1. Introduction

An annealing process for steel manufacturing is needed to prevent the hardening of steel and to improve its quality. This process continuously proceeds through web transport systems, and is known as a ‘continuous annealing process’. In order to achieve optimum processing, the maintenance of the desired tension of the web within the process is required. There are a number of problems with maintaining the desired tension of the web in a CAP. The change in temperature of the web is extensive during a CAP1) and a WTS in a CAP is a very large-scale plant. This means that a broad workspace is needed for a CAP. Thus, in order to save workspace, steel mills generally employ a vertical WTS. Consequently, a WTS in a CAP affects the temperature and gravity on the web, consequently making it difficult to maintain the desired tension of the web in a WTS.

Fukushima presented an overview of the trend for a CAP, the history of continuous annealing and the technology of a high speed CAP.2) He also explained the C-camber and L-camber phenomena of the web due to the furnace temperature. In order to avoid thermal deformations, he mentioned that the equipment design for a CAP must be carefully considered throughout the actual operations. Brandenburg investigated the steady-state and transient behaviors of the tensile force, stress, strain and register error of the moving web as functions of important process variables such as the position and speed of the driven rollers, and the density, cross-sectional area, modulus of elasticity and temperature of the web.3) In particular, he explained that the web is influenced by the various temperatures in the steady-state and transient conditions through the relationship between the tension-dependent strain and the temperature-dependent strain. However, he did not use the concept of the equivalent strain for deriving the model equation while considering the temperature effect of the web. He also did not consider the deformation of the web due to the gravity effect.

In this paper, a new mathematical model of a WTS is proposed that is capable of considering thermal and gravity effects in a CAP. A wide range of the variation of temperature and deformation, which is caused by the properties of a CAP, is considered for the mathematical model. In addition, the feed-forward velocity compensator for the web tension control system was suggested, which was based on the proposed mathematical model with thermal and gravity effects. In order to evaluate the validity of the proposed mathematical model for a vertical WTS and the proposed web tension control system in a CAP, computer simulations that considered thermal and gravity effects were executed and an experiment was implemented.

A new mathematical model of a web transport system (WTS) in a continuous annealing process (CAP) is proposed. In general, temperature and gravity effects influence the web of a vertical WTS in a CAP. However, these effects are not systematically considered in the conventional mathematical model of a WTS. Disregard for these effects causes a low quality of webs in the CAP. Therefore, in order to improve the quality of webs, a precise web tension control is required based on a mathematical model with thermal and gravity effects. Thus, the mathematical model with thermal and gravity effects in the CAP was established in this paper. In addition, the feed-forward velocity compensator for the web tension control system was suggested, which was based on the proposed mathematical model with thermal and gravity effects. In order to evaluate the validity of the proposed mathematical model for a vertical WTS and the proposed web tension control system in a CAP, computer simulations that considered thermal and gravity effects were executed and an experiment was implemented.

KEY WORDS: web transport system; continuous annealing process; feed-forward tension control; thermal effect; gravity effect.

Modeling and Feed-forward Velocity Compensation of Multi-span Web Transport Systems with Thermal and Gravity Effects

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tems with and without feed-forward velocity compensation based on the proposed mathematical model. From the results of the experiment and computer simulation for the web tension control systems, the validity of the proposed mathematical model of a WTS in a CAP is proven.

In Sec. 2, the problems of a WTS in a CAP caused by thermal and gravity effects are introduced, and a new mathematical web model of a WTS in a CAP is systematically derived. In Sec. 3, the design procedure of a feed-forward compensator, which is derived from the proposed model, is introduced and the velocity compensation logic of the system is explained. In Sec. 4, by comparing the computer simulation with the experiment, the validity of the proposed model is verified and the web tension control systems for a vertical WTS, with and without the feed-forward velocity compensator, are evaluated by computer simulation. The conclusion of the study is described in Sec. 5.

2. Mathematical Model of WTS with Thermal and Gravity Effects

The CAP consists of the pre-heating section (PHS), the heating section (HS), the soaked section (SS), the cooling section (CS), the over-aging section (OAS) and the final cooling section (FCS). The web in the PHS is preheated from 30 to 280°C by using the wasted gas of the HS. The non-oxidizing direct heating method or the indirect heating method using the radiant tube is employed in the HS and the web is heated from 700 to 900°C. The temperature of the web in the SS is regulated to the annealing temperature and the temperature in the CS is cooled from 520 to 500°C by using the ambient gas. The temperature of the web in the OAS is kept at the cooled state during the pre-set time. In the FCS, the temperature of the web reduces to the normal temperature or the proper temperature, which is between 460 and 480°C for the galvanizing processes.

The WTS in a CAP has a number of problems for the web tension control, because the wide range variation of temperature changes the property of steel as is shown in Young's modulus. In addition, the processes are composed by a vertical WTS in order to reduce the equipment area, which causes the gravity effect due to the weight of the web.

Figures 1 and 2 show the temperature distribution of the web on spans at the PHS and HS parts in a CAP, and the variations of Young's modulus and the thermal coefficient according to the temperature variation, respectively. The wide range in the variation of temperature causes the change in Young's modulus and in the thermal coefficient of the web. Figure 3 shows the deflection of a vertical WTS caused by the weight of the web itself. Thus, in this section, the web model with temperature and gravity effects will be systematically derived.

2.1. Basic Model of the 1-Span Web in a WTS

Figures 4 and 5 show the schematic diagram of the 1-span web model in a WTS and an infinitesimal mass element for the web, respectively. Temperature and gravity effects are not considered in the control volume as shown in Fig. 4. The continuity of the mass for the control volume can be represented as

\[
\frac{d}{dt} \int_{0}^{L} \rho(x,t) A(x,t) dx = \rho_1(t) A_1(t) v_1(t) - \rho_2(t) A_2(t) v_2(t) \quad \cdots \cdots \quad (1)
\]

Fig. 1. Temperature distribution of the web on spans in a CAP.

Fig. 2. Variations of Young's modulus and thermal coefficient according to the temperature variation.

Fig. 3. The deflection of a vertical WTS.

Fig. 4. Schematic diagram of the 1-span web model.

Fig. 5. Infinitesimal mass element for the web.
where the subscripts 1 and 2 denote the inlet and outlet of the control volume, respectively.

In order to derive the dynamic equations of the 1-span web model in a WTS, the following assumptions are considered\textsuperscript{1) }:

(1) The web has a perfectly elastic behavior.
(2) There is no slip between the web and the roller.
(3) The web has a constant tension at the same span.
(4) The width and thickness of the web span are constant at all the points in the process.
(5) There are no changes in either the density, Young’s modulus or the cross sectional area of the web span.
(6) The strain of the web span in the longitudinal direction is minimal.
(7) The strain is uniform within the web span.

Considering the infinitesimal web element as shown in Fig. 5, the stretched and outstretched states of the web are represented as the following equations:

\[ dx = (1 + \varepsilon_x) dx_u, \quad w = (1 + \varepsilon_z) w_u, \]
\[ h = (1 + \varepsilon_y) h_u, \quad dm = \rho w dx = \rho_w \varepsilon_z dx_u \]

From assumption (4), \( \varepsilon_x = \varepsilon_z = 0 \). Thus, from Eq. (2), the equation for \( \rho, A \), and \( \varepsilon \) can be obtained as:

\[ \rho A = \frac{1}{1 + \varepsilon} \rho_w A_w \]

where \( \varepsilon = \varepsilon_x \).

From Eqs. (1) and (3), the following equation can be obtained:

\[ \frac{d}{dt} \int_0^t \frac{1}{1 + \varepsilon_x(x,t)} dx = \frac{1}{1 + \varepsilon_x(t)} v_1(t) - \frac{1}{1 + \varepsilon_z(t)} v_2(t) \]

From Eq. (4) and the assumptions listed above, the relationship between the strain and the velocity of the web can be obtained as follows:

\[ L_2 \frac{d}{dt} \varepsilon_{eq,2}(t) = (\varepsilon_x(t) - 1)v_1(t) - (\varepsilon_z(t) - 1)v_2(t) \]

where \( \varepsilon_{eq} \) is the average strain in the control volume.

If assumption (7) is applied to Eq. (5), then Eq. (5) can be expressed as follows:

\[ L_2 \frac{d}{dt} \varepsilon_{eq,2}(t) = -\varepsilon_{eq,2}(t)v_2(t) + \varepsilon_x(t)v_1(t) + v_2(t) - v_1(t) \]

Applying Hooke’s law to Eq. (6), and to assumptions (4) and (5), the relationship between the web tension and the web velocity can be expressed as

\[ L_2 \frac{d}{dt} T_2(t) = -T_2(t)v_2(t) + T_1(t)v_1(t) + AE(v_2(t) - v_1(t)) \]

From Eq. (7), it can be found that the tension in the web span is created by the velocity difference between the web spans and the tension depends on the incoming web tension. Equation (7) can be usefully applied for the WTS in the steady-state. However, Eq. (7) is not suitable for the WTS in a CAP in which the effects of temperature and gravity of the web should be considered.

2.2. Model of a 1-Span Web with Thermal and Gravity Effects

When thermal and gravity effects on the web span are reflected, either Eq. (6) or (7) should be applied in order to develop the web model. For the web model with thermal and gravity effects, the following additional assumptions should be considered\textsuperscript{1) }:

(1) The temperature distribution of the web span is in the steady state.
(2) The internal temperature of the web span varies with the web length, but the minute variation of temperature at the steady state can be disregarded.
(3) The strain within a web span is represented as the summation of elastic, thermal and gravity strains.
(4) The web tension model that considers the change in temperature is derived by using the equivalent strain and the equivalent Young’s modulus, as the web tension model for constant temperature.
(5) The variation of the web length due to the web gravity in a vertical WTS is defined by the equivalent strain and the equivalent Young’s modulus of the web.

The total strain in the infinitesimal displacement \( dx \) can be defined as

\[ \varepsilon' = \varepsilon(x, t) + \varepsilon_t(\theta(x) - \theta) + \varepsilon_n(x) \]

From Eq. (8), the equivalent strain for a 1-span web can be described from assumptions (1) and (3) as

\[ \varepsilon_{eq,2} = \varepsilon_{eq,2}(t) + \varepsilon_n(t) + \varepsilon_{eq,2} \]

where

\[ \varepsilon_{eq,2} = \frac{1}{L_2} \int_0^t \varepsilon_2(x, t) dx, \quad \varepsilon_{eq,2} = \frac{1}{L_2} \int_0^t \varepsilon_2(\Delta \theta(x)) dx \]

By substituting Eq. (9) into Eq. (6), the following equation is obtained

\[ L_2 \frac{d}{dt} \varepsilon_{eq,2'}(t) = -\varepsilon_{eq,2}(t)v_2(t) + \varepsilon_1(t)v_1(t) + [v_2(t) - v_1(t)] \]

The total equivalent strain can be obtained by the equivalent Young’s modulus, which is given by the equivalent strain equation and Hooke’s law as follows\textsuperscript{5) }:

\[ \varepsilon_{eq,2} = \frac{1}{L_2} \int_0^t \varepsilon_2(x, t) dx \]

From Eq. (11), the equivalent Young’s modulus for a 1-span web can be defined as
Equation (12) cannot be applied to obtain the equivalent elastic strain in Eq. (9) because it does not reflect the thermal effect.

The Young’s modulus reflecting the thermal effect will now be considered. The Young’s modulus of the web span according to the variation of temperature has a nonlinear characteristic in the whole operating range, as shown in Fig. 2. However, it can be assumed that, for a small operating range, the relationship between Young’s modulus and temperature has a linear characteristic. The Young’s modulus in the individual span can then be represented as

\[ E = \frac{E_2(\theta) - E_1(\theta)}{L_2} + E_1(\theta) \ldots (13) \]

From Eqs. (12) and (13), the equivalent elastic Young’s modulus according to the temperature variation can be obtained as

\[ E_{eq}^e = \frac{L_2}{\int_0^{L_2} 1/E(x) \ dx} = \frac{E_2 - E_1}{\ln(E_2) - \ln(E_1)} \ldots (14) \]

Therefore, by using Eqs. (11) and (14) the equivalent elastic strain considering the temperature variation can be expressed as

\[ \varepsilon_{eq}^e = \frac{T_s(t)}{AE_{eq}^2} \ldots (15) \]

The web model with the thermal effect is now considered, which is shown in Fig. 6. The strain variation of the length through the thermal effect can be represented as

\[ \varepsilon_{th} = \alpha(\theta - \theta_1) \ldots (16) \]

With the assumption that \( \alpha \) and \( \theta \) has linearity in the individual span section, \( \alpha \) and \( \theta \) can be represented, respectively, as

\[ \alpha = \frac{\alpha_2 - \alpha_1}{L} \cdot x + \alpha_1 \ldots (17) \]

\[ \theta = \frac{\theta_2 - \theta_1}{L} \cdot x + \theta_1 \ldots (18) \]

Using Eqs. (16) through to (18), the thermal strain \( \varepsilon_{th} \) is obtained as

\[ \varepsilon_{th} = \frac{(\alpha_2 - \alpha_1)(\theta_2 - \theta_1)}{L^2} x^2 + \frac{\theta_2 - \theta_1}{L} x \ldots (19) \]

Also, from Eq. (19), the equivalent thermal strain \( \varepsilon_{eq}^th \) can be obtained as

\[ \varepsilon_{eq}^th = \frac{1}{L_2} \int_0^{L_2} \varepsilon_{th}^e \ dx = \frac{1}{3} (\alpha_2 - \alpha_1)(\theta_2 - \theta_1) + \frac{1}{2} (\theta_2 - \theta_1) \ldots (20) \]

Finally, we consider the web model with the gravity effect, which is shown in Fig. 7. The imposed weight for the element length is represented as

\[ W_s = \rho g Ax = \sigma \bar{A} = \frac{W_{eq}}{E_{eq}^2 A} \ldots (21) \]

Thus, the strain by the gravity effect is obtained from Eq. (21) as follows:

\[ \varepsilon_{eq}^w = \frac{W_s}{E_{eq}^2 A} = \frac{\rho g x}{E_{eq}^2} \ldots (22) \]

The equivalent strain by the gravity in a span can be expressed as

\[ \varepsilon_{eq}^w = \frac{1}{L_2} \int_0^{L_2} \varepsilon_{eq}^w(x) \ dx = \frac{1}{L_2} \int_0^{L_2} \rho g x \ dx \ldots (23) \]

Substituting the general web tension model Eqs. (15), (20) and (23) into Eq. (10), the web tension model reflecting thermal and gravity effects can be obtained. By multiplying \( AE_{eq}^2/L_2 \) with Eq. (10), the relationship between the tension and the velocity of the web tension model considering the total strain, which consists of elastic, thermal and gravity strains, can be expressed as

\[ \frac{d}{dt} T_0 = \frac{E_{eq}^2 \bar{V}_2}{E_{eq}^2} + AE_{eq}^2 (1 - \varepsilon_{eq}^th - \varepsilon_{eq}^w) \bar{V}_2 \]

\[ + AE_{eq}^2 \left\{ \frac{T_0}{AE} - 1 \right\} \bar{V}_1 \ldots (24) \]
The general dynamics of the roller of a vertical type 1-span web transport system, which is shown in Fig. 8, can be expressed as

\[
\frac{J_N}{R_N} \frac{dv_N}{dt} = -\frac{b_N}{R_N} v_N + R_N (T_N - T_{N-1}) + K_{mot} U_N
\]

(26)

where \((T_N - T_{N-1})\) indicates the load tension for the roll motor.

3. Design of the Feed-forward Velocity Compensator

When the unexpected variation of the web strain due to thermal and gravity effects in a CAP exists on the web, the web tension control in a WTS becomes difficult. Figure 9 shows part of the control structure of a real system that has a mixed structure of the open loop and closed loop controls. This control structure can be distinguished by the two groups based on tension meters 1 and 2. The information of the two tension meters which are attached to the 6th and 22nd spans are applied to the twenty-two velocity controllers (ASR1–ASR6 and ASR7–ASR22) by the velocity information sent to all the rollers through the two tension controllers (ATR1 and ATR2). The ATR1 offers its output information from the ASR1 to the ASR6 and the ATR2 ofers its output information from the ASR7 to the ASR22. In the 1st group, the information of tension meter 1 only directly affects the ASR6 through the ATR1 and the others, with the exception of the ASR6, are indirectly affected. The 2nd group works in the same manner as the 1st group. Therefore, only the 6th and 22nd rollers can effectively control the tension of the web. From these reasons, the tension control loops with a tension meter can be explained by the closed loop structures and the other tension loops without a tension meter can be explained by the open loop structure. Thus, it is very difficult to achieve the web tension control for the WTS with the limited number of tension sensors. In order to construct a precise web tension control system by using only a software algorithm without modifying the hardware structure of the WTS, it is desirable to use an independent feed-forward velocity compensation for each velocity loop as well as feedback control schemes.

Figure 10 shows the simplified block diagram of the PI and web tension control system with feed-forward velocity compensation in a CAP. This is a cascade control system, which consists of an automatic speed regulator (ASR) of the inner loop for the web speed control and an automatic tension regulator (ATR) of the outer loop for the web tension control. This control system can improve the control system performance compared with the single-loop control. To linearize the nonlinear Eq. (7), it is assumed that the state variables in Eq. (7) have small perturbations from the steady-state operating value, namely, \(\delta v_N = v_N - v_{N,0}\) for the web strain and \(\delta v_N = v_N - v_{N,0}\) for the web velocity, where the subscript ‘0’ denotes the steady-state operating value. The relationship between the linearized strain and velocity of the web is then given by:

\[
L_2 \frac{d}{dt} \delta v_N(t) = -v_{2,0} \delta \varepsilon_1(t) + v_{1,0} \delta \varepsilon_2(t) + \delta v_1(t) - \delta v_2(t)
\]

(27)

Applying Hooke’s law as \(T_1 = AE_1 \delta \varepsilon_1\) and \(T_2 = AE_2 \delta \varepsilon_2\) to Eq. (27), where \(E_1\) and \(E_2\) have the relationship of \(E = E_1 = E_2\) because their temperatures do not vary, the relationship between the linearized tension and velocity of the web can be expressed as
where \( v_{1,0} \) and \( v_{2,0} \) indicate the constant inlet and outlet velocities, respectively, of the web at the nominal operating condition.

After linearizing the nonlinear Eq. (7), the bandwidths of the inner and outer loops of the web tension control system are selected at approximately 20 rad/s and 3 rad/s, respectively. The feedback control laws for the ASR and the ATR are the proportional-integral controls (PIC) as follows:

\[
U_{\text{ASR}} = K_p v_v + K_i \int e_v \, dt \quad \text{.................(29)}
\]

where \( e_v = v_{\text{ref}} - v_N \),

\[
U_{\text{ATR}} = K_p v_r + K_i \int e_r \, dt \quad \text{.................(30)}
\]

where \( e_r = T_{\text{ref}} - T_N \).

The strain of the web in a CAP is influenced by thermal and gravity effects, as previously stated in Sec. 2. Thus, the tension of the web in a CAP becomes an undesirable state. In this situation, it is impossible to achieve the desired web tension state by using PI feedback controllers that are only given by Eqs. (29) and (30). In order to more effectively prevent the variation of the undesirable web tension, the feed-forward velocity compensator should be designed. In the case where there is a temperature variation of the web in a CAP, in order to effectively reject the disturbance due to the variation of temperature, it is important to apply the PI for the ATR and ASR tension feedback controls and feed-forward velocity compensators. The feed-forward velocity compensators can be introduced, which are derived from the proposed model of Eq. (25). To design the feed-forward velocity compensator, it is assumed that the web tension value in Eq. (25) is at a steady state. Then,

\[
\frac{1}{T_N} \left[ \frac{E_{\text{eq},N} v_N}{E_{\text{eq},N}} T_N + AE_{\text{eq},N} (1 - \varepsilon_{\text{eq},N}^{th} - \varepsilon_{\text{eq},N}^{w} ) v_N \right. + \left. AE_{\text{eq},N} ( \frac{T_{N-1}}{AE_{N-1}} - 1 ) v_{N-1} \right] = 0 \quad \text{...............(31)}
\]

or

\[
v_{N-1} = \frac{\frac{T_N}{AE_{\text{eq},N}^{w}} - 1 + \varepsilon_{\text{eq},N}^{th} + \varepsilon_{\text{eq},N}^{w}}{\frac{T_{N-1}}{AE_{N-1}} - 1} v_N \quad \text{...............(32)}
\]

Equation (32) is related to the velocity of the 23rd roller, as shown in Fig. 9, which is called the ‘speed master roller’. The role of the speed master roller provides the reference velocity to the 22nd roller. For instance, if there is a velocity difference between the 22nd and the 23rd rollers, then the variation of tension appears in the 22nd span (between the 22nd and the 23th rollers). Specifically, if the web tension is varied by temperature and gravity effects at the 22nd span, the 22nd roller needs to compensate for the velocity difference between the 22nd and the 23rd rollers. These processes continuously progress from the 21st span to the 1st span. Thus, the feed-forward velocity compensation value can be obtained by the velocity difference between the rollers as follows:

\[
v_{\text{feedforward}} = v_{N-1} - v_{\text{ref}}
\]

\[
\frac{T_N}{AE_{\text{eq},N}^{w}} - 1 + \varepsilon_{\text{eq},N}^{th} + \varepsilon_{\text{eq},N}^{w}
\]

\[
\frac{T_{N-1}}{AE_{N-1}} - 1
\]

\[
v_N - v_{\text{ref}}
\]

\[
\text{...............(33)}
\]

4. Experiment and Simulation of Web Tension Control Systems

In order to prove the validity of the proposed model, an experiment was implemented for the total 22 spans with the temperature change from 30 to 790°C, as shown in Fig. 1. A real WTS for a continuous annealing of steel strip was used for this experiment. The variation of the web tension due to the temperature variation necessarily exists in this system. To prevent the excessive variation of the web tension, the drooping loop was used. This loop connects the input and the output of the ASR part with the drooping gain, as shown in Fig. 10. Also, the tension signals are measured only at the 6th and 22nd spans among the total 22 spans and the current signals are measured with angular velocity meters at all the rollers. This data acquired through the experiment is compared with the results of the computer simulation. The parameters used for computer simulation are shown in Table 1.

Figures 11(a) and 11(b) show the motor currents of spans obtained by experiment and simulation at the 1st, 3rd

<table>
<thead>
<tr>
<th>Table 1. System parameters of the WTS.</th>
</tr>
</thead>
<tbody>
<tr>
<td>System parameter</td>
</tr>
<tr>
<td>Cross-sectional area (m²)</td>
</tr>
<tr>
<td>Drooping gain</td>
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<tr>
<td>Equivalent moment of inertia of the roller (kg·m²)</td>
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<tr>
<td>Length of the web span (m)</td>
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<tr>
<td>Radius of the roller (m)</td>
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<tr>
<td>Viscous friction coefficient of the 1st and 2nd groups (Nm·sec/рад)</td>
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<tr>
<td>Torque constant of the motor (Nm/A)</td>
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<td>Density (kg/m³)</td>
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and 5th spans in the 1st group and at the 7th, 12th, 17th and 22nd spans in the 2nd group, respectively. In Figs. 11(a) and 11(b), the ripple of currents is caused by unknown disturbances such as the vibration of the web and the deformation of the roll due to a high temperature variation. These disturbances are not considered in the computer simulation. Figure 12 shows the mean value of currents excited at each roll motor obtained by experiment and simulation. From Figs. 12(a) and 12(b), the current value excited at each roller is repeated up and down, which is necessary to support the weight of the web in a vertical WTS. Figure 13 shows the web tension at the 6th and 22nd spans where the tension meters are equipped. From Figs. 11, 12 and 13, the results of the proposed mathematical model are closely matched to the results of the experiment. Because the responses of the proposed mathematical model replicates those of the real system, the proposed model can be applied to evaluate the performance of the real system with the feed-forward velocity compensator through computer simulation.

A computer simulation based on the verified mathematical model is executed by using the MATLAB software. Two mathematical models according to the temperature of the web are considered, which are the constant temperature model (CTM) and the varying temperature model (VTM). The temperature of the CTM was fixed at 30°C in all the spans, and the temperature of the VTM varied from 30 to 790°C, as shown in Figs. 1 and 2. In addition, the variation of Young’s modulus and the thermal coefficient due to the temperature change, as shown in Fig. 2, is the measured values in a real plant. Therefore, in order to reflect these...
values to the feed-forward velocity compensator, the values in Fig. 2 are used as a lookup table for computer simulation. The value of the desirable web tension for all the models was 1.329 kgf. Also, their desirable web velocities were applied at 1.5 m/s, as shown in Fig. 2. The numbers of the ATR and the ASR were 2 and 22, respectively, as shown in Fig. 9. Control gains were then selected that would be suitable for those bandwidths as mentioned in Sec. 3. The system parameters of the WTS, which are shown in Table 1, are applied to the same values in the 22-spans, with the exception of Young’s modulus, the thermal coefficient due to the temperature variation, and the viscous friction coefficient due to the mechanical property.

Figures 15(a), 15(b), 15(c) and 15(d) show the results of tension errors of the web tension control systems. Figure 15(a) shows that the tension error is regulated to around 0 under the isothermal condition (30°C) for the CTM. Figure 15(b) shows that the tension error is not regulated and that it diverges after 20 s for the VTM. Figure 15(c) shows that the tension error is satisfactorily regulated by using only the drooping loop without the feed-forward velocity compensation. The difference between Figs. 15(b) and 15(c) is in the existence of the drooping loop for the temperature change. The temperature change on the web causes a current variation of roll motors due to the changes in Young’s modulus and the thermal coefficient. The variation of tension on the web then occurs. Therefore, to prevent the divergence of the tension due to the temperature variation on the web, as shown in Fig. 15(b), the variation of the current should be limited by the drooping loop, the result of which can be shown in Fig. 15(c). This method has been used for industrial plants that are sensitively affected by temperature variation, and is used for preventing the divergence of motor current for each span.

On the other hand, if the feed-forward velocity compensation in Eq. (33), and the drooping loop are applied to the VTM, the tension error occurs in the allowable range, as shown in Fig. 15(d). From this result, it is found that the effects of the temperature and the gravity are reflected in the WTS model and they can be effectively rejected by the feed-forward velocity compensation. Figure 16 shows the feed-forward velocity compensation values for the VTM with the drooping loop at the 7th, 12th, 17th and 22nd spans in the 2nd group. The increase of temperature on the web, as shown in Fig. 1, causes a decrease in the feed-forward velocity compensation value for each span. Thus, the velocities of the 17th, 12th, and 7th spans are reduced in order of precedence from the velocity of the 22nd span.
Particularly, the velocity compensation value at the 22nd span is 0, because the temperature at the 22nd and 23rd spans is isothermal, as shown in Fig. 1.

In Figs. 15(a) and 15(c), it is found that there are minute offset values in the 2nd group spans. These offset values can be explained by the number of tension meters used to feedback the tension state (roller numbers 6 and 22) as shown in Fig. 9, and in the change of Young’s modulus and the gravity effect in Eq. (25). The 1st tension meter is used to execute the tension feedback control of the 1st group spans (roller numbers 1 through 6) and the 2nd tension meter is used to execute the tension feedback control of the 2nd group spans (roller numbers 7 through 22). In the section of the 2nd group, the other rollers (7th through 21st rollers), with the exception of the 22nd roller in the 2nd group, are dependently controlled from the 22nd roller, which has the tension meter and is directly controlled for the web tension. Thus, the web tension control result of the 22nd roller is more accurate than the others. This is the first reason for the minute offset value. The temperature in the VTM causes the changes in Young’s modulus and the thermal coefficient. These changes mean that the characteristics of the systems have changed. This is the second reason for the minute offset value. From the results of Fig. 15, it is found that the VTM with the drooping loop and feed-forward velocity compensation has an improved performance.

**Figure 17** shows the responses of the tension error and the web velocity for the 7th span of the VTM.

When the reference velocity input in Fig. 14 is given to the inner loop, the web velocity output is converged to 1.5 m/s after having a small overshoot due to the characteristic of the roll motor. A variation in the tension then occurs in the region of the transient velocity. From this result, it can be found that there is a coupling of the web velocity and tension.

**Figure 18** shows the tension errors at the 12th, 17th and 22nd spans in the 2nd group of the VTM for the perturbation of Young’s modulus.

When the reference velocity input in Fig. 14 is given to the inner loop, the web velocity output is converged to 1.5 m/s after having a small overshoot due to the characteristic of the roll motor. A variation in the tension then occurs in the region of the transient velocity. From this result, it can be found that there is a coupling of the web velocity and tension.

**Figure 18** shows the tension errors at the 12th, 17th and 22nd spans in the 2nd group of the VTM with the drooping loop and feed-forward compensation for the parameter perturbation of Young’s modulus of the web. For the 12th and 17th spans without the feedback structure or tension feedback signal, the trend in the tension errors are similar, as shown in Figs. 18(a) and 18(b), although there are a number of perturbations of the Young’s modulus. For the 22nd span with the feedback structure, the tension errors are satisfactorily regulated, as shown in Fig. 18(c), even though there
are a number of perturbations of the Young’s modulus on the VTM of the WTS. From these results, it is found that the VTM of the WTS with the drooping loop and feed-forward velocity compensation can satisfactorily regulate the web tension.

However, in order to apply the feed-forward control strategy to real systems, it is very important to accurately model the real plant. If some perturbation exists in the partial tension feedback structure, the PI feed-forward control strategy may be not suitable. This is confirmed by computer simulation results. Therefore, in order to overcome the shortcoming of the PI feedback and feed-forward control strategies in this study, adaptive-robust control schemes need to be applied to the WTS with the partial feedback structure or it should be changed to the full feedback structure with a tension observer. This provides a sufficient framework for future studies.

### 5. Conclusion

This paper presents a new mathematical model of a vertical WTS with thermal and gravity effects in a CAP. This model has a number of problems such as the variation of Young’s modulus due to temperature change and gravity effects. The proposed WTS model is expressed by the equivalent Young’s modulus, the equivalent thermal strain and the equivalent gravity strain. In addition, the total strain in the single span is composed of a linear combination of the elastic, thermal and gravity strains and it is extended to the multi-span WTS in a CAP. In order to prove the validity of the proposed mathematical model, an experiment is implemented. Furthermore, a feed-forward velocity compensator based on the proposed VTM is designed in order to improve the performance of web tension control systems that have a structural shortcoming according to the limitation of the number of tension sensors and for verifying the effectiveness of the proposed VTM. The computer simulations are executed for a nonlinear 22-span WTS with and without the drooping loop and feed-forward velocity compensation. It was found that the feed-forward web tension control system based on the proposed VTM in a CAP has an improved performance.

### Nomenclature

- \( b_{1g}, b_{2g} \): Viscous friction coefficient (N m s/rad) of the 1st and 2nd groups
- \( dm \): Mass of an infinitesimal web element (kg)
- \( e_T \): Web tension error (N m)
- \( e_v \): Web velocity error (m/s)
- \( g \): Gravity acceleration (m/s²)
- \( k \): Coulomb friction torque (N m)
- \( v \): Velocity of the web (m/s)
- \( x \): Displacement of the web (m)
- \( A \): Cross-sectional area of the web span (m²)
- \( D_g \): Drooping gain
- \( E \): Young’s modulus of the web (N/m²)
- \( J \): Equivalent moment of inertia of the roller (kg m²)
- \( K_i \): Integral control gain
- \( K_{mot} \): Torque constant of the motor (N m/A)
- \( K_p \): Proportional control gain
- \( L \): Length of the web span (m)
- \( R \): Radius of the roller (m)
- \( T \): Tension of the web span (N)
- \( U \): Control input (A)
- \( \alpha \): Thermal coefficient of the web span (°C⁻¹)
- \( \varepsilon \): Web strain
- \( \rho \): Density (kg/m³)
- \( \sigma \): Stress (N/m²)
- \( \theta \): Temperature of the web (°C)

### Superscript

- \( e \): Elastic
- \( t \): Total
- \( th \): Thermal
- \( w \): Gravity

### Subscript

- \( eq \): Equivalent
- \( o \): Steady state operating value
- \( ref \): Reference input
- \( u \): Unstretched state of the web

\[ 1, 2, \ldots, N \]: Number of the web span

### REFERENCES