1. Introduction

Electric Arc Furnace (EAF) is an example of a low stirring metallurgical reactor. The extent of turbulence generated from the motion of the fluid is directly related to the melting rate and temperature homogenization, consequently, a better understanding of fluid flow and mixing phenomena is critical to improve the melting process. Fluid flow results from: (a) buoyancy forces due to thermal gradients inside the furnace, which in turn constitute the driving force for natural convection, (b) CO bubbling from the decarburization reaction; (c) jet impingement from the oxygen lance on the metal bath, (d) electromagnetic forces around the pitch circle, (e) shear stresses from the arc column impinging on the metal bath, and (f) bottom gas bubbling, when porous plugs are installed.

Szekely et al.1) investigated fluid flow and temperature stratification in a cylindrical DC furnace under the different driving forces for momentum transfer. Their work excluded electromagnetic forces, gas injection and the slag phase. They reported a maximum linear velocity, below the arc, of 20 cm · s⁻¹ and a maximum radial velocity, at the surface, of 40 cm · s⁻¹.

Kurimoto et al.2) reported a mathematical model to predict fluid flow patterns and temperature distribution in the bath region of cylindrical DC furnaces due to electromagnetic body forces. They concluded that fluid flow was controlled by the Lorenz forces, reporting a maximum velocity of 220 cm · s⁻¹ and a negligible contribution from natural convection.

Deneyes and Robertson3) reported experimental observations in a laboratory scale DC arc furnace for slag cleaning. By video photography it was possible to observe radial streamlines flowing from the center towards the crucible wall. The driving force for the flow from a cold zone to a hot zone was attributed to surface tension gradients. The authors also reported a large thermal stratification, within 5 cm, which represents the slag height employed in the experiments, resulting in temperature differences from 150–200°C. As the input power was increased the surface temperature was also increased, from 1375°C for 3.5 kW to 1500°C at 6.8 kW arc power.

Murthy et al.4) reported experimental measurements describing the velocity field and the turbulent electromagnetically-driven flow, employing DC current. In this work a new approach to the classification of the flow as laminar or turbulent for recirculating flow systems was presented. This approach is given by the ratio of effective viscosity to molecular viscosity.

Ramírez et al.5) described fluid flow, heat transfer and electromagnetic phenomena in the bath region of a DC-EAF. They reported that the slag decreases mixing in the bath, furthermore, in order to improve mixing, the furnace should be operated under short arc lengths and high arc currents.

DC systems present strong electromagnetic stirring since electrons flow from the cathode graphite electrode through the electric arc and then through the steel bulk to the anode usually located at the bottom of crucible; consequently, electromagnetic body forces promote extra stirring in the entire melt. On the contrary, in AC systems cathode and anode are graphite electrodes and then electrons travel from the cathode to the anode through the top surface of steel bath, and therefore electromagnetic stirring is restricted...
only at the pitch circle.

Numerous investigations have been carried out to describe fluid flow due to bottom stirring.\textsuperscript{6–13} Irons \textit{et al.}\textsuperscript{5–8} investigated fluid flow involving a two phase system due to bottom gas injection, oxygen injection and CO generation. They found a small, localized stirring effect due to the separate effects of bottom gas injection, oxygen lancing and CO generation. This behavior was attributed to the presence of a highly viscous slag which dissipated the motion of the fluid; however, the magnitude of the injected gas was not indicated. They also found that stirring was significant when bottom gas injection and CO generation operated simultaneously. This behavior was attributed to the absorption of CO in the argon bubbles. They reported a maximum velocity vector of 152 cm \textcdot s\textsuperscript{-1} for the combined effect of argon injection and CO absorbed by the argon bubbles, for a system including the slag phase. Their model also reported a small affected area due to the generation of CO during oxygen lancing, even if the decarburization rate was increased from 0.002 to 0.02%/min.

Caffery \textit{et al.}\textsuperscript{8} reported the combined effect of oxygen injection and bottom blowing, with variations in lance position. The velocity profile in the balcony region for an industrial EAF indicated values ranging from 6 to 15 cm \textcdot s\textsuperscript{-1}. When the gas flow rate was increased from 2.5 to 10 L \textcdot min\textsuperscript{-1}, bath homogeneity decreased, suggesting an upper limit of 2.5 L \textcdot min\textsuperscript{-1}, above this value the flow structure was highly complex, resulting in cold spots.

Tanski\textsuperscript{10} reported numerical computations from a single, axially aligned, bottom blowing air jet. His model indicated a higher recirculation region as the injection rate increased; the maximum centerline velocity increased from 104 to 427 cm \textcdot s\textsuperscript{-1} when the air flow rate was increased from 3 to 12 L \textcdot min\textsuperscript{-1}, respectively.

Zhang and Fruehan\textsuperscript{11} reported that mixing time is not a strong function of tuyere pattern in spite that in two arrangements the mixing time decreased from 120 to 20 s when the gas flow rate increased from 5–40 L/Min. Li\textsuperscript{12} recommends increasing the number of plugs to improve stirring conditions. Banerjee and Irons\textsuperscript{13} reported that mixing time under isothermal conditions cannot be applied to top heating vessels, because the thermal effects have a significant influence on the flow field within the furnace.

In previous works, emphasis on the heating source has been neglected. In this work results on velocity and temperature fields due to natural convection inside a three-phase AC electric arc furnace are reported, assuming that liquid steel occupies the entire computational domain with emphasis on the influence of arc length on both buoyancy forces and mixing time in an industrial crucible of complex geometry.

2. Description of Mathematical Model

Figure 1 represents the computational domain and general dimensions of the furnace. In order to simplify the computations, the following assumptions are made:

- Liquid steel occupies the entire computational domain (this model handles only one phase and neglects the presence of the top slag phase as well as the formation of CO bubbles).
- Constant thermo-physical properties for liquid steel.\textsuperscript{11}
- Walls at a constant temperature of 1 500°C. Those temperatures are assumed at the furnace walls because they are close to the liquidus temperature of low-carbon steels.\textsuperscript{11}
- Radiation from the walls and roof is not taken into account.
- Steady state conditions.
- Driving forces are limited to buoyancy forces neglecting Marangoni forces. Marangoni forces are important in welding operations because the extreme temperature differences and the small pool size create extremely high gradients in surface tension. On the contrary, in the present case those forces were ignored because of the much larger scale of the liquid pool in EAF gives much smaller temperature gradients and smaller surface tension gradients in EAF baths than in weld pools.
- Electromagnetic forces are neglected for a AC-EAF.
- Stirring due to oxygen injection or bottom gas injection is not taken into account.

Fluid flow patterns due to buoyancy forces allow the computation of velocity fields and temperature profile inside the computational domain. Temperature gradients create density differences and hence buoyancy driven flows. The Boussinesq approximation is employed to compute density differences.

Body fitted coordinates (BFC) are used to represent the bath region to represent realistically the complex geometry of the real furnace.

The governing equations describing fluid flow include the conservation of energy, the equation of continuity and the turbulent equations of Navier–Stokes.

2.1. Governing Equations

The governing equations correspond to the conservation of energy, momentum and mass; all of them can be represented by the following general expression.
\[ \nabla \cdot \Gamma \phi \nabla \phi - \nabla \cdot (\rho \Gamma \phi) + S = -\frac{\partial \phi}{\partial t} \] 

Where:
- \( \phi \) = dependent variable, for example velocity and temperature.
- \( \Gamma \) = diffusion coefficient in kg \cdot m\(^{-1}\) \cdot s\(^{-1}\).
- \( S \) = source term(s) different to convective, diffusive or transient terms.

- **Mass Conservation (Continuity) for a Fluid of Constant Density**
  \[ \nabla \cdot \mathbf{v} = 0 \] 
  Where \( \mathbf{v} \) represents the velocity vector.

- **Momentum Conservation**
  For a fluid at constant pressure and density independent of temperature
  \[ \nabla \cdot \frac{k}{C_p} \nabla T - \nabla \cdot p \mathbf{v} T = \frac{\partial T}{\partial t} \] 
  Where: \( k \), \( C_p \), and \( \rho \) represent thermal conductivity, heat capacity and density of liquid steel, respectively.

- **Energetic Conservation**
  The source term for the turbulent kinetic energy, \( G \), is given below.
  \[ G = \mu_{eff} \left\{ \frac{1}{2} \left( \frac{\partial v_x}{\partial x} \right)^2 + \frac{\partial v_y}{\partial y} \left( \frac{\partial v_y}{\partial y} \right)^2 + \frac{\partial v_z}{\partial z} \left( \frac{\partial v_z}{\partial z} \right)^2 \right\}^{\frac{3}{2}} \] 

  The effective viscosity, \( \mu_{eff} \), is defined as:
  \[ \mu_{eff} = \mu + \mu_t \] 
  \( \mu \) is the molecular viscosity and \( \mu_t \) the turbulent viscosity, which is computed by combining \( k \) and \( \varepsilon \) as follows:
  \[ \mu_t = \frac{\rho C_{\mu} k^2}{\varepsilon} \] 
  Where: \( C_{\mu} \), \( C_t \), \( \sigma_k \), \( \sigma_\varepsilon \), and \( C_\mu \) are constants with the following values:
  \[ C_1 = 1.44, \ C_2 = 1.92, \ C_{\mu} = 0.09, \ \sigma_k = 1.0, \ \sigma_\varepsilon = 1.3 \]

2.2. **Boundary Conditions**

Figure 1 shows all boundaries of the computational domain, i.e. lateral and bottom walls and a top free surface that has three zones affected by the electric arcs named “arc attached zones”. A suitable coordinate system is also depicted for the “body fitted coordinates” employed in this work.

**Bottom Wall:** Zero velocities are assigned at this boundary due to the non-slip condition and impermeability of the crucible:
\[ v_x = v_y = 0 \text{ non slip condition} \]
\[ v_z = 0 \text{ impermeability} \]

Turbulence parameters \( k \) and \( \varepsilon \) are zero at the walls since close to the walls the flow regime has to be laminar. Extrapolation from these zero values at the walls to the fully turbulent regime in the bulk of liquid steel is given by the well known “universal wall functions”:
\[ k = 0 \text{ and } \varepsilon = 0 \text{ at the bottom wall} \]

Heat flows from the melt to the surroundings through the furnace walls. If we assume that the refractory lining defines this amount of heat and that the heat flow is one-dimensional perpendicular to the wall, then Fourier’s law may be used to compute the heat flow at the bottom wall. Temperature at the inner walls is assumed to be constant at \( 1600^\circ C (\text{1873} \text{K}) \) and the external temperature was set at a constant value of \( 300^\circ C (\text{573} \text{K}) \). Thus heat flow from the
bottom wall ($Q_{bw}$) through a MgO refractory ($k_{MgO} = 1.9 \text{ W/m} \cdot \text{K}$) of total area $A_b$ and a thickness of 45 cm, is computed as follows:

$$Q_{bw} = -k_{MgO} \cdot A_b \frac{\partial T}{\partial z} \ldots \ldots \ldots (14)$$

Lateral Wall: Similar to the bottom wall (Eq. (12)) zero velocities are also assigned at this wall due to the non-slip condition and impermeability of the crucible:

$$v_x = 0 \quad \text{non slip condition}$$
$$v_y = v_z = 0 \quad \text{impermeability} \ldots \ldots (15)$$

Turbulence is also zero. To connect these zero values with the fully turbulent regime in the liquid steel “universal wall functions” were also employed:

$$k=0 \quad \text{and} \quad \varepsilon=0 \quad \text{at the lateral wall} \ldots \ldots (16)$$

Heat flow from the melt to the surroundings through the lateral furnace walls ($Q_{lw}$) of area $A_w$ was computed similar to the bottom wall (Eq. (14)), as Eq. (17) shows below:

$$Q_{lw} = \left( -k_{MgO} \cdot A_w \frac{\partial T}{\partial x} \right) + \left( -k_{MgO} \cdot A_w \frac{\partial T}{\partial y} \right) \ldots \ldots (17)$$

Free Surface: Since no slag is considered in the system, the gas atmosphere above the melt does not shear the liquid steel. Then, shear ($\tau_{xy}$ and $\tau_{yz}$) from the gas phase to the liquid bath is set to be zero, as indicated below.

Zero shear at the free surface in the $y$-direction:

$$\tau_{xy} = -\mu_{eff} \frac{dv_y}{dz} = 0 \ldots \ldots (18)$$

Zero shear at the free surface in the $x$-direction:

$$\tau_{zx} = -\mu_{eff} \frac{dv_x}{dz} = 0 \ldots \ldots (19)$$

Liquid steel does not flow out at the free surface:

$$v_z = 0 \ldots \ldots (20)$$

Zero gradients of turbulence at the free surface:

$$\frac{\partial k}{\partial z} = \frac{\partial \varepsilon}{\partial z} = 0 \ldots \ldots (21)$$

Radiation exchange ($Q_{rad}$) from the free surface of the melt with an area $A_{ds}$ to the surroundings at a temperature $T_{surr}$ is defined as follows:

$$Q_{rad} = -\frac{A_{ds} \varepsilon'}{\cos \theta} (T^4 - T_{surr}^4) \ldots \ldots (22)$$

where $\sigma$ is the Stefan–Boltzmann’s constant ($\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$) and $\varepsilon'$ is the emissivity of the liquid steel (in this calculation $\varepsilon' = 0.8$).

Arc Attached Zones: The three arc attached zones are dissipating heat by radiation as the rest of the free surface (given by Eq. (22)) but additionally these zones receive a great amount of heat flowing from the electric arcs. For a given set of electric parameters (voltage and arc length), power delivery ($P_d$) represents a heat input, as indicated below:

$$-k_{eff} \frac{\partial T}{\partial z} = P_d \ldots \ldots (23)$$

Where $k_{eff}$ represents the effective thermal conductivity (the sum of molecular and turbulent conductivities) and $P_d$ is computed by the Channel Arc Model (CAM). Values of arc power ($P_d$) of 120 MW and 95 MW result when the power system operates at 1210 V with arc lengths of 45 cm and 25 cm respectively. A full description of this model is given elsewhere.16)

**Table 1** indicates the physical properties of molten steel used in the computations.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$7.2 \times 10^3 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>Specific heat, $c_p$</td>
<td>670 J/kg·K</td>
</tr>
<tr>
<td>Viscosity, $\mu$</td>
<td>$6.5 \times 10^{-3} \text{ kg/m} \cdot \text{s}$</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>15 W/m·K</td>
</tr>
<tr>
<td>Emittance of bath surface</td>
<td>0.8</td>
</tr>
<tr>
<td>Thermal coefficient of volume expansion,</td>
<td>$1.4 \times 10^{-4} \text{ K}^{-1}$</td>
</tr>
</tbody>
</table>

3. Results and Discussion

The commercial CFD code PHOENICS(17) version 3.4 was used to solve the complete set of partial differential equations, describing mass, energy and momentum conservation, using the technique volume of control (VOC). Fortran subroutines for mixing time calculations were developed and added to the main computational code.

An industrial electric arc furnace with a nominal capacity of 210 tons was employed to carry out the simulations. This furnace is equipped with a transformer of 155 MV·A. The system operates at 1 210 V with arc lengths of 45 cm and 25 cm respectively. A full description of this model is given elsewhere.16)

**Table 2** indicates the physical properties of molten steel used in the computations.

**Figure 2** shows velocity fields in a cross sectional view in the electric arc furnace due to thermal gradients, as a function of arc length. It is observed streamlines ascending from the bottom, until they reach the free surface, at this point the streamlines separate into two circulating loops in opposite directions, the right loop follows a clockwise direction. The eyes of these circulating loops are closer to the furnace walls. This pattern is opposite to DC electric arc furnaces. In a DC furnace, the flow also follows two circulating loops, however the streamlines are descending at the central axis, this behavior is due to the influence of...
electromagnetic forces on the flow. In AC furnaces the electromagnetic forces are negligible since the electric current does not flow through the bath and consequently contribute little to fluid flow. The maximum velocities are located in the middle of the central axis; these velocities slightly change with arc length, from 11.9 to 12.6 cm·s\(^{-1}\), when the arc length is changed from 25 to 45 cm.

Figure 3 shows velocity fields at the free surface from the top view for two conditions of arc length. It is observed that streamlines move in the radial direction, from the pitch circle towards the furnace walls. It can be noticed that the velocity vectors are fairly symmetric along the furnace, which means that the extended section towards the balcony does not show any particular difference with the rest of the furnace. This scenario has proven to change dramatically when additional forces are incorporated, due to oxygen lancing and gas bottom injection.

Figure 4 shows the temperature profiles of liquid steel in a cross sectional view at different arc lengths. In general, there are three temperature zones in the bath region based on both thermal gradients and temperature ranges; a cold zone with temperatures from 1 559–1 583°C, a hot zone with temperatures from 1 583–1 665°C and a superheated zone with temperatures from 1 665–1 750°C, this last zone also display hot spots of approximately 2 600°C which correspond to the arc attached zones.

Typically a short arc operation can be set below 25 cm, meanwhile a long arc operation is above 45 cm. It is quite interesting to observe how arc length can contribute to increase temperature homogenization. Operating under short arc conditions, there is the formation of a large volume of cold liquid steel, with temperatures ranging from 1 559–1 583°C, however if the arc length is increased this volume is drastically decreased.

The maximum thermal gradient between the cold and hot zones is in the range of 80–160°C. This thermal gradient is independent of the arc length; however the volume of the hot zone markedly increases when the arc length increases.
Such a large thermal gradient in the bulk of liquid steel is an indication of the poor mixing conditions prevailing in the electric arc furnace. It is evident that the stirring conditions in the Electric Arc Furnace are rather poor. Efforts to enhance mixing times are mandatory, however, similar techniques employed in the ladle furnace such as bottom gas injection are not efficient in the EAF due to the low height to diameter ratio, and probably a new furnace design with a higher aspect ratio would be a way to solve this problem. One of the main benefits of achieving temperature homogenization is a shorter melting time.

Table 2 summarizes the average velocities and temperatures as a function of arc length. The values shown in this table confirm the influence of arc length on fluid flow and heat transfer described previously in Figs. 2, 3 and 4.

Figure 5 shows the temperature profiles at the free surface of liquid steel in a top view at different arc lengths. In this view it can be observed the size of the superheated zone as a function of arc length. One of the important features is related to the position of injection of Direct Reduced Iron (DRI) particles. This position is located between phases 2 and 3, as close as possible to the pitch circle. The melting rate of DRI particles will be increased if those particles are injected in the superheated zone, which can be guaranteed if the furnace is operated under long arc length.

The following results have been obtained for the standard case, defined for an arc length of 45 cm and a maximum arc voltage of 1210 V.

Figure 6 shows the results for the convective heat transfer coefficient; $h$. Figure 6(a) shows a cross sectional view while Fig. 6(b) shows a top view of the furnace. Heat transfer coefficients were computed from the following relationship:

$$h = \frac{2\pi k}{d_p} \left( 2 + 0.6 \frac{Re^{1/2} \ Pr^{1/3}}{} \right)$$

Where: $d_p$ is the particle diameter, Re and Pr are the Reynolds and Prandtl numbers, respectively.

In this computation a mean particle size of 12 mm was used. The magnitude of the convective heat transfer coefficient is related to the extent of turbulence present in the system. The range reported in this work fluctuates from 0.5 to 2.2 cal/cm²·s·°C. Low stirring conditions correspond to 0.5–0.7 cal/cm²·s·°C, medium stirring conditions from 0.7–1.1 cal/cm²·s·°C, and high stirring conditions above 1.5 cal/cm²·s·°C. These values are of great importance in the melting process. The numbers are one order of magnitude above those reported by Elliot et al., however, in previous works no data has been reported in regard to the electric power supplied, furthermore Elliot’s experiments were carried out using a laboratory crucible with liquid slag. In our work an industrial furnace was employed containing liquid steel. In Fig. 6(b), it can be noticed the low values in the center of the pitch circle. This region is usually considered as one of the hottest zones in the furnace, however, in this zone the convective heat transfer coefficient is low, this...
is due to the low velocity fields.

The extent of turbulence can be assessed with the aid of several parameters, such as the magnitude of the turbulent kinematic viscosity or the turbulent kinematic energy. In this work, the turbulence criteria suggested by Szekely et al. has been followed, using a viscosity ratio. This ratio relates the effective viscosity with the molecular viscosity. Figure 7 shows lines of iso-viscosity ratios for the standard case. The results shown indicate a wide range, from \(2 \times 10^{-3} - 3 \times 10^{-2}\). As this ratio increases, is an indication of a much higher value of the turbulent viscosity with respect to molecular viscosity. It is observed that the viscosity ratio decreases, either, from the furnace center to the walls and from the top to the bottom. Szekely’s work reported values in the order of \(1 \times 10^3\) for a small plasma furnace. Their values were associated with strong convection and a high level of turbulent mixing, resulting in a fairly uniform temperature distribution. In our case, the magnitude for the ratio \(\mu_{eff}/\mu\) is much higher than the ratio reported by Szekely. These differences are attributed to different conditions of furnace size (1 m in radius) and arc power (18 MW) in Szekely’s work, in contrast to 8 m diameter and 120 MW for the industrial furnace of the present simulations. In spite of higher magnitudes for the viscosity ratio, the thermal stratification reported in this work suggests that those values are not representative of a well-stirred industrial furnace.

Mixing phenomena was analyzed by injecting a tracer and monitoring its concentration throughout the furnace in 10 different positions. Figure 8 shows the position of tracer injection as well as the position of 10 sensors. Figure 9 displays the typical concentration profiles resulting from a tracer injection, as a function of time. The shape of the curves gives information about the mixing mechanisms. In the circulation loop located below the point of injection, the tracer circulates due to convection. On the other hand, the sensors placed to the left (2, 4, 6, 8) experience mixing by diffusion and turbulence and yield a continuous increase in concentration over time.

For the purpose of defining the mixing time, every node was compared with the equilibrium concentration until steady state conditions were achieved. This analysis was carried varying the arc length. Figure 10 shows the mixing time as a function of arc length. It shows that as arc length increases there is a decrease in the mixing time, thus, adding another reason to operate with long arc length. The mixing time decreases from 9.5 min for a short arc operation to 9.0 min for a long arc operation.

4. Conclusion

Fluid flow and mixing phenomena have been investigated in a three phase AC-Electric Arc Furnace, for a configuration where liquid steel occupies the entire computational domain and when the main driving forces are due to buoy-
The main variable investigated was arc length. The results show a large dependency of fluid flow on arc length. It was observed the formation of two circulatory loops with maximum velocities in the order of 12 cm/s, liquid steel flows from the center to the walls, ascending in the central axis and descending at the walls. The balcony region doesn’t show any particular differences with the rest of the flow. Velocity fields are not sensitive to changes in arc length; however, the temperature profiles show a marked effect, in this case, as arc length increases the volume of hot and superheated steel increases. What is relevant of this effect is the possibility to inject DRI in the zone of superheated steel, operating under long arc conditions. Strong thermal stratification has also been confirmed in this reactor, especially when other forces such as mechanical stirring due to gas injection or CO formation are absent.

The computation of the convective heat transfer coefficient allowed distinguishing three regimes from low to high stirring energies, with values from 0.5 to 2.2 cal/cm²·s·°C, respectively. The central part of the pitch circle is superheated, however, due to the low velocity fields in that region, yields a low value for the convective heat transfer coefficient.

Finally, it has been shown that by increasing the arc length, the mixing time is decreased, which adds another reason to operate under long arc conditions.

REFERENCES