Outflow Characteristics of a Pressure Medium during Sheet Hydroforming

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The outflow characteristics of a pressure medium during a sheet hydroforming process have been studied experimentally by directly measuring the hydraulic pressure distribution. Initial measurements of the outflow of the pressure medium through the gap between two clamped dies were carried out in order to verify the method for measuring the hydraulic pressure distribution employed in this study and to investigate the basic properties of the outflow. The upward forces were calculated by integrating the measured hydraulic pressure during the outflow and found to be in satisfactory balance with the clamping forces, which demonstrates the validity of this measurement method. Furthermore, we suggest that the critical outflow pressure can be predicted by considering the force equilibrium. An experimental investigation of a square-cup sheet-hydroforming process was then carried out. The upward forces were calculated by integrating the measured hydraulic pressure and again found to be in satisfactory balance with the forming forces at the beginning of the process, which shows that the critical outflow pressure can also be predicted for square-cup deep-drawing processes. The hydraulic pressure distributions in the flange area and the chamber change significantly as the sheet conforms to the die shoulder, and the friction loss of hydraulic pressure at the die shoulder becomes large. These results indicate that the magnitude of the fluid-lubrication effect on the drawability of the sheet can vary with the stage of sheet deformation.

KEY WORDS: press forming; sheet hydroforming; deep drawing; outflow of pressure medium; pressure distribution.

1. Introduction

Sheet hydroforming (SHF) is a deep-drawing process in which hydraulic pressure is employed instead of a female die. Because of the high hydraulic counter pressure, the sheet can not only be firmly formed to the shape of the punch, but three main effects that improve the drawability also arise, *i.e.*, the friction-increasing effect, the fluid-lubrication effect, and the reverse-bending effect.^{1,2)} The friction force between the sheet and the punch increases because of the hydraulic pressure, preventing both large elongations and eventual fracture at the punch shoulder. When the pressure medium in the chamber is pressurized to a certain magnitude, the pressure medium often pushes up the blank holder and flows out through the gap between the die and the blank holder. The outflow of the pressure medium results in the fluid-lubrication effect between the die and the blank holder (*i.e.*, in the flange area), thus allowing a large draw-in. In addition, the sheet is sometimes subjected to reverse bending at the die shoulder due to the hydraulic pressure. Reverse bending prevents the sheet from contacting the die shoulders, which enables smooth draw-in and work hardening. SHF has many advantages because of these effects, such as a reduction in tooling costs, improvements in dimensional accuracy, and increases in the drawing ratio.

Applications of SHF are found in the automobile, aerospace, and kitchen industries.^{2,3)}

The forming conditions, including the loading paths of the hydraulic pressure and the blank holding force, need to be appropriately determined and precisely controlled if the above-mentioned effects are to be achieved. It is, however, much more difficult to determine the appropriate forming conditions for SHF than for conventional press forming because the number of parameters of SHF is larger. To overcome this difficulty, SHF has been extensively studied both experimentally and theoretically.

A brief survey of the literature on SHF is now presented. Crucial basic research into SHF was actively carried out by Japanese researchers from the 1960s to the 1980s. Kasuga *et al.*^{4–8)} studied a cylindrical-cup hydroforming process both experimentally and theoretically, and analyzed the deformation process in detail. They also proposed analytical models for estimating the variation of the hydraulic pressure in the chamber and the critical outflow pressure. Nakamura *et al.*^{1,2,9)} studied the fracture mechanisms of sheet hydroforming processes in detail. They classified such fractures into five types and described the causes of, and solutions to, each type. Moreover, they also demonstrated the three main effects described above that improve drawability. Since then, many research groups have carried out various

studies of hydroforming processes (e.g. Refs. 10–16)), including the hydroforming of sheet metal pairs,¹¹⁾ viscous pressure forming,¹³⁾ and warm hydroforming.¹⁶⁾ With the rapid increase in computation power, numerical simulations have become easier to implement so various studies using finite-element methods (FEMs) have recently been reported.^{10,13,17–20)} The authors are developing a new static FEM program for modeling SHF.²¹⁾ The simulation of an elliptical-cup deep-drawing process in which there is no outflow of the pressure medium has been carried out using this program, and it was shown that the results of the simulation are in good agreement with the results of experiments.

However, problems remain for finite-element simulations of SHF. One problem is that it is difficult to account for the outflow of the pressure medium during sheet deformation²¹⁾ because outflow characteristics, such as the critical outflow pressure and the variation of the hydraulic pressure distribution, are not well-known. Since the fluid-lubrication effect due to the outflow of the pressure medium plays an important role in drawability as described above, the outflow characteristics of the pressure medium need to be understood and modeled in order to carry out accurate numerical simulations. Although several analytical models of outflow characteristics have been proposed,^{7,17,18)} most are limited to specific shapes, and adequate experimental testing of these models has not yet been performed. Further, the hydraulic pressure distributions in the flange area during and after outflow have only rarely been examined.

This paper describes our measurements of the outflow characteristics of a pressure medium in a square-cup SHF process. The hydraulic pressure distributions in the flange area and the chamber were measured directly during the process, and their variations were studied in detail in order to determine the fundamental outflow characteristics. We propose numerical models for the critical outflow pressure and the hydraulic pressure distribution in the flange area that can be used to take outflow characteristics into account in finite-element simulations of sheet hydroforming processes.

2. Basic Characteristics of the Outflow

2.1. Experimental Conditions

The main focus of this study was to examine the variation of the hydraulic pressure distribution during SHF processes. To verify the method for measuring the hydraulic pressure employed in this study and to examine the basic characteristics of the outflow of the pressure medium, we initially carried out experiments on the outflow of the pressure medium. **Figures 1** and **2** show a schematic diagram of the experimental set-up and the geometry of the lower die respectively. Seven holes with a diameter of 2 mm run through the lower die from the top to bottom surfaces. Pressure sensors (Kyowa Electronic Instruments, PGL-A series) that are used to measure the hydraulic pressure on the die surface are mounted at the bottom ends of the holes. The holes are denoted a, b, c, d, e, f, and g in Fig. 2. Point a is in the chamber, whereas the other six points are on the die surface.

The experimental procedure was as follows. After clamp-

Fig. 1. Schematic diagram of the experimental set-up for the initial measurements.

Fig. 2. Geometry of the lower die for the initial measurements.

ing the dies at the cross head of the testing machine with a prescribed force, an electric pump was used to pressurize the pressure medium in the chamber. When the upward force exerted by the hydraulic pressure exceeded the clamping force, the pressure medium pushed the upper die up and flowed out through the gap between the upper and lower dies. The variation of the hydraulic pressure on the die surface was measured with the pressure sensors throughout the process. A displacement sensor (Keyence, EX-110V) was also mounted on the lower die to measure the variation of the gap between the upper and lower dies. Experimental data were recorded with a data logger approximately every 2 ms. In this experiment, clamping forces of 10, 20, 30, and 40 kN were used. A mineral hydraulic oil with a kinetic viscosity of 32 cSt at 40°C was used as the pressure medium.

2.2. Results and Discussion

2.2.1. Variation of the Hydraulic Pressure

Figure 3 shows the variations of the hydraulic pressure and the gap between the upper and lower dies; since the trends in these variations are independent of the clamping force, only the results for a clamping force of 20 kN are shown. Similarly, only the hydraulic pressures at points a, b, c, and d are shown in Fig. 3, since these results are the same as those for the points e, f, and g. The hydraulic pressure in the chamber (at point a) increases and reaches a peak im-

Fig. 3. Experimental results for a clamping force of 20 kN. (a) Hydraulic pressure variations, (b) variation of the gap between the upper and lower dies.

mediately after the pump is activated, whereas the hydraulic pressures on the die surface (at points b, c, and d) start increasing when the pressure in the chamber reaches this peak. Similarly, the gap starts increasing at this peak. These results show that the upper die is pushed up and the pressure medium starts flowing out through the gap when the pressure in the chamber reaches its peak. After this peak, the hydraulic pressures in the chamber and on the die surface remain constant until the end of the experiment. The region in which the hydraulic pressures remain constant is hereafter called the stability region. The same trend was observed for the gap. Note that the oscillation in the hydraulic pressures during the process is due to the performance of the pump.

Figure 4 shows the hydraulic pressure distributions along the radial direction in the stability region for clamping forces of 10, 20, 30, and 40 kN; since the hydraulic pressures oscillate during the process, as can be seen in Fig. 3(a), mean pressures are shown in this figure. The theoretical pressure distributions $P(r)$ given by Eq. (1) (see Appendix) are also included, for which an axi-symmetric steady flow through a constant gap was assumed:

$$P(r) = P_c (1 - \ln(r/R) / \ln(R_{\text{end}}/R)) \dots \dots \dots (1)$$

where r is the distance from the center of the die and P_c is the hydraulic pressure in the chamber. R and R_{end} are the radii of the chamber and the lower die respectively; $R = 25$ mm and $R_{\text{end}} = 93$ mm. The theoretical curves were modified so that the hydraulic pressures at the distances of ($R =$) 25 mm and ($R_{\text{end}} =$) 93 mm correspond to those in the chamber and the atmospheric pressure respectively. The hydraulic pressure decreases with distance from the chamber. The trend in the distribution is independent of the clamping force, whereas the magnitude of the hydraulic pressure increases with the clamping force. The measured pressure distributions are in good agreement with the theoretical distributions under all conditions.

Fig. 4. Experimental and theoretical distributions in the stability region of the hydraulic pressure along the radial direction.

Fig. 5. Comparison of the calculated upward force and the clamping force.

2.2.2. Critical Outflow Pressure

In the stability region, the upward force due to the hydraulic pressure is balanced by the clamping force. Since the experimental hydraulic pressure distribution along the radial direction is in satisfactory agreement with the theoretical distribution (Fig. 4), the upward force can be approximately calculated by integrating the theoretical pressure distribution with respect to the area. The calculated upward force and the clamping force are shown in **Fig. 5**. Note that the clamping forces in the stability region are slightly higher than the initial prescribed forces because the gap between the upper and lower dies increases during the outflow (Fig. 3(b)). The upward forces are in balance with the clamping forces for all conditions, which confirms the validity of the hydraulic pressure measurements.

The ability to predict the critical outflow pressure in SHF is valuable because the outflow of the pressure medium results in the fluid-lubrication effect in the flange area as described in the introduction. As shown in Fig. 3, the pressure medium starts flowing out through the gap when the pressure in the chamber is at its peak. However, the pressure peak is a transient phenomenon; presumably the peak pressure does not affect the deformation of the sheet. A practical approach is to assume that the pressure medium starts flowing out when the hydraulic pressure reaches the stability region magnitude, and then the hydraulic pressure in the

stability region can be treated as a pseudo-critical outflow pressure. With this assumption, the results in Fig. 5 suggest that the pseudo-critical outflow pressure can be predicted by utilizing the force equilibrium in the stability region.

Note that the upward force at the pressure peak is much weaker than the clamping force for all conditions examined in this study. This inconsistency is probably due to several factors, such as minute shape defects in the tools and the fact that the peak pressure could not be measured precisely owing to the oscillation of the hydraulic pressure. Hence the force equilibrium at the peak pressure needs to be carefully examined in future research that takes these various factors into account.

3. Outflow Characteristics of a Square-cup Deep-drawing Process

3.1. Experimental Conditions

An experimental study of a square-cup SHF process was carried out in which the variation of the hydraulic pressure on the die surface (*i.e.* in the flange area) during the process was studied in detail.

Figures 6 and 7 show a schematic diagram of the experimental set-up and the geometry of the lower die respectively. The hydraulic pressure distribution was measured with the procedure used in the experiment discussed above. Four holes with a diameter of 2 mm were present on the die surface. Figure 7 shows holes denoted a, b, c, d, and e: point a is in the chamber, and the other four points are on the die surface.

A mineral hydraulic oil was used for lubrication. The lubricant oil was removed from the punch to enable the friction-increasing effect to occur.^{1,2)} The blank holding force was supplied by springs with a spring constant of 471 N/mm and attached with screw nuts, as shown in Fig. 6. Blank holding forces of 4.5, 9.0, 12.5, 18.0, and 22.5 kN were applied. The sheet was drawn at a constant punch speed of 0.25 mm/s. In this experiment, the electric pump was activated throughout the process in order to pressurize the pressure medium in the chamber. For safety reasons, however, the pump stopped pressuring when the hydraulic pressure in the chamber exceeded 40 MPa. The volume of flow was not examined in this experiment since it is governed by the pump performance. Further, the gap between the die and the blank holder was not measured because it is strongly affected not only by the volume of the flow but also by changes in the thickness of the sheet.

Mild steel was used as the sheet material; its mechanical properties are shown in Table 1. Each square sheet blank was 100 mm long and 0.7 mm thick.

For comparison, conventional press forming was also carried out, in which hydraulic pressure was not employed.

3.2. Results and Discussion

3.2.1. Deformation Profiles

Figure 8 shows the deformation profiles obtained with SHF and conventional press forming for a blank holding force of 22.5 kN. The critical punch strokes for fracture were found to be 19.5 mm for SHF and 11.7 mm conventional press forming. Fractures were observed in the sidewall in the product obtained with SHF and at the punch

Fig. 6. Schematic diagram of the experimental set-up for the square-cup deep-drawing process.

Fig. 7. Geometry of the lower die for the square-cup deep-drawing process.

Table 1. Mechanical properties of the sheet.*

E / GPa	σ_y / MPa	F / MPa	n	ϵ_0	r
210	150	530	0.24	0.0032	2.1

* E is Young's modulus, σ_y is the yield stress, σ_b is the tensile strength, and r is the Lankford value. The true stress-true plastic strain curve can be approximated with Swift's law $\bar{\sigma} = F(\epsilon_0 + \bar{\epsilon}^n)^m$ in the range of $0.1 \leq \bar{\epsilon}^n \leq 0.2$.

shoulder in the product obtained with conventional press forming. It is well known that (a) a better critical punch stroke is achieved with SHF when appropriate forming conditions are present, and (b) fractures can occur with conventional press forming at a punch shoulder and at a sidewall with SHF if the friction-increasing effect occurs satisfactorily.^{1,2)} These typical features of SHF are clear in the deformation profiles shown in Fig. 8, demonstrating the adequacy of the forming conditions.

3.2.2. Variation of the Hydraulic Pressure

Figure 9 shows the variation of the hydraulic pressure for a blank holding force of 22.5 kN. Note that the trend in the variation is independent of the blank holding force, so only the results for 22.5 kN are shown.

The hydraulic pressure in the chamber (at point a) rapidly increases and reaches a peak immediately after the

(a) SHF
at punch stroke of 19.5 mm

(b) Conventional press forming
at punch stroke of 11.7 mm

Fig. 8. Deformation profiles obtained with a blank holding force of 22.5 kN. (a) SHF at punch stroke of 19.5 mm, (b) conventional press forming at punch stroke of 11.7 mm.

Fig. 9. Variation of the hydraulic pressure for a blank holding force of 22.5 kN. The lower figure shows a magnified image of the punch stroke range 0–7 mm.

process begins, whereas the hydraulic pressures in the flange area (at points b, c, d, and e) start increasing when this peak pressure arises. After the peak is reached, a stability region is observed for both the chamber and the flange area up to a punch stroke of about 3 mm. It is clear that the trends in the variations are very similar to those shown in Fig. 3(a) up to a punch stroke of about 3 mm, *i.e.* the pressure medium starts to flow out at the peak pressure.⁷⁾ This hydraulic pressure variation is obtained because the sheet is not yet completely conformed to the die shoulder, as shown in Fig. 10(a), so the flow of the pressure medium is stable. This conclusion is supported by the fact that the stability region persists up to a punch stroke of about 3 mm, which is in good agreement with the die shoulder radius of 4 mm. Note that the hydraulic pressure in the chamber gradually increases even in the stability region because the volume of

Fig. 10. Schematic diagram of sheet deformation at the die shoulder. (a) Stage I, (b) stage II.

the pressure medium is gradually compressed by the penetration of the punch.

The hydraulic pressure in the chamber then starts to increase rapidly. At the same time, the hydraulic pressures in the flange area suddenly decrease and then remain constant. These variations arise because the sheet has conformed to the die shoulder as shown in Fig. 10(b), and hence the pressure medium is sealed in the chamber.⁷⁾ Note that the pressure medium continues to flow out even after the pressure medium is sealed. It is clear that there is a very large difference between the hydraulic pressures in the chamber and in the flange area during this stage. It is also interesting that the hydraulic pressure at point b starts to decrease slightly earlier than that at point c. This difference presumably arises because the sheet conforms to the straight part of the die shoulder (*i.e.* the OA direction) more easily than to the corner part (*i.e.* the OB direction), which has double curvature.

After the hydraulic pressure in the chamber reaches 40 MPa, it oscillates significantly and becomes unstable once the pump has stopped pressuring. Finally, a fracture occurs at the sidewall and the hydraulic pressure in the chamber decreases rapidly.

3.2.3. Hydraulic Pressure Distribution

The process can be categorized into stages I and II according to the variation of the hydraulic pressure in the chamber, as shown in Fig. 9. We now examine the hydraulic pressure distribution at each stage in detail.

(a) Stage I

Figure 11 shows the hydraulic pressure distributions along the lines OA and OB (see Fig. 7) for a punch stroke of about 0.5 mm, which corresponds to the beginning of the stability region. It is assumed in Fig. 11 that the hydraulic pressure in the chamber (at point a) is constant up to the end of the die shoulder since the sheet has not yet conformed to the die shoulder at this stage (see Fig. 10(a)). Since the outflow from the square-shaped chamber is complicated, the theoretical pressure distribution cannot be described with a simple equation. An alternative linear approximation is shown in this figure. It is convenient for numerical simulations to model the hydraulic pressure distribution in the flange area with a linear approximation. The lines are drawn linearly such that the pressures at the end of the die shoulder (a distance from the center of 25 mm) and at the sheet edge (that of 50 mm) agree with that in the chamber and with the atmospheric pressure respectively.

Fig. 11. Experimental hydraulic pressure distributions in the flange area for a punch stroke of 0.5 mm (stage I). (a) OA direction, (b) OB direction.

Although small discrepancies are observed for the line OB, linear approximations to the hydraulic pressure distributions in the flange area are acceptable.

Using the linearized pressure distributions, the force equilibrium at the beginning of the stability region between the upward force due to the hydraulic pressure and the forming force given by the sum of the blank holding force and the punch force was examined. The upward force was calculated by integrating the linearized pressure distribution with respect to the initial sheet area. The initial sheet area is used since the amount of draw-in at this stage is negligible. The forming force and the calculated upward force are shown in Fig. 12. The upward forces are in balance with the forming forces for all conditions, indicating that the pseudo-critical outflow pressure can also be predicted appropriately from the force equilibrium for square-cup deep-drawing processes even when a linearized pressure distribution is employed. Since the variation of the forming force can be obtained with numerical simulations, the pseudo-critical outflow pressure can also be determined directly with numerical simulations by utilizing the force equilibrium.

(b) Stage II

The hydraulic pressure distributions along the lines OA and OB for a punch stroke of 7.0 mm are shown in Fig. 13. Since the sheet has already conformed to the die shoulder (see Fig. 10(b)) and the pressure medium is sealed as described in Sec. 3.2.2, it is assumed in Fig. 13 that the hydraulic pressure in the chamber (at point a) is constant up to

Fig. 12. Comparison of the forming force and the calculated upward force at the beginning of the stability region.

Fig. 13. Experimental hydraulic pressure distributions in the flange area for a punch stroke of 7.0 mm (stage II). (a) OA direction, (b) OB direction.

the end of the chamber. As mentioned in Sec. 3.2.2, the hydraulic pressures in the flange area are much smaller than that in the chamber. This result arises because the sheet has conformed to the die shoulder, *i.e.* the distance between the sheet and the die shoulder has drastically decreased; hence the friction loss of the hydraulic pressure is significant at the die shoulder. Clearly, the hydraulic pressure distributions in the flange area cannot be approximated as linear for both lines OA and OB, since they change markedly as the sheet deforms in transition from stage I to stage II. This change could lead to a change in the magnitude of the fluid-lubrication effect on the drawability of the sheet. Therefore it is suggested that this phenomenon should also be taken into account in numerical simulations. The amount of fric-

tion loss at the die shoulder needs to be adequately modeled if this phenomenon is to be incorporated into numerical simulations. We plan to investigate this issue in future research.

4. Conclusions

This study focused on the outflow characteristics of a pressure medium in a sheet hydroforming process (SHF). Initial measurements of the outflow of a pressure medium through a gap between two clamped dies were carried out in order to study the basic properties of the outflow. We drew the following conclusions:

(1) Initially, the hydraulic pressure in the chamber rapidly increases and reaches a peak. The hydraulic pressures in the chamber and on the die surface then remain constant.

(2) The pressure medium starts to flow out from the chamber at the peak in the pressure in the chamber. Since this peak in the pressure is a transient phenomenon, we suggest that the subsequent constant pressure can be regarded as a pseudo-critical outflow pressure.

(3) The measured distributions of the hydraulic pressure on the die surface are in good agreement with the theoretical distributions. Moreover, the upward forces calculated by integrating the theoretical hydraulic pressure are in balance with the clamping forces during the outflow, which demonstrates the validity of the hydraulic pressure measurements.

(4) The pseudo-critical outflow pressure can be predicted by utilizing the force equilibrium between the upward and clamping forces.

A square-cup deep-drawing process was then carried out and the variation of the hydraulic pressure during the process was studied. We reached the following conclusions from this experiment.

(5) A similar hydraulic pressure variation to that found in the initial experiments was observed until the sheet is completely conformed to the die shoulder. For this stage, the hydraulic pressure distribution in the flange area can be satisfactorily approximated as linear. The upward forces calculated by integrating the linearized hydraulic pressure are in balance with the forming forces, which indicates that the pseudo-critical outflow pressure can also be predicted for square-cup deep-drawing processes. This estimation of the pseudo-critical outflow pressure can be directly utilized in numerical simulations.

(6) After the sheet has conformed to the die shoulder, the hydraulic pressure in the flange area decreases markedly due to the large friction loss at the die shoulder. This decrease could result in changes in the magnitude of the fluid-lubrication effect on the drawability of the sheet, suggesting that this phenomenon should be taken into account in numerical simulations.

We summarize the future research proposed in this paper as follows. The hydraulic pressure is generally well controlled at a prescribed value in SHF. Thus the variation and distribution of the hydraulic pressure in the flange area under such conditions need to be studied. Since variation in the gap between the die and the blank holder can affect the fluid-lubrication effect, this issue should also be examined in detail, in particular by taking changes in the thickness of

the sheet into consideration. Changes in the hydraulic pressure distribution in the flange area need to be understood in order to incorporate this phenomenon into numerical simulations.

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Appendix

The purpose of the present Appendix is to develop the theoretical hydraulic pressure distribution along the radial direction, Eq. (1). In this study, an axisymmetric flow is assumed. Moreover, to simplify the problem, we assume that; (a) the flow of the pressure medium between the upper and lower dies is steady, (b) the gap between the dies h is constant, (c) the velocity component in the z direction is zero, (d) the volume forces are neglected, and (e) the Reynolds

number $Re \ll 1$. With these assumptions, the Navier–Stokes equations of motion and the equation of continuity can be given in the forms

$$\frac{\partial P}{\partial z} = 0 \quad \text{.....(A-1)}$$

$$-\frac{dP}{dr} + \mu \left(\frac{\partial^2 v}{\partial z^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial r^2} \right) \quad \text{.....(A-2)}$$

$$\frac{\partial v}{\partial r} + \frac{v}{r} = 0 \quad \text{.....(A-3)}$$

where v is the velocity component in the r directions, and μ is the coefficient of viscosity. Differentiating Eq. (A-3) partially with respect to r and substituting into Eq. (A-2) yields

$$\frac{dP}{dr} = \mu \frac{\partial^2 v}{\partial z^2} \quad \text{.....(A-4)}$$

Integrating Eq. (A-4) with respect to z yields

$$v = \frac{1}{2\mu} \frac{dP}{dr} z^2 + C_1 z + C_2 \quad \text{.....(A-5)}$$

where C_1 and C_2 are the integral constants. Considering the

boundary conditions of $v=0$ at $z=0$ and $z=h$, we have

$$v = \frac{1}{2\mu} \frac{dP}{dr} (z^2 - hz) \quad \text{.....(A-6)}$$

The mean velocity v_m can be represented in the form

$$v_m = \frac{1}{h} \int_0^h \frac{1}{2\mu} \frac{dP}{dr} (z^2 - hz) dz = -\frac{h^2}{12\mu} \frac{dP}{dr} \quad \text{....(A-7)}$$

Equation (A-7) is arranged in the form

$$\frac{dP}{dr} = -\frac{6\mu Q}{\pi h^3} \frac{1}{r} \quad \text{.....(A-8)}$$

where $Q=2\pi r h v_m$ is the volume flow rate. Considering the hydraulic pressure $P=P_C$ at $r=R$ and that at $r=R_{\text{end}}$ is atmosphere pressure and is given zero offset, integrating Eq. (A-8) gives

$$P(r) = P_C \left(1 - \frac{\ln(r/R)}{\ln(R_{\text{end}}/R)} \right) \quad \text{.....(A-9)}$$

where P_C is the hydraulic pressure in the chamber, and R and R_{end} are the radii of the chamber and the lower die respectively.