity of the lower phase is insignificant. This is because the flow phenomena in the bulk liquid under conditions typical of ladle metallurgy practice are turbulent, and viscous forces are only of secondary importance in gas-stirred systems.^{6,7)} The available experimental data from recent studies^{4,5)} indicate that the viscosity of the upper phase has little appreciable effect on the steady state eye sizes. Apparently, viscous dissipation effects and drag between the phases under the flow configu-

rations in the eye formation process are relatively small. It was shown previously that the effect of interfacial tension between the liquids is only minor.⁵⁾ Due to the relatively large length scales in the eye formation process, the forces arising from interfacial tension are very small in comparison with the effects of inertia and gravity. Thus, interfacial tension is excluded from the analysis. It has also been proven that the physical properties of the injected gas and the size and type of injector have little influence on the flow in the buoyancy-induced flow regime of the plume, typically found to start a short distance above the injection point.^{8,9)} Thus, these parameters are not expected to influence the eye formation process significantly, so they are omitted from the analysis.

On the basis of these considerations, the characteristic eye size, represented by the area of the open eye (A_e), is expressed in terms of the relevant independent variables in a general functional relationship:

$$A_e = f_1(Q, H, h, D, g, \rho_u, \rho_l) \dots\dots\dots(1)$$

In the above equation, ρ is the density and the subscripts u and l stand for the upper and lower phases, respectively.

The system represented by Eq. (1) has 8 variables with three fundamental units. Adopting the well-known Buckingham Π theorem, five different dimensionless Π groups can be obtained from Eq. (1). The ones selected are:

- The non-dimensional eye area, as $A_e^* = \frac{A_e}{H^2} \dots\dots\dots(2)$

- The non-dimensional gas flow rate, as $Q^* = \frac{Q}{g^{0.5} H^{2.5}} \dots\dots\dots(3)$

- The density ratio of the liquids, as $\rho^* = \frac{\rho_u}{\rho_l} \dots\dots\dots(4)$

- The non-dimensional slag height, as $h^* = \frac{h}{H} \dots\dots\dots(5)$

- The geometrical aspect ratio of the bath, as $D^* = \frac{D}{H} \dots\dots\dots(6)$

In the above analysis, the bath height has been used as the characteristic system dimension to non-dimensionalize other quantities. Thus, Eq. (1) can be recast into the non-dimensional form:

$$A_e = f_2(Q^*, h^*, \rho^*, D^*) \dots\dots\dots(7)$$

It is interesting to note that most of the variables in Eq. (7) are simple ratios of length scales and densities in the system.

3. Improved Model for the Eye Size

In a previous article by the present authors, an extensive experimental study of slag eye formation and a mathematical model for the eye size were presented.⁵⁾ The model expressed the eye area as a function of the plume cross-sectional area and a Froude number defined using the plume velocity and the height of the upper phase liquid (slag):

$$\frac{A_e}{A_p} = a + b Fr_D^{1/2} \dots\dots\dots(8a)$$

$$Fr_D = \left(\frac{\rho_l}{\Delta \rho} \right) \frac{U_p^2}{gh} \dots\dots\dots(8b)$$

where A_e is the area of the eye, A_p is the cross-sectional area

of the plume at the bath surface, U_p is the average plume velocity, h is the height of the upper phase, ρ_l is the density of the bulk liquid, $\Delta\rho$ is the density difference between the liquids and a and b are numerical constants. However, explicit values of a and b were not provided, in order to account for the deviations from ideality that may be encountered in actual practice.

In Eqs. (8a) and (8b), the plume area (which is smaller than the eye area) and plume velocity are not among the independent variables in Eq. (1) and as such, the model cannot be applied in a straightforward manner to predict the eye size from the operating variables of the system. To overcome this limitation, an extended model is derived in terms of the non-dimensional quantities obtained from the preceding dimensional analysis.

In the study from which Eq. (8) was obtained, the ladle diameter was large enough to not influence the eye size. The force balance from which Eq. (8) was derived simply used the nominal height of the slag.⁵⁾ However, as the eye size increases slag is pushed to the periphery, increasing the effective slag height; so the actual slag height (h') should be used in the force balance. Thus, Eq. (8) is modified to:

$$\frac{A_e}{A_p} = a + b \left(\frac{\rho_l}{\Delta\rho} \right)^{1/2} \frac{U_p}{\sqrt{gh'}} \quad \text{.....(9)}$$

The actual slag height is related to the nominal height (h) by a mass balance:

$$h' = \left(\frac{A_b}{A_b - A_e} \right) h \quad \text{.....(10)}$$

In the above equation, A_b is the cross-sectional area of the ladle, which can be expressed in terms of the ladle diameter:

$$A_b = \frac{\pi D^2}{4} \quad \text{.....(11)}$$

Using Eqs. (10) and (11), Eq. (9) takes the form:

$$\frac{A_e}{A_p} = a + b \left(\frac{\rho_l}{\Delta\rho} \right)^{1/2} \frac{U_p}{\sqrt{gh}} \left(1 - \frac{4}{\pi} \frac{A_e}{D^2} \right)^{1/2} \quad \text{.....(12)}$$

Using bath height (H) to non-dimensionalize the plume area in Eq. (12), one obtains:

$$A_p^* = \frac{A_p}{H^2} \quad \text{.....(13)}$$

Using Eqs. (2)–(6) and Eq. (13), Eq. (12) can be manipulated to:

$$\frac{A_e^*}{A_p^*} = a + b(1 - \rho^*)^{-1/2} \frac{U_p}{\sqrt{gH}} (h^*)^{-1/2} \left[1 - \frac{4}{\pi} \frac{A_e^*}{D^{*2}} \right]^{1/2} \quad \text{.....(14)}$$

In a recent work on the similarity analysis in two-phase plumes, Krishnapisharody and Irons¹⁰⁾ correlated the non-dimensional plume velocity to the non-dimensional gas flow rate (Q^*) as:

$$\frac{U_p}{\sqrt{gH}} = U_p^* = 1.16(Q^*)^{0.32} \quad \text{.....(15)}$$

Using Eq. (15), Eq. (14) is modified to the general non-dimensional form:

$$\frac{A_e^*}{A_p^*} = \alpha + \beta(1 - \rho^*)^{-1/2} (Q^*)^{1/3} (h^*)^{-1/2} \left[1 - \frac{4}{\pi} \frac{A_e^*}{D^{*2}} \right]^{1/2} \quad \text{.....(16a)}$$

where α and β are numerical constants. Further, the non-dimensional plume area can be quantified solely in terms of Q^* as¹⁰⁾:

$$A_p^* = 1.41(Q^*)^{0.4} \quad \text{.....(16b)}$$

Equations (16a) and (16b) together prescribe the non-dimensional eye area in terms of other dimensionless quantities derived from the primary variables of the system. These new model equations are fully consistent with the form of Eq. (7) suggested by dimensional analysis and are in line with the laws of physical modeling.

It can be further noticed that in Eq. (16a), A_e^* appears on both sides of the equation in such a form that its direct evaluation is unwieldy and cumbersome. However, Eq. (16) can be further simplified to a form suitable for practical engineering applications, based on the following considerations.

If we use the diameter of the eye, denoted by d_e , to represent the eye size on the Right Hand Side of Eq. (16a), this equation takes up the alternate form:

$$\frac{A_e^*}{A_p^*} = \alpha + \beta \left[1 - \left(\frac{d_e}{D} \right)^2 \right]^{1/2} (1 - \rho^*)^{-1/2} (Q^*)^{1/3} (h^*)^{-1/2} \quad \text{.....(17)}$$

In the above equation, the rate at which the eye size (A_e^*) changes with other parameters (Q^* , ρ^* , h^*) depends on the eye size itself, through the coefficient factor of the second term on the R. H. S., $\beta[1 - (d_e/D)^2]^{1/2}$. To simplify Eq. (17), some scaling analysis of this coefficient factor with respect to d_e/D is required. Let us take an upper limit of the eye diameter as half the diameter of the ladle in any realistic operating practice. Then, it is easily seen that, as d_e/D is varied from 0 to 1/2, the coefficient factor varies from the value of 1.0β to 0.87β . This is indeed a relatively minor change for such a large variation in the eye size; thus the effect of vessel diameter is weak for relatively large eyes. Hence, for practical purposes, the coefficient factor in Eq. (17) may be replaced by an average effective value of 0.94β ($=\gamma$), absorbing the effect of D^* . This results in:

$$\frac{A_e^*}{A_p^*} = \alpha + \gamma(1 - \rho^*)^{-1/2} (Q^*)^{1/3} (h^*)^{-1/2} \quad \text{.....(18)}$$

The simplified model, Eq. (18), captures all the appropriate influences on the eye formation process. The model constants will be obtained from the analysis of available experimental data on the eye size. The complete model will be presented [Eq. (19)] in the next section.

4. Discussion

The adequacy of the model can be ascertained by testing it against the eye size data taken from the literature. The model is valid only for conditions simulating ladle metal-lurgy operations (usually with a thin slag layer), hence only such data where $h/H < 0.05$ were selected for the analysis. The results are discussed below.

Figures 1, 2 and 3 show plots of the variation of the ratio of the non-dimensional areas (A_e^*/A_p^*) with one-third power of the non-dimensional gas flow rate for three different systems. In each of these plots, the results generated by Eq. (19) are also given as a series of straight lines. The variation of the data generally follows a linear trend. These figures also indicate the negative influence of the non-dimensional upper layer thickness on the slag eye size.

In **Fig. 4**, the ratio of the non-dimensional eye area as in Eq. (18) is plotted against the parameter $(Q^*)^{1/3}(h^*)^{-1/2}$, to isolate the effect of density ratio of the liquids (ρ^*) on the eye formation behavior. The available experimental data

Fig. 1. Variation of non-dimensional eye area with dimensionless gas flow rate in water-paraffin oil system (K & I: Krishnapisharody and Irons⁵).

Fig. 2. Variation of non-dimensional eye area with dimensionless gas flow rate in CaCl_2 -paraffin oil system (K & I: Krishnapisharody and Irons⁵).

Fig. 3. Variation of non-dimensional eye area with dimensionless gas flow rate in an industrial steel-slag system (Y & S: Yonezawa and Schwerdtfeger¹).

Fig. 4. Effect of relative density of liquids on the eye size in gas-stirred systems (K & I: Krishnapisharody and Irons⁵, Y & S: Yonezawa and Schwerdtfeger¹, H: Han *et al.*¹¹).

Fig. 5. Variation of the dimensionless eye area in different liquid-liquid systems with other non-dimensional parameters as in Eq. (18) (K & I: Krishnapisharody and Irons⁵, Y & S: Yonezawa and Schwerdtfeger¹, H: Han *et al.*¹¹).

from several liquid-liquid systems, including a full-scale steel-slag system, are used in Fig. 4. It can be noted that the data from different systems fall into distinct envelopes in the plot, each of which can be approximated by a linear fit. Such a behavior is expected from the model [Eq. (18)], as the density ratios of the fluids in the system are vastly different. Finally, these different sets of data are combined in a single plot by plotting the non-dimensional area ratio (A_e^*/A_p^*) vs. the parameter $(1-\rho^*)^{-1/2} (Q^*)^{1/3} (H/h)^{1/2}$, as shown in Fig. 5. It can be clearly observed from Figs. 1–5 that Eq. (18) captures the trends in the eye size variation in a variety of systems with different physical properties reasonably well. Most of the data were found to lie within $\pm 30\%$ of the linear fit.

The above results clearly demonstrate that Eq. (18) presents a reasonable macroscopic model for the slag eye formation in gas-stirred liquid systems. However, for predictive applications, the numerical constants in the model (α and γ) must be evaluated. This is better obtained from the analysis of experimental data.

A linear regression analysis of all the experimental data in Fig. 5 yields the relationship:

$$\frac{A_e^*}{A_p^*} = (-0.54 \pm 0.25) + (5.07 \pm 0.26)(1-\rho^*)^{-1/2} (Q^*)^{1/3} (h^*)^{-1/2} \dots (19)$$

Using Eq. (16b), the above relationship can be modified to the simpler form, after omitting the standard error terms, as:

$$A_e^* = -0.76(Q^*)^{0.4} + 7.15(1-\rho^*)^{-0.5} (Q^*)^{0.73} (h^*)^{-0.5} \dots (20)$$

The preceding relationship deduced in the form of Eq. (7)

can be easily applied to obtain estimates of eye sizes, given the primary operating variables of the process. This makes it useful for engineering applications, such as preliminary design calculations in ladle metallurgy.

5. Conclusions

A new mathematical model for the slag eye size in gas-stirred systems is presented, in the basic non-dimensional form [Eqs. (19) and (20)], as an extension to a previously derived mechanistic model. The model computes the eye size from the primary operating variables of the ladle and demonstrated reliable predictive abilities in a variety of multi-phase systems. It may be employed for engineering and design purposes in ladle metallurgy.

REFERENCES

- 1) K. Yonezawa and K. Schwerdtfeger: *Metall. Mater. Trans. B*, **30B** (1999), 411.
- 2) Subagyo, G. A. Brooks and G. A. Irons: *ISIJ Int.*, **43** (2003), 262.
- 3) D. Mazumdar and J. W. Evans: *Metall. Mater. Trans. B*, **35B** (2004), 400.
- 4) M. Iguchi, K. Miyamoto, S. Yamishita, D. Iguchi and M. Zeze: *ISIJ Int.*, **44** (2004), 636.
- 5) K. Krishnapisharody and G. A. Irons: *Metall. Mater. Trans. B*, **37B** (2006), 763.
- 6) D. Mazumdar and R. I. L. Guthrie: *Metall. Trans. B*, **16B** (1985), 83.
- 7) D. Mazumdar: *Metall. Trans. B*, **21B** (1990), 925.
- 8) K. Krishnapisharody and G. A. Irons: *Metall. Mater. Trans. B*, **38B** (2007), 377.
- 9) V. Sudhakar and D. Mazumdar: *Metall. Mater. Trans. B*, **27B** (1996), 704.
- 10) K. Krishnapisharody and G. A. Irons: Scanmet III—Proc. 3rd Int. Conf. on Process Development in Iron and Steelmaking, Vol. 2, Mefos, Lulea, (2008), 589.
- 11) J. W. Han, S. H. Heo, D. H. Kam, B. D. You, J. J. Pak and H. S. Song: *ISIJ Int.*, **41** (2001), 1165.