The Theoretical and Experimental Research on the Forward Slip Coefficient in Rail Universal Rolling

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(Received on April 21, 2008; accepted on December 10, 2008)

In rail universal rolling, the forward slip coefficient is one of the most important parameters. For simplifying the analytic model, the vertical roll with box pass has been simplified as an equivalent flat roll firstly. Then the equation of neutral line and the area of forward slip zone on the flank of horizontal roll have been derived. Furthermore, the horizontal resultant force acting on the rolled workpiece has been obtained and the forward slip coefficient of rail universal rolling has been proposed. For verifying the theoretical model, the universal rolling process of 60 kg/m heavy rail and 18 kg/m light rail have been simulated by the Rigid-Plastic FEM software DEFORM-3D V5.0, and the universal rolling experiments of 18 kg/m light rail has been accomplished in the Yanshan University Rolling Laboratory. Moreover, the theoretical results and numerical simulation results of forward slip coefficient are in agreement with the experimental data basically. So, this theoretical model can be applied in rail universal rolling.

KEY WORDS: rail; universal mill; neutral angle; neutral line; backward slip; forward slip; rigid-plastic FEM; simulation; experiments.

1. Introduction

Since 1970s, the universal rolling method has been applied in H-beam rolling widely and the corresponding theoretical research has been carried out,^{1–8)} moreover, the numerical simulation by FEM has been used in universal rolling process.^{9–11)} Then the theory of universal rolling method has been developed and improved. Although the universal rolling method has been also applied in heavy rail rolling for 30 years, there are few theoretical research about the rail rolling by universal mills. At present, the application of universal rolling method becomes more and more prevalent in producing the high-precision rail, and it has been replacing conventional producing methods gradually. For the certain similarity of rail rolling and H-beam rolling, the result of theoretical research about H-beam rolling can be applied as an available reference in rail rolling.

The universal mill comprises four rolls, two horizontal drive rolls and two vertical driven rolls (**Fig. 1**). The horizontal roll and the vertical roll of rolling rail base is flat, but the box pass has been grooved on the vertical roll of rolling rail head.

At present, the following methods have been applied in the derivation of forward slip coefficient of universal rolling generally: (1) force equilibrium method; (2) analogical method; (3) regression method; (4) theoretical-empirical equation method. By the force equilibrium method, many influence factors have been considered in the derivation process of forward slip coefficient. So the forward slip coefficient of rail universal rolling is derived by the force equilibrium method in this paper.

2. The Simplified Model of Vertical Roll for Rolling the Top of Rail

In rod (or bar) rolling process, the roll surface is not flat for the groove on the roll, so previous researchers^{12–14)} pro-

posed the concept of “mean roll radius”, which has been used as equivalent radius to replaces the varying roll radius along the roll axis. In earlier work by Yong-gang Dong and Wen-zhi Zhang,¹⁵⁾ the concept “critical point on the contact boundary” was proposed, moreover, a new analytical model for the calculation of mean roll radius in Round-Oval-Round pass sequence was represented.

In rolling the top of the rail, the roll radius is not constant along the direction of vertical roll axis. For simplifying the model of rail universal rolling, the vertical roll with box pass can be simplified as an equivalent flat roll, the radius of which is equal to the mean roll radius of box pass.

As can be seen in **Fig. 2**, the stress free surface of the outgoing workpiece, which does not contact with the side wall of box pass, can be predicted as an oblique plane. The angle θ of this plane varies from 0 to α with the change of the maximum spread Δb_f in the box pass. So θ can be shown as

$$\theta = w_f \cdot \alpha_f \dots \dots \dots (1)$$

Fig. 1. Schematic plan of rail universal rolling.

Fig. 2. Schematic plan of deformation zone.

where α_t is the angle of side wall of box pass, w_t is a weighting function, it can be shown as

$$w_t = \frac{\Delta b_t}{2H_f \cdot \tan \alpha_t} \quad (2)$$

$$\Delta b_t = W_{t1} - W_{t0} \quad (3)$$

where H_f is the height of box pass, W_{t0} is the width of incoming workpiece, W_{t1} is the width of outgoing workpiece, B_0 is the minimum width of box pass, B_f is the maximum width of the box pass. it can be shown as

$$B_f = B_0 + 2H_f \tan \alpha_t \quad (4)$$

$$H_f = G_v + H_p \quad (5)$$

The width of the top of rail at the exit section W_{t1} can be calculated by the Modified B. K. Smirnov equation⁽⁶⁾

$$W_{t1} = \beta_t \cdot W_{t0} \quad (6)$$

$$\beta_t = c_{wt} \left[1 + 0.0714 \left(\frac{1}{\eta_t} - 1 \right)^{0.862} A^{0.746} a_0^{0.763} \varphi^{0.160} \tan \alpha^{0.362} c_m \right] \quad (7)$$

$$c_{wt} = 1 + \frac{\eta_t - \eta_w}{\eta_w} w_f \quad (8)$$

$$A = \frac{R_{vt} - H_f + G_{vt}}{H_f} \quad (9)$$

$$a_0 = \frac{H_{t0}}{W_{t0}} \quad (10)$$

$$\frac{1}{\eta_t} = \frac{H_{t0}}{H_f} \quad (11)$$

where β_t is the spread coefficient of the top of rail, c_m is the influence factor of the material on the spread (Table 1), c_{wt} is the deformation conformity factor between the web of rail and the top of rail, w_f is the proportion between the volume of metal flowing to the cross section and the total volume of metal flow, G_{vt} is the roll gap, R_{vt} is the maximum radius of vertical roll rolling the top of rail, φ is the friction factor of box pass.

The width of the top of rail at the exit section W_{b1} can be calculated by the modified S. Ekelund spread equation

 Table 1. The component influence coefficient m of different alloyed steel.

Steel type	Brand of steel	The component influence coefficient m
Plain carbon steel	Q195	1.0
Ledeburite	W18Cr4V	1.10
Pearlite-martensite	GCr15, 40Cr, 38GCrAl	1.24-1.30
Austenite	Cr13Ni4Mn9	1.40
Ferrite	1Cr17Al	1.55
Carbide austenite	Cr15Ni60	1.60

$$W_{b1} = c_m \cdot c_{wb} \left(\sqrt{4c_h^2 (H_{b0} + H_{b1}) \left(\frac{l_b}{W_{b0}} \right) + W_{b0}^2 + 4c_h l_b (3H_{b0} - H_{b1})} - 2c_h (H_{b0} + H_{b1}) \left(\frac{l_b}{W_{b0}} \right) \right) \quad (12)$$

$$c_{wb} = 1 + \frac{\eta_b - \eta_w}{\eta_w} w_f \quad (13)$$

$$c_h = \frac{1.6\mu_{vb}L_b - 1.2(H_{b0} - H_{b1})}{H_{b0} + H_{b1}} \quad (14)$$

$$l_b = \sqrt{R_{vb}(H_{b0} - H_{b1})} \quad (15)$$

where c_{wb} is the deformation conformity factor between the web of rail and the top of rail, l_b is the length of contact zone of the base of rail.

In this paper, the critical point is defined as the point on the contact boundary at the exit section. It affects the contact status and the shape of the outgoing workpiece directly, so it is indispensable for determining the mean roll radius.

At the exit cross-section, the critical point (C_y, C_z) can be determined by the intersecting point between the stress free surface and the side wall of box pass. So the critical point (C_y, C_z) can be obtained from the following equations:

$$2(C_z + C_y \cdot \tan \theta) = W_{t1} \quad (16)$$

$$\frac{H_f - C_y}{H_f} = \frac{2C_z - B_0}{B_f - B_0} \quad (17)$$

When $0 < \theta < \alpha$, solving the Eq. (16) and the Eq. (17) yields

$$C_z = \frac{W_{t1} - \frac{\tan \theta}{\tan \alpha_t} \cdot B_f}{2 \left(1 - \frac{\tan \theta}{\tan \alpha_t} \right)} \quad (18)$$

$$C_y = \frac{0.5B_f - C_z}{\tan \alpha_t} \quad (19)$$

As shown in Eq. (18) and Eq. (19)

$$\text{when } \theta = 0, \quad C_z = B_0/2 \quad \text{and} \quad C_y = H_f \quad (20)$$

$$\text{when } \theta = \alpha, \quad C_z = B_f/2 \cdot G_{vt} \cdot \tan \alpha \quad \text{and} \quad C_y = G_{vt} \quad (21)$$

As can be seen in Fig. 2, the little round angle is neglected. Since the small amount of area is neglected, the area of available contact section (hatching zone) A_h at the exit section can be determined by

$$A_h = 2C_z \cdot H_f - \left(C_z - \frac{B_0}{2} \right) (H_f - C_y) + C_z \cdot (W_{t1} - G_h) \tan \alpha_{h1} - \left(C_z - \frac{G_h}{2} \right)^2 \tan \alpha_{h1} \quad (22)$$

If the profile of the top of rail at the exit cross-section is simplified as a rectangular cross-section which width is $2C_z$ and height is $\overline{H_{t1}}$, then the area of equivalent contact section (rectangular zone $a'b'c'd'$) $A_{a'b'c'd'}$ can be obtained by

$$A_{a'b'c'd'} = A_h = 2C_z \cdot \overline{H_{t1}} \quad \text{.....(23)}$$

The equivalent height of outgoing workpiece $\overline{H_{t1}}$ can be calculated by

$$\begin{aligned} \overline{H_{t1}} = \overline{G_{vt}} = \frac{A_h}{2C_z} = \frac{1}{2}(H_f + C_y) + \frac{B_0}{4C_z}(H_f - C_y) \\ + \left(\frac{W_{t1}}{2} - \frac{C_z}{2} - \frac{G_h^2}{8C_z} \right) \tan \alpha_{h1} \quad \text{.....(24)} \end{aligned}$$

where G_h is the horizontal roll gap, α_{h1} is the oblique angle of the right flank of horizontal roll.

The equivalent radius of the vertical roll for rolling the top of rail $\overline{R_{vt}}$ can be obtained by

$$\overline{R_{vt}} = R_{vt} + G_{vt} - \overline{G_{vt}} \quad \text{.....(25)}$$

So the vertical roll for rolling the top of rail can be simplified as a flat roll which radius is $\overline{R_{vt}}$ and the gap is $\overline{G_{vt}}$.

As shown in Fig. 3, the profile of the base of rail at the exit cross-section can be simplified as a equivalent rectangular cross-section which width is W_{b1} and height is H_{b1} . The equivalent height H_{b1} can be expressed as

$$H_{b1} = \overline{G_{vb}} = G_{vb} + \left(\frac{W_{b1}}{4} - \frac{G_h^2}{4W_{b1}} \right) \tan \alpha_{h2} \quad \text{.....(26)}$$

Where α_{h2} is the oblique angle of the left flank of horizontal roll.

So, the simplified model of rail rolling can be shown as Fig. 4.

Based on the simplified model of rail universal rolling mentioned above, there are 5 contact zone existing in the deformed workpiece (Fig. 5): (1) the contact zone I between the vertical roll and the top of rail; (2) the contact zone II between the horizontal roll and the web of rail; (3) the contact zone III between the vertical roll and the base of rail; (4) the contact zone IV between the horizontal roll and the top of rail; (5) the contact zone V between the horizontal roll and the top of rail.

3. Neutral Angle of Horizontal Roll and Vertical Roll

The neutral angle is an important parameter since it de-

finer the relative-movement condition between the roll and the deformed metal in the deformation zone, and it has to be considered in computing the relative friction between the roll and the deformed metal.

3.1. Determination of the Neutral Angle of the Vertical Roll

According to the research of W. Zhang,⁵⁾ the neutral angle of the vertical roll can be derived from the equilibrium conditions of the torque acting on the vertical roll. the friction torque M_{vb} acting on the vertical roll from the base of rail can be expressed as

$$M_{vb} = \overline{p_{vb}} \overline{W_b} R_{vb} \mu_{vb} \gamma_{vb} - \overline{p_{vb}} \overline{W_b} R_{vb} \mu_{vb} (\alpha_{vb} - \gamma_{vb}) \quad \text{.....(27)}$$

where $\overline{W_b}$ is the mean width of the base of rail in deformation zone.

The friction torque M_{ob} acting on the vertical roll from the bearing of the vertical roll for rolling the base of rail can be obtained by

$$M_{ob} = -\overline{p_{vb}} \overline{W_b} \alpha_{vb} \mu_{ob} R_{ob} \quad \text{.....(28)}$$

Since the vertical roll is driven, the equilibrium equation for the torque acting on the vertical roll can be shown as

$$M_{ob} + M_{vb} = 0 \quad \text{.....(29)}$$

Solving the Eq. (29), the neutral angle of vertical roll for rolling the base of rail γ_{vb} can be shown as

$$\gamma_{vb} = \frac{\alpha_{vb}}{2} \left(1 + \frac{\mu_{ob} R_{ob}}{\mu_{vb} R_{vb}} \right) \quad \text{.....(30)}$$

By the same method mentioned above, the neutral angle of vertical roll for rolling the top of rail γ_{vt} can be shown as

$$\gamma_{vt} = \frac{\alpha_{vt}}{2} \left(1 + \frac{\mu_{ot} R_{ot}}{\mu_{vt} R_{vt}} \right) \quad \text{.....(31)}$$

3.2. The Neutral Angle of the Horizontal Roll

The neutral angle of horizontal roll can be obtained by the law of constant volume of metal flow (Fig. 6)

$$H_n V_h \cos \gamma_h = H_{w1} V_1 \quad \text{.....(32)}$$

where H_n is the height of the web of rail at the neutral plane of deformation zone, it can be shown as

$$H_n = H_{w1} + 2R_h(1 - \cos \gamma_h) \quad \text{.....(33)}$$

The velocity of rail at the exit section can be shown as

$$V_1 = V_h \delta_w \quad \text{.....(34)}$$

Fig. 3. Schematic plan of simplifying the vertical roll.

Fig. 4. The simplification of the cross-section of the base of rail.

Fig. 5. The simplified geometrical model of the rail universal rolling.

Fig. 6. The neutral line on the flank of horizontal roll.

A new equation can be obtained by Eqs. (32)–(34)

$$2R_h \cos^2 \gamma_h - (H_{w1} + 2R_h) \cos \gamma_h + H_{w1} \gamma_w = 0 \quad (35)$$

The Eq. (36) and Eq. (37) under mentioned can be used to simplify the Eq. (35)

$$\cos^2 \gamma_h \approx 1 - \gamma_h^2 \quad (36)$$

$$\cos \gamma_h \approx 1 - \gamma_h^2/2 \quad (37)$$

So the neutral angle of horizontal roll can be obtained from

$$\gamma_h = \sqrt{\frac{(\delta_w - 1)}{(R_h / H_{w1}) - 1/2}} \approx \sqrt{\frac{H_{w1}}{R_h} (\delta_w - 1)} \quad (38)$$

4. Forward Slip Coefficient of the Web

4.1. The Equation of Neutral Line on the Flank of Horizontal Roll

In the dissertation of Zhe Wang²⁾ and Xiao-guang Jin³⁾ for studying the forward slip coefficient of H-beam universal rolling, the concept “neutral line on the flank of horizontal roll” was proposed. For determining the equation of neutral line, a random point, which is on the contact zone between the flank of horizontal roll and the base of rail, has been chosen (Fig. 6).

On the flank of horizontal roll, the velocity of this point V_f can be obtained from

$$V_f = \frac{V_h}{R_h} \sqrt{x^2 + \left(\frac{H_{w1}}{2} + R_h - y \right)^2} \quad (39)$$

The horizontal component of the velocity V_f can be shown as

$$V_{fx} = \frac{V_h}{R_h} \left(\frac{H_{w1}}{2} + R_h - y \right) \quad (40)$$

On the surface of the base of rail, the velocity of this point V_b can be determined by the condition of constant flow volume of metal

$$V_{bx} = \frac{V_1 H_{b1}}{H_{bx}} = \frac{V_h \delta_w H_{b1}}{H_{bx}} \quad (41)$$

where H_{bx} is the height of the base of rail at the cross-section where this point locates, H_{b1} is the height of the base of rail at the exit section, V_1 is the velocity of rail at the exit section.

As can be seen in Fig. 7, H_{bx} can be expressed as

$$H_{bx} = H_{b1} + (R_{vb} - \sqrt{R_{vb}^2 - x^2}) \quad (42)$$

Substituting Eq. (42) into Eq. (41) yields

Fig. 7. The velocity of the base and top of rail on the flank of horizontal roll.

Fig. 8. The velocity vector of the point P.

 Fig. 9. Schematic plan of forward slip zone on the flank of horizontal roll ($\delta_w \leq 1$).

 Fig. 10. Schematic plan of forward slip zone on the flank of horizontal roll ($\delta_w > 1$).

$$V_{bx} = \frac{V_h \delta_w H_{b1}}{H_{b1} + (R_{vb} - \sqrt{R_{vb}^2 - x^2})} \quad (43)$$

As can be seen in Fig. 8, the relative velocity of this point between the flank of horizontal roll and the base of rail ΔV_x can be shown as

$$\Delta V_x = V_{bx} - V_{fx} \quad (44)$$

Since the relative velocity ΔV_x of any point on the neutral line is 0, the equation of neutral line on the left flank of horizontal roll can be expressed as

$$z_b = \frac{H_{w1}}{2} + R_h - \frac{\delta_w H_{b1} R_h}{H_{b1} + (R_{vb} - \sqrt{R_{vb}^2 - x^2})} \approx \frac{H_{w1}}{2} + R_h - \frac{\delta_w H_{b1} R_h}{H_{b1} + x^2 / D_{vb}} \quad (45)$$

So, the intersecting point (x_{b0}, y_{b0}) between neutral line and the boundary of the base of rail are expressed as (Figs. 9, 10).

$$z_{b0} = (W_{b0} + W_{b1})/4 \quad (46)$$

$$x_{b0} = \sqrt{\frac{H_{b1} \cdot D_{vb} [W_{b0} - H_{w1} - D_h (1 - \delta_w)]}{D_h - W_{b0} + H_{w1}}} \approx \sqrt{\frac{H_{b1} \cdot D_{vb} (W_{b0} - H_{w1})}{D_h - W_{b0} + H_{w1}}} \quad (47)$$

By the same method mentioned above, the neutral line on the right flank of horizontal roll can be shown as

$$z_t \approx \frac{H_{w1}}{2} + R_h - \frac{\delta_w H_{t1} R_h}{H_{t1} + x^2 / D_{vt}} \quad (48)$$

The intersecting point (x_{t0}, y_{t0}) between neutral line and the boundary of top of rail are expressed as

$$z_{t0} = (W_{t0} + W_{t1}) / 4 \quad (49)$$

$$x_{t0} = \sqrt{\frac{H_{b1} \cdot D_{vb} [W_{b0} - H_{w1} - D_h (1 - \delta_w)]}{D_h - W_{b0} + H_{w1}}} \approx \sqrt{\frac{H_{b1} \cdot D_{vb} (W_{b0} - H_{w1})}{D_h - W_{b0} + H_{w1}}} \quad (50)$$

4.2. The Area of Forward Slip Zone and Backward Slip Zone on the Flank of Horizontal Roll

As can be seen Fig. 9 and Fig. 10, the total area of contact zone between the flank of horizontal roll and the base of rail A_{fb} can be shown as

$$A_{fb} = \int_0^{L_b} \left[\frac{W_{b0} - H_{w1}}{2} - (R_h - \sqrt{R_h^2 - x^2}) \right] dx = \frac{L_b}{2} \cdot [(W_{b0} - H_{w1}) - (R_h - \sqrt{R_h^2 - L_b^2})] \quad (51)$$

By the same method mentioned above, the area of contact zone between the flank of horizontal roll and the top of rail A_{ft} can be shown as

$$A_{ft} = \int_0^{L_t} \left[\frac{W_{t0} - H_{w1}}{2} - (R_h - \sqrt{R_h^2 - x^2}) \right] dx = \frac{L_t}{2} \cdot [(W_{t0} - H_{w1}) - (R_h - \sqrt{R_h^2 - L_t^2})] \quad (52)$$

where L_b, L_t is the length of deformation zone in rolling the base of rail and top of rail (Fig. 5), respectively.

4.2.1. The Area of Forward Slip Zone on the Flank of Horizontal Roll ($\delta_w \leq 1$)

As can be seen in Fig. 9, when $\delta_w \leq 1$ the neutral line and forward slip zone on the left flank of horizontal roll have been indicated.

If $x_{b0} < L_b$, the area of forward slip zone on the left flank of horizontal roll A'_{fb} can be shown as

$$A'_{fb} = \int_0^{x_{b0}} \left(\frac{W_{b0} - H_{w1}}{2} - R_h + \frac{\delta_w R_h}{1 + x^2 / (H_{b1} \cdot D_{vb})} \right) dx = \frac{W_{b0} - H_{w1}}{2} x_{b0} - R_h x_{b0} (1 - \delta_w) \quad (53)$$

By the same method, when $x_{t0} < L_t$ the area of forward slip zone on the right flank of horizontal roll A'_{ft} can be shown as

$$A'_{ft} = \int_0^{x_{t0}} \left(\frac{W_{t0} - H_{w1}}{2} - R_h + \frac{\delta_w R_h}{1 + x^2 / (H_{t1} \cdot D_{vt})} \right) dx = \frac{W_{t0} - H_{w1}}{2} x_{t0} - R_h x_{t0} (1 - \delta_w) \quad (54)$$

If $x_{b0} \geq L_b$, the area of forward slip zone on the left flank of horizontal roll A'_{fb} can be shown as

$$A'_{fb} = \frac{W_{b0} - H_{w1}}{2} L_b - R_h L_b (1 - \delta_w) \quad (55)$$

By the same method, when $x_{t0} \geq L_t$ the area of forward slip zone on the right flank of horizontal roll A'_{ft} can be shown as

$$A'_{ft} = \frac{W_{t0} - H_{w1}}{2} L_t - R_h L_t (1 - \delta_w) \quad (56)$$

4.2.2. The Area of Forward Slip Zone on the Flank of Horizontal Roll ($\delta_w > 1$)

As can be seen in Fig. 10, when $\delta_w > 1$ the neutral line and forward slip zone on the left flank of horizontal roll have been indicated.

If $x_{b0} < L_b$, the area of forward slip zone on the left flank of horizontal roll A'_{fb} can be shown as

$$A'_{fb} = x_0 \cdot \frac{W_{b0} - H_{w1}}{2} + \int_{x_0}^{x_{b0}} \left(\frac{W_{b0} - H_{w1}}{2} - R_h + \frac{\delta_w R_h}{1 + x^2 / (H_{b1} \cdot D_{vb})} \right) dx \quad (57)$$

where x_0 is the distance from the bottom of neutral line to the outgoing cross-section, it can be shown as

$$x_0 = R_h \gamma_h \quad (58)$$

By the same method, when $x_{t0} < L_t$ the area of forward slip zone on the right flank of horizontal roll A'_{ft} can be shown as

$$A'_{ft} = x_0 \cdot \frac{W_{t0} - H_{w1}}{2} + \int_{x_0}^{x_{t0}} \left(\frac{W_{t0} - H_{w1}}{2} - R_h + \frac{\delta_w R_h}{1 + x^2 / (H_{t1} \cdot D_{vt})} \right) dx \quad (59)$$

On the contrary, when $x_{b0} \geq L_b$ the area of forward slip zone on the left flank of horizontal roll A'_{fb} can be shown as

$$A'_{fb} = x_0 \cdot \frac{W_{b0} - H_{w1}}{2} + \int_{x_0}^{L_b} \left(\frac{W_{b0} - H_{w1}}{2} - R_h + \frac{\delta_w R_h}{1 + x^2 / (H_{b1} \cdot D_{vb})} \right) dx \quad (60)$$

By the same method, when $x_{t0} \geq L_t$ the area of forward slip zone on the right flank of horizontal roll A'_{ft} can be shown as

$$A'_{ft} = x_0 \cdot \frac{W_{t0} - H_{w1}}{2} + \int_{x_0}^{L_t} \left(\frac{W_{t0} - H_{w1}}{2} - R_h + \frac{\delta_w R_h}{1 + x^2 / (H_{t1} \cdot D_{vt})} \right) dx \quad (61)$$

4.3. The Area of Forward Slip Zone in the Deformation Zone of the Web of Rail

The total area of contact zone A_w can be shown as

$$A_w = b_w \cdot L_w \quad (62)$$

when $\delta_w \leq 1$, the area of forward slip zone A'_w is 0. So, the area of backward-slip zone A''_w can be obtained by

$$A''_w = A_w = b_w \cdot L_w \quad (63)$$

Consequently, when $\delta_w \leq 1$, the deformation zone is complete backward-slip zone.

When $\delta_w < 1$, the area of forward slip zone can be obtained by

$$A'_w = b_w R_h \gamma_h = b_w \sqrt{H_{w1} R_h (\delta_w - 1)} \quad (64)$$

4.4. The Horizontal Resultant Force Acting on the Deformed Workpiece

The horizontal resultant force X_v acting on the top and base of rail from the vertical roll can be obtained by

$$\begin{aligned} X_v = X_{vt} + X_{vb} = W_{t0} \overline{p_{vt}} \mu_{vt} L_t \left(1 - \frac{2\gamma_{vt}}{\alpha_{vt}} - \frac{\tan(\alpha_{vt}/2)}{\mu_{vt}} \right) \\ + W_{b0} \overline{p_{vb}} \mu_{vb} L_b \left(1 - \frac{2\gamma_{vb}}{\alpha_{vb}} - \frac{\tan(\alpha_{vb}/2)}{\mu_{vb}} \right) \quad (65) \end{aligned}$$

The horizontal resultant force X_{fb} acting on the base of rail from the flank of horizontal roll can be obtained by

$$X_{fb} = 2 \overline{p_{hb}} \mu_{hb} (A''_{fb} - A'_{fb}) = 2 \overline{p_{hb}} \mu_{hb} (A_{fb} - 2A'_{fb}) \quad (66)$$

For simplifying the deriving process, the horizontal resultant force X_{fi} acting on the top of rail from the flank of horizontal roll can be obtained by

$$\frac{X_{fi}}{X_{fb}} = \frac{\overline{p_{ht}} \mu_{ht} (A_{fi} - 2A'_{fi})}{\overline{p_{hb}} \mu_{hb} (A_{fb} - 2A'_{fb})} = \rho \quad (67)$$

where the coefficient ρ is a proportion factor about horizontal resultant force X_{fi} and X_{fb} . Then

$$X_f = X_{fi} + X_{fb} = (1 + \rho) X_{fb} \quad (68)$$

If $\delta_w \leq 1$, the horizontal resultant force X_w acting on the web of rail from the horizontal roll can be obtained by

$$\begin{aligned} X_w = 2b_w \left(\overline{p_h} L_w \mu_{hw} - \overline{p_h} \frac{L_w}{\cos(\alpha_h/2)} \sin\left(\frac{\alpha_h}{2}\right) \right) \\ \approx 2b_w \overline{p_h} L_w \mu_{hw} \left(1 - \frac{\alpha_h}{2\mu_{hw}} \right) \quad (69) \end{aligned}$$

If $\delta_w > 1$, the horizontal resultant force X_w can be obtained by

$$X_w = 2 \overline{p_h} \mu_{hw} (A_w - 2A'_w) - 2b_w \overline{p_h} L_w \mu_{hw} \tan\left(\frac{\alpha_h}{2}\right) \quad (70)$$

4.5. The Forward Slip Coefficient of the Web of Rail ($\delta_w \leq 1$)

The equilibrium condition of horizontal resultant force acting on the deformed workpiece can be shown as

$$\sum X = X_w + X_f + X_v = 0 \quad (71)$$

when $x_{b0} < L_b$, solving the Eq. (71) yields

$$\delta_w = 1 - \frac{(W_{b0} - H_{w1}) - a^{2/3}}{D_h} \quad (72)$$

$$\begin{aligned} a = \frac{X_w + X_{vt} + X_{vb} - (1 + \rho) \overline{p_{hb}} L_b \mu_{hb} W_{b0} - H_{w1}}{2 \overline{p_{hb}} \mu_{hb} \sqrt{\frac{H_{b1} \cdot D_{vb}}{D_h - W_{b0} + H_{w1}}} (1 + \rho)} \\ - \frac{L_b}{2} [W_{b0} - H_{w1} - (R_h - \sqrt{R_h^2 - L_b^2})] \quad (73) \end{aligned}$$

when $x_{b0} \geq L_b$, solving the Eq. (71) yields

$$\delta_w = 1 - \frac{1}{2D_h} \left[\frac{X_w + X_v}{(1 + \rho) \overline{p_{hb}} \mu_{hb} L_b} + W_{b0} - H_{w1} - (R_h - \sqrt{R_h^2 - L_b^2}) \right] \quad (74)$$

4.6. The Forward Slip Coefficient of the Web of Rail ($\delta_w > 1$)

When $x_{b0} < L_b$, solving the Eq. (71) yields

$$\begin{aligned} \delta_w = 1 + \frac{1}{b} \left[\frac{\overline{p_h} b_w L_w \mu_{hw}}{\overline{p_{hb}} \mu_{hb}} \left(1 - \frac{\alpha_h}{2\mu_{hw}} \right) + (1 + \rho) L_f (W_{b0} - H_{w1}) \right. \\ \left. \times \left(1 - \frac{2}{L_f} \sqrt{\frac{H_{b1} \cdot D_{vb} (W_{b0} - H_{w1})}{D_h - W_{b0} + H_{w1}}} \right) + \frac{X_v}{\overline{p_{hb}} \mu_{hb}} \right] \quad (75) \end{aligned}$$

where b is expressed as

$$\begin{aligned} b = (1 + \rho) \left\{ 50 \sqrt{\frac{(0.02 D_h + W_{b0} - H_{w1})^2 H_{b1} \cdot D_{vb}}{D_h - W_{b0} + H_{w1}}} \right. \\ \left. - 0.28 R_h \sqrt{R_h H_{w1}} - 50 (W_{b0} - H_{w1}) \sqrt{\frac{H_{b1} \cdot D_{vb} (W_{b0} - H_{w1})}{D_h - W_{b0} + H_{w1}}} \right. \\ \left. + \frac{14 \overline{p_h} b_w \mu_{hw}}{\overline{p_{hb}} \mu_{hb}} \sqrt{H_{w1} R_h} \right\} \quad (76) \end{aligned}$$

when $x_{b0} \geq L_b$, solving the Eq. (71) yields

$$\begin{aligned} \delta_w = 1 + \left\{ \frac{\overline{p_h} b_w L_w \mu_{hw}}{\overline{p_{hb}} \mu_{hb}} \left(1 - \frac{\alpha_h}{2\mu_{hw}} \right) + (1 + \rho) L_f (W_{b0} - H_{w1}) \right. \\ \left. \times \left(1 - \frac{2}{L_f} \sqrt{\frac{H_{b1} \cdot D_{vb} (W_{b0} - H_{w1})}{D_h - W_{b0} + H_{w1}}} \right) + \frac{X_v}{\overline{p_{hb}} \mu_{hb}} \right\} / \\ \left\{ 2(1 + \rho) (R_h L_f - 0.14 R_h \sqrt{H_{w1} R_h}) + \frac{14 \overline{p_h} b_w \mu_{hw}}{\overline{p_{hb}} \mu_{hb}} \sqrt{H_{w1} R_h} \right\} \quad (77) \end{aligned}$$

5. Forward Slip Coefficient of the Top and Base of Rail

According to the condition of constant flow volume of metal, the following equation can be obtained (Fig. 7)

$$V_{vb} = \frac{V_1 H_{b1}}{(H_{b1} + R_{vb} (1 - \cos \gamma_{vb})) \cos \gamma_{vb}} \quad (78)$$

So the forward coefficient of the base of rail can be shown as

$$\delta_{vb} = \frac{V_1}{V_{vb}} = \cos \gamma_{vb} \left[(1 - \cos \gamma_{vb}) \frac{R_{vb}}{H_{bl}} + 1 \right] \approx 1 + \left(\frac{R_{vb}}{2H_{bl}} - 1 \right) \gamma_{vb}^2 \quad (79)$$

By the same method, the forward slip coefficient of the top of rail can be expressed as

$$\delta_{vt} = \cos \gamma_{vt} \left[(1 - \cos \gamma_{vt}) \frac{R_{vt}}{H_{tl}} + 1 \right] \approx 1 + \left(\frac{R_{vt}}{2H_{tl}} - 1 \right) \gamma_{vt}^2 \quad (80)$$

6. The Numerical Simulation of Rail Universal Rolling

For verifying the forward slip theoretical model of rail universal rolling, the three-dimensional solid model of rail universal rolling are built by the software SolidWorks 2005 according to corresponding equipment parameters. The universal rolling process of 60 kg/m heavy rail and 18 kg/m light rail are simulated by the Rigid-plastic FEM software DEFORM-3D V5.0.

The rigid-plastic FEM model of rail universal rolling is built as Fig. 11 for the symmetry of workpiece. The horizontal is defined as a rigid body, the workpiece is defined as rigid-plastic body, and the driven vertical roll are jacketed on the supporting spindle and they are defined as elastic body, the supporting spindles are defined as rigid body. The size of initial workpiece is shown as Fig. 11. The FEM model of universal mill is shown as Fig. 12(a). For biting into the gap easily, the head of initial workpieces are cut as Figs. 12(b) and 12(c). So the web of rail will contact the horizontal roll before it contact the vertical roll.

The material of heavy rail is rare-earth rail steel BNbRe, and its resistance to the deformation can be obtained by the physical simulation experiments, the details can be seen in Ref. 18). The material of light rail is plain carbon steel Q235, and its resistance to the deformation can be obtained by the Ref. 19). The chemical composition of the heavy rail steel and light rail steel are shown in Table 2 and Table 3 respectively.

The rolling schedule of 60 kg/m heavy rail universal rolling and 18 kg/m light rail universal rolling are shown as Table 4 and Table 5.

The coefficient of heat transfer for convection between the workpiece and the environment is chosen as $0.02 \text{ N/(s} \cdot \text{mm} \cdot ^\circ\text{C)}$, and the coefficient of heat transfer for contact between the workpiece and the roll is chosen as $11 \text{ N/(s} \cdot \text{mm} \cdot ^\circ\text{C)}$. The conversion coefficient from mechan-

ical energy to heat energy is chosen as 0.9. The environment temperature is defined as 25°C , and the initial temperature of heavy rail is defined as 1150°C . The friction type is defined as shear friction and the friction coefficient of heavy rail universal rolling is chosen as 0.25, and the friction coefficient of light rail universal rolling is chosen as 0.35.

A part of numerical simulation result is shown as Figs. 13–16. The velocity of workpiece No. 2-1 in the 18 kg/m light rail universal rolling is shown as Fig. 13, and the corresponding velocity of vertical rolls in the 18 kg/m light rail universal rolling are shown as Figs. 14(a) and 14(b). The velocity of workpiece in the pass UR1 during the 60 kg/m heavy rail rolling is shown as Fig. 15, and the velocity of vertical rolls in pass UR1 during the 60 kg/m heavy rail rolling are shown as Figs. 16(a) and 16(b). According to the

(a) The size of initial workpiece for rolling the 60kg/m heavy rail

(b) The size of initial workpiece for light rail universal rolling experiments

Fig. 11. The size of initial workpieces for rail universal rolling.

(a) The pass schedule of rail universal rolling

(b) The initial workpiece of 60kg/m heavy rail

(c) The initial workpiece of 18kg/m light rail

Fig. 12. The rigid-plastic FEM model of rail universal rolling.

Table 2. The chemical composition range of 60 kg/m heavy rail steel BNbRe.

Chemical composition	C	Si	Mn	P	S	Nb	Re
Range (%)	0.7–0.82	0.6–0.9	0.9–1.2	≤0.04	≤0.04	0.02–0.05	0.02–0.05

Table 3. The chemical composition of light rail steel Q235.

Chemical composition	C	Si	Mn	P	S
Content (%)	0.17	0.21	0.83	0.019	0.012

Table 4. The rolling schedule for producing 60 kg/m heavy rail.

Pass	Stand	Horizontal roll gap (mm)	Reduction(mm)			Rolling velocity (mm/s)	Horizontal roll rotating velocity (rad/s)
			The web of rail	The base of rail	The head of rail		
1	UR	32.0	11	7.0	14.0	4910	9.043
2	E	31.5	0.5	—	—	5000	13.619
3	UR	24.0	7.5	4.5	11.5	5200	11.308
4	UR	18.3	5.7	4.0	9.0	4950	9.128
5	E	18.3	0	—	—	5070	13.50
6	UF	16.7	1.6	1.5	2.5	5500	10.511

Table 5. The rolling schedule for 18 kg/m light rail universal rolling experiments.

Workpiece No.	Reduction (mm)			Main motor speed (rpm)	Horizontal roll rotating velocity (rad/s)	Workpiece's temperature(°C)	
	The web of rail	The head of rail	The base of rail			Tapping temperature	Initial rolling temperature
1-1,1-2	1.5	4.0	2.0	1200	3.9873	1050	1000
2-1,2-2	1.5	4.5	2.0	1000	3.3228	1040	995
3-1,3-2	2.0	3.5	2.0	1000	3.3228	1060	1005
4-1,4-2	2.0	3.0	2.0	800	2.6582	1070	1015
5-1,5-2	2.5	2.5	2.0	800	2.6582	1100	1035
6-1,6-2	2.5	2.5	2.0	600	1.9937	1100	1047

Fig. 13. The velocity of workpiece No. 2-1 in the 18 kg/m light rail universal rolling.

Fig. 14. The velocity of vertical roll in the workpiece No. 2-1 in the 18 kg/m light rail universal rolling.

Fig. 15. The velocity of workpiece in pass UR1 during the 60 kg/m heavy rail rolling.

Fig. 16. The velocity of vertical roll in pass UR1 during the 60 kg/m heavy rail rolling.

numerical simulation results, the forward slip coefficient based on the Rigid-Plastic FEM can be obtained.

6. The Experiments of Light Rail Universal Rolling

The light rail universal rolling experiments are accomplished by the three-stand universal mills in the Yanshan University Rolling Laboratory. As can be seen in Fig. 17,

the main experimental equipments are the data acquisition system, the infrared thermometer, the three-stand universal mills and the box type heater.

The initial workpiece for the rail universal rolling is a 18 kg/m light rail which length is 6 m. This light rail is divided into 20 pieces and the length of every pieces is 300 mm long. For biting into the gap easily, the head of workpiece is cut as same as the profile of FEM model.

A part of rolled pieces are shown as Fig. 18.

For obtaining the value of forward slip coefficient, some pits are chiseled on the surface of the horizontal roll and vertical roll with the same distance, and some humps on the surface of workpiece can be obtained after the workpiece is rolled in the universal mill. As can be seen in Figs. 19(a), 19(b) and 19(c), the forward slip coefficient can be calculated by measuring the distance between two adjacent hump.

7. Results and Discussions

The field data, the Rigid-Plastic FEM simulation results and the theoretical model results of forward slip coefficient

(a) The three-stand universal mills

(b) The box type heater

Fig. 17. The main equipments for rail universal rolling.

in 60 kg/m heavy rail universal rolling is listed in Table 6, and the field data the experimental results, the Rigid-Plastic FEM simulation results and the theoretical model results of forward slip coefficient in light rail universal rolling experiments is listed in Table 7.

As can be seen in Table 6 and Table 7, the forward slip coefficient of the web of rail is always less than 1 in rail universal rolling. That's to say, the deformation zone of the web of rail is complete backward slip zone. On the contrary, the forward slip coefficient of the base and top of rail are always greater than 1 in rail universal rolling. Because the horizontal roll is driving and the vertical roll is driven, the metal flow of the web of rail will be restricted by the base and top of rail and the outgoing velocity of the web of rail is less than the linear velocity of the horizontal roll. So the forward slip coefficient of the web of rail is always less than 1. Since the corresponding vertical roll is actually driven by the base of rail or top of rail, the outgoing velocity of the base or top of rail is greater than the linear velocity of corresponding vertical roll.

8. Conclusion

In this paper, the method of force equilibrium has been employed to derive a series of equations for forward slip coefficient, which is applicable to different size of workpiece and different pass schedule.

(1) In general, the forward slip coefficient of the web is less than 1 in rail universal rolling. If the reduction of the base and top of rail is small enough in rail universal rolling, the forward slip coefficient of the web can be greater than 1.

(2) Whatever are the rolling conditions, the forward slip coefficient of the base and top of rail relative to the vertical roll is always greater than 1.

(3) With the increment of web reduction and rotational speed of the horizontal roll, the forward slip coefficient of

Fig. 18. A part of rolled piece.

(a) The web of rail

(b) The base of rail

(c) The head of rail

Fig. 19. The measurement of the forward slip coefficient.

Table 6. The results comparison of forward slip coefficient in 60 kg/m heavy rail universal rolling.

Pass	δ_w			δ_{vb}			δ_{vt}		
	Theoretical model	FEM	Field data	Theoretical model	FEM	Field data	Theoretical model	FEM	Field data
UR1	0.982	0.957	0.99	1.10	1.095	1.15	1.065	1.097	1.12
UR2	0.951	0.929	0.97	1.094	1.078	1.13	1.076	1.088	1.11
UR3	0.939	0.949	0.96	1.076	1.065	1.12	1.059	1.074	1.10

Table 7. The results comparison of forward slip coefficient in the light rail universal rolling experiments.

Workpiece No.	δ_w			δ_{vb}			δ_{vt}		
	Experimental data	FEM	Theoretical model	Experimental data	FEM	Theoretical model	Experimental data	FEM	Theoretical model
1-1	0.987	0.969	0.991	1.090	1.125	1.145	1.120	1.165	1.148
1-2	0.976			1.100			1.115		
2-1	0.977	0.961	0.988	1.088	1.072	1.096	1.115	1.150	1.139
2-2	0.950			1.050			1.110		
3-1	0.975	0.959	0.981	1.068	1.095	1.089	1.110	1.135	1.142
3-2	0.977			1.073			1.095		
4-1	0.975	0.960	0.984	1.050	1.085	1.075	1.090	1.126	1.112
4-2	0.967			1.065			1.085		
5-1	0.962	0.954	0.976	1.060	1.076	1.085	1.090	1.105	1.116
5-2	0.957			1.063			1.080		
6-1	0.958	0.947	0.971	1.050	1.070	1.087	1.085	1.098	1.105
6-2	0.960			1.081			1.079		

the web of rail increases. With the reduction's increment of the base and top of rail, the forward slip coefficient of the web of rail decreases. Moreover, the rotational speed of the horizontal roll influences the forward slip coefficient more than else factors.

(4) The theoretical results and simulation results are in agreement with the experimental data basically. So, this theoretical model can be applied in rail rolling.

Nomenclature

α_h : Bite angle of the horizontal roll
 α_{vb} : Bite angle of the vertical roll for rolling the base of rail
 α_{vt} : Bite angle of the vertical roll for rolling the top of rail
 V_h : Linear velocity of the horizontal roll
 V_{vt} : Linear velocity of the vertical roll for rolling the top of rail
 V_{vb} : Linear velocity of the vertical roll for rolling the base of rail
 H_{w0} : The height of the web of rail at the entrance section
 H_{w1} : The height of the web of rail at the exit section
 H_{b0} : The height of the web of rail at the entrance section
 H_{b1} : The height of the base of rail at the exit section
 W_{b0} : The width of the base of rail at the entrance section
 W_{b1} : The width of the base of rail at the exit section
 R_h : The radius of horizontal roll
 R_{vb} : The radius of vertical roll for rolling the base of rail
 $\bar{G}_{vt}$: The equivalent gap of the vertical roll rolling the top of rail
 G_{vt} : The gap of the vertical roll rolling the top of rail
 D_{vb} : The diameter of vertical roll for rolling the base of rail
 $\bar{D}_{vt}$: The equivalent diameter of vertical roll for rolling the base of rail
 $\bar{P}_h$: Mean unit pressure acting on the web from the horizontal roll
 $\bar{P}_{vt}$: Mean unit pressure acting on the top of rail from the vertical roll
 $\bar{P}_{vb}$: Mean unit pressure acting on the rail base from the vertical roll
 $\bar{P}_{ht}$: Mean unit pressure acting on the top of rail from the flank of horizontal roll
 $\bar{P}_{hb}$: Mean unit pressure acting on the rail base from the flank of horizontal roll
 μ_{hw} : Friction coefficient between the web and horizontal roll
 μ_{ht} : Friction coefficient between the top of rail and the flank of horizontal roll

μ_{hb} : Friction coefficient between the rail base and the flank of horizontal roll
 μ_{vt} : Friction coefficient between the top of rail and vertical roll
 μ_{vb} : Friction coefficient between the top of rail and vertical roll
 μ_{ob} : The equivalent friction coefficient of the bearing of the vertical roll for rolling the base of rail
 R_{ob} : The equivalent radius of the bearing of vertical roll for rolling the base of rail.
 μ_{ot} : The equivalent friction coefficient of the bearing of the vertical roll for rolling the top of rail
 R_{ot} : The equivalent radius of the bearing of vertical roll for rolling the top of rail.

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