Microwave Penetration Depth in Materials with Non-zero Magnetic Susceptibility

Zhiwei PENG,¹⁾ Jiann-Yang HWANG,¹⁾ Joe MOURIS,²⁾ Ron HUTCHEON²⁾ and Xiaodi HUANG¹⁾

1) Department of Materials Science and Engineering, Michigan Technological University, Houghton, USA.

E-mail: jhwang@mtu.edu

2) Microwave Properties North (MPN), Deep River, Ontario, Canada.

(Received on July 14, 2010; accepted on August 10, 2010)

A simplified equation for determining the transverse electromagnetic mode (TEM) power penetration depth of microwaves in materials having both magnetic and dielectric response was derived. The penetration depths for a magnetite concentrate were calculated using this “full-response” equation, and a significant difference is shown compared with the penetration depth obtained using only the dielectric response (*i.e.*, assuming no imaginary part of complex relative permeability). The temperature dependence of the power penetration depth, up to 1 000°C, was determined using measured values of the real and imaginary parts of complex relative permittivity, ϵ'_r , ϵ''_r and permeability, μ'_r , μ''_r . The accurate determination of penetration depths can help in optimizing the dimensions of a load in a microwave furnace, producing more uniform heating under microwave irradiation and avoiding thermal runaway.

KEY WORDS: microwave irradiation; penetration depth; magnetic susceptibility; permittivity; permeability.

1. Introduction

Microwave processing of materials has been attracting more and more attention since the early 1980s.^{1–11)} The main advantage of this technology, compared with other conventional methods (*e.g.* infrared radiation or convective transfer), is that microwaves penetrate into materials, heating volumetrically, and significantly improving the heat transfer to the interior of a sample.¹²⁾ According to the equations for propagation of a TEM electromagnetic wave in materials, a fraction $1/e$ (that is, 63.2%) of the traveling wave energy (or power) is deposited in the material over the distance of α “TEM power penetration depth”, $D_p = 1/(2\alpha)$, where α is the TEM field attenuation factor.^{13,14)} This penetration depth of microwaves is a key parameter in evaluating microwave heating.

In conventional furnaces, heat is transferred by thermal wavelength electromagnetic radiation, and the penetration depth of infrared radiation ($f=10^{13}$ Hz) is very small (much less than 10^{-4} m) in the majority of solids.¹⁵⁾ Thus, only a very thin surface layer of the material will be heated, and heating of the remainder of the material will depend on heat transfer within the material (mainly heat conduction in the material). Since this process requires a relatively long time, rapid heating is difficult to attain in most materials using infrared heating. Conversely, in microwave heating, the penetration depth varies from meters to millimeters because the frequencies used for heating, generally 915 MHz and 2450 MHz, are much lower than those used for infrared heating. This means that, for a material that is appropriately sized relative to its penetration depth under microwave irradiation, large surface temperature gradients can be avoided.

Relatively uniform rapid heating (volumetric heating) is, therefore, much easier to achieve.

The penetration depth is influenced by both the electric and magnetic properties of non-metallic materials.^{16–19)} Microwave heating work has tended to focus on non-magnetic dielectric materials (*e.g.* ceramics), with limited attention paid to the heating of mixed property materials (*e.g.* magnetite and other ferrites) under microwave irradiation.^{19–24)} However, since magnetic properties can enhance absorption of microwaves, some research has been conducted to take advantage of this property.^{10,25–28)} A typical example is microwave assisted steelmaking, where magnetite concentrate is used as material.^{27,28)} Magnetite is known as a good absorber of microwaves, with rapid heating and great energy conservation being achieved,²⁸⁾ but the mechanism is not well quantified, as the high temperature microwave properties have not been measured! Although the determination of the TEM power penetration depth in non-magnetic materials has been reported, the measurements giving the high temperature penetration depth of microwave in materials with non-zero susceptibility (magnetic dielectrics) are still lacking.^{19,29,30)}

In this paper, a simple equation was developed to quantitatively determine the penetration depth of microwave in magnetic dielectrics, which has not been reported. Through determination of the penetration depth, the optimization of dimensions of materials in microwave heating can be achieved, improving the energy efficiency.

2. Microwave Heating Fundamentals

The following outlines the basic defined variables related

with the interaction between microwaves and materials. For non-magnetic dielectrics, the absorption of microwaves is related to its permittivity (ϵ), which can be defined as follows:

$$\epsilon = \epsilon_0 \epsilon_r = \epsilon_0 (\epsilon'_r - j\epsilon''_r) \dots\dots\dots(1)$$

where ϵ_0 is the permittivity of free space (8.854×10^{-12} F/m), and j is imaginary unit, $j^2 = -1$. The complex relative permittivity (ϵ_r) is used to describe the constitutive relation between the electric flux density and the electric field intensity in lossy dielectrics. It is comprised of two components: the real part of complex relative permittivity or the relative dielectric constant (ϵ'_r), and the imaginary part of complex relative permittivity or the relative dielectric loss factor (ϵ''_r). The real part of complex relative permittivity is a measure of the ability of the dielectrics to store electrical energy, while the imaginary part of complex relative permittivity represents the loss of electrical energy in dielectrics. The energy lost from the electric field to the dielectric is eventually converted into thermal energy or heat. Thus, for dielectrics with no magnetic properties, the imaginary part of complex relative permittivity determines the heating rate when microwave energy is applied.

For non-metallic magnetic materials, such as ferrites, the absorption of microwaves depends on both the permittivity and permeability (μ). The permeability is defined by the following equation:

$$\mu = \mu_0 \mu_r = \mu_0 (\mu'_r - j\mu''_r) \dots\dots\dots(2)$$

where μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ H/m), and μ_r is the complex relative permeability, which is used to describe the constitutive relation between the magnetic flux density and the magnetic field intensity where magnetic field based loss mechanisms exist. The complex relative permeability is also comprised of two components: the real part of complex relative permeability or the relative magnetic constant (μ'_r), and the imaginary part of complex relative permeability or the relative magnetic loss factor (μ''_r). The real part of complex relative permeability is a measure of the ability of the dielectrics to store magnetic energy, while the imaginary part of complex relative permeability represents the loss of magnetic field energy. The energy lost from the magnetic field is again eventually converted into thermal energy or heat. Thus, for magnetic materials, the imaginary part of complex relative permeability heavily influences the heating rate under microwave irradiation.

The ratio of the imaginary to real parts of the permittivity and permeability, define another parameter, the loss tangent ($\tan \delta$), which is commonly used to indicate the efficiency of conversion of microwave energy into thermal energy within the dielectrics. For non-magnetic materials, the dielectric loss tangent is defined as:

$$\tan \delta_\epsilon = \epsilon''_r / \epsilon'_r \dots\dots\dots(3)$$

Similarly, for magnetic materials, the magnetic loss tangent is defined as follows:

$$\tan \delta_\mu = \mu''_r / \mu'_r \dots\dots\dots(4)$$

Neglecting magnetic effects (*i.e.*, $\mu_r = 0$), the TEM power penetration depth, D_p , is defined as the distance from the surface into the dielectric at which the traveling wave power

drops to e^{-1} from its value at the surface.³⁰⁾ It can be expressed as

$$D_p = \frac{\lambda_0}{2\pi(2\epsilon'_r)^{1/2}} \left\{ \left[1 + \left(\frac{\epsilon''_r}{\epsilon'_r} \right)^2 \right]^{1/2} - 1 \right\}^{-1/2} \dots\dots\dots(5)$$

When the magnetic susceptibility is significant, as in magnetic dielectrics such as ferrites, this equation must be modified. Several formulations are available, but have not been expressed in simple terms. We have developed an appropriate simple form using Maxwell's equations.

3. Derivation of Simplified Microwave Power Penetration Equation

In a homogeneous medium and without convection or external currents, the differential Maxwell's equations in lossy dielectrics are given by

$$\nabla \times E = -j\omega\mu H \dots\dots\dots(6)$$

$$\nabla \times H = j\omega\epsilon E \dots\dots\dots(7)$$

$$\nabla \cdot E = 0 \dots\dots\dots(8)$$

$$\nabla \cdot H = 0 \dots\dots\dots(9)$$

where E is the electric field intensity (V/m), H is the magnetic field intensity (A/m) and ω is the angular frequency of microwave (rad/s).

Considering vector multiplication, for Maxwell's equations, we have

$$\begin{aligned} \nabla \times \nabla \times E &= \nabla(\nabla \cdot E) - \nabla^2 E \\ &= -j\omega\mu \nabla \times H = \omega^2 \mu \epsilon E = -\gamma^2 E \dots\dots(10) \end{aligned}$$

$$\begin{aligned} \nabla \times \nabla \times H &= \nabla(\nabla \cdot H) - \nabla^2 H \\ &= -j\omega\epsilon \nabla \times E = \omega^2 \mu \epsilon H = -\gamma^2 H \dots\dots(11) \end{aligned}$$

where γ is the propagation constant. Thus, we obtain

$$\nabla^2 E - \gamma^2 E = 0 \dots\dots\dots(12)$$

$$\nabla^2 H - \gamma^2 H = 0 \dots\dots\dots(13)$$

The two equations above are also known as Helmholtz equations. Assuming a TEM plane wave traveling in the z direction with x component of electric field intensity, Helmholtz equation would yield

$$\frac{d^2 E_x}{dz^2} - \gamma E_x = 0 \dots\dots\dots(14)$$

The general solution to this wave equation is

$$\begin{aligned} E_x(z) &= C_1 e^{-\gamma z} + C_2 e^{+\gamma z} \\ &= C_1 e^{-\alpha z} e^{-j\beta z} + C_2 e^{\alpha z} e^{j\beta z} \dots\dots\dots(15) \end{aligned}$$

where C_1 and C_2 are constants.

In time domain, the representation of E is

$$\begin{aligned} E_x(z, t) &= \text{Re}\{E_x(z) e^{j\omega t}\} \\ &= C_1 e^{-\alpha z} \cos(\omega t - \beta z) + C_2 e^{\alpha z} \cos(\omega t + \beta z) \dots\dots\dots(16) \end{aligned}$$

Here, the propagation constant (γ) is a complex number and can be expressed as

$$\gamma = \alpha + j\beta \quad \dots\dots\dots(17)$$

where α is the field attenuation factor (Np/m), and β is the phase constant (rad/m).

From the definition, we have

$$\begin{aligned} \gamma &= j\omega(\mu\epsilon)^{1/2} \\ &= j\omega(\mu_0\mu_r\epsilon_0\epsilon_r)^{1/2} \\ &= j\omega[\mu_0(\mu_r' - j\mu_r'')\epsilon_0(\epsilon_r' - j\epsilon_r'')]^{1/2} \\ &= [-\omega^2\epsilon_0\mu_0(\epsilon_r'\mu_r' - \epsilon_r''\mu_r'') + j\omega^2\epsilon_0\mu_0(\epsilon_r'\mu_r'' + \epsilon_r''\mu_r')]^{1/2} \end{aligned} \quad \dots\dots\dots(18)$$

By separating the real part and imaginary part of Eq. (18) and equating the real part with α , we have

$$\begin{aligned} \alpha &= \omega \left(\frac{\epsilon_0\mu_0}{2} \right)^{1/2} \{ \epsilon_r''\mu_r'' - \epsilon_r'\mu_r' + [(\epsilon_r'\mu_r')^2 + (\epsilon_r''\mu_r'')^2 \\ &\quad + (\epsilon_r'\mu_r'')^2 + (\mu_r'\epsilon_r'')^2]^{1/2} \}^{1/2} \\ &= 2\pi f \left(\frac{\epsilon_0\mu_0}{2} \right)^{1/2} \{ \epsilon_r''\mu_r'' - \epsilon_r'\mu_r' + [(\epsilon_r'\mu_r')^2 + (\epsilon_r''\mu_r'')^2 \\ &\quad + (\epsilon_r'\mu_r'')^2 + (\mu_r'\epsilon_r'')^2]^{1/2} \}^{1/2} \\ &= \frac{\sqrt{2}\pi}{\lambda_0} \{ \epsilon_r''\mu_r'' - \epsilon_r'\mu_r' + [(\epsilon_r'\mu_r')^2 + (\epsilon_r''\mu_r'')^2 \\ &\quad + (\epsilon_r'\mu_r'')^2 + (\mu_r'\epsilon_r'')^2]^{1/2} \}^{1/2} \end{aligned} \quad \dots\dots\dots(19)$$

The wave is attenuated as it traverses the medium and therefore the power is dissipated. According to the definition of the (1/e) power penetration depth, we obtain

$$\begin{aligned} D_p &= \frac{1}{2\alpha} \\ &= \frac{\lambda_0}{2\sqrt{2}\pi} \{ \epsilon_r''\mu_r'' - \epsilon_r'\mu_r' + [(\epsilon_r'\mu_r')^2 + (\epsilon_r''\mu_r'')^2 \\ &\quad + (\epsilon_r'\mu_r'')^2 + (\mu_r'\epsilon_r'')^2]^{1/2} \}^{-1/2} \end{aligned} \quad \dots\dots\dots(20)$$

or, in terms of loss tangent,

$$\begin{aligned} D_p &= \frac{\lambda_0}{2\pi(2\epsilon_r'\mu_r')^{1/2}} [(1 + \tan^2\delta_\epsilon \tan^2\delta_\mu + \tan^2\delta_\mu \tan^2\delta_\epsilon)^{1/2} \\ &\quad + \tan\delta_\epsilon \tan\delta_\mu - 1]^{-1/2} \end{aligned} \quad \dots\dots\dots(21)$$

Equation (20) is, therefore, the equation for the determination of penetration depth of magnetic dielectrics. Assuming $\mu_r''=0$, Eq. (20) simplifies to

$$D_p = \frac{\lambda_0}{2\pi(2\epsilon_r')^{1/2}} \left\{ \left[1 + \left(\frac{\epsilon_r''}{\epsilon_r'} \right)^2 \right]^{1/2} - 1 \right\}^{-1/2} \quad \dots\dots\dots(22)$$

which is the same as Eq. (5). For magnetic dielectrics, Eq. (20) should be applied in the calculation of the power penetration depth.

4. Cavity Perturbation Measurement Techniques

Since both permittivity and permeability are important to characterize the penetration depth of microwave in magnetic materials, both parameters were measured at MPN using “cavity perturbation technique”^{31–34)} (see www.mpn.ca for details). This method measures the difference in the microwave cavity response between a cavity with an empty sample-holder and the same cavity with a sample-holder plus the sample, and uses this to calculate either the permittivity or permeability, depending on the field type (electric or magnetic) in the region of the cavity in which the sample is placed. The technique measures the frequency shift and the change in loaded Q of a resonant mode of the cavity caused by inserting a sample.

4.1. Permittivity Measurements

The permittivity was measured by placing the sample in the central region of maximum electric field in a TM_{0n0} cavity (Fig. 1), and measuring the frequency shift and quality factors. The electric susceptibility ($\chi_e = \chi_e' - j\chi_e''$) can be calculated through the following equation:

$$\frac{\Delta f}{f_e} + j \left(\frac{1}{2Q_{L,S}} - \frac{1}{2Q_{L,E}} \right) = \frac{-\chi_e}{1 + F_{sh}\chi_e} \left(\frac{V_s}{V_c} \right) A \dots\dots(23)$$

where, in the present case, f_e is the specific cavity mode frequency ($915 \pm 0.5\%$ MHz or $2450 \pm 0.5\%$ MHz), $Q_{L,E}$ is the loaded cavity quality factor with the empty holder, Δf is the frequency shift produced by the sample, $Q_{L,S}$ is the loaded cavity Q with the holder and sample, F_{sh} is a real number dependent only on the sample shape, V_s and V_c are the respective sample and cavity volumes, A is a real calibration constant dependent only on the shape of the electric

Fig. 1. Schematic diagram of the TM_{0n0} cavity system (in cross-section) showing the linear actuator with the quartz sample holder and a sample located on axis in the centre of the cavity. The quartz base that supports the sample in the holder has a small hole in it to allow gas flow upwards using a (not shown) metered gas flow control system. The components of the system: (1) high temperature resistance furnace, (2) entry hole, (3) TM_{0n0} right cylindrical cavity, (4) sample in quartz holder, (5) quartz sample holder, (6) sample stage on linear actuator, and (7) support stand.

fields in the absence of the sample. The two calibration constants in the formula, A and F_{sh} can be determined either by comparison with computer simulations or by comparison with known samples. Solid or liquid calibration samples are used, depending upon whether the sample material is liquid (or a tamped-in powder) or solid (carved or core-drilled sample or pressed pellets). The absolute calibration is normally done at room temperatures, and uses measurements on known high purity materials (similar to ASTM Standard Test Method, D2520-86, Method C).

It should be noted that for very large values of electric susceptibility, χ_e , the value of the fractional frequency shift approaches an asymptotic value $(-A/F_{sh}) \cdot (V_s/V_c)$. Thus the fractional error in determination of the susceptibility is proportional to the susceptibility.

The measurement system is illustrated in **Fig. 1**. The main system components are a high temperature resistance furnace and a cylindrical TM_{0n0} cavity with diameter of 580 mm and length of 50 mm. To do a measurement at a specific temperature, the top section of the holder which has the sample in it is raised into the furnace, and held at least 5 min to ensure that the equilibrium furnace temperature is achieved in the sample. Then the hot sample and holder are rapidly lowered into the central, maximum electric field, region of thick-walled, well-cooled copper TM_{0n0} cavity. The resonant frequencies and loaded quality factor, $Q_{L,S}$, of the cavity modes are rapidly measured by a Hewlett-Packard 8753 network analyzer and stored for off-line analysis (which includes subtraction of hot empty sample holder effects which were previously measured). The sample and holder are typically out of the furnace for ~ 1.5 s for each frequency measured—in the present case of two frequencies, which means about 3 s out of the furnace. The sample temperature decrease during this period is small (typically <10 degrees up to 1000°C) and the cooling rates are known and included in the calculation of the exact temperature of a measurement. The present permittivity measurements were done from room temperature (24°C) in $\sim 50^\circ\text{C}$ steps to 1030°C , and then cooled down.

After the completion of the sequence of measurements, the mass and dimension of the sample are re-measured. The whole measurement sequence is pre-programmed and computer-controlled over the desired set of temperatures, with furnace, linear motion actuator (sample movement), and network analyzer all controlled by a PC running Labview control software.

4.2. Permeability Measurements

The complex permeability was measured using a ridge-loaded-WR284 waveguide-based resonant cavity³⁴⁾ which produces a region of maximum magnetic field strength at the shorted end of the waveguide. The sample was moved rapidly into this maximum field, and again the frequency shift and change in loaded Q are measured. The complex permeability is calculated using a formula identical in form to Eq. (23), but with the appropriate frequency and Q and shape factors and calibration constants for the magnetic cavity.

The permeability measurements are, in practice, less accurate than permittivity measurements because large corrections have to be made for the frequency and Q shift in

the magnetic cavity that are caused by the small, but significant, electric field that exists in the high magnetic field region. Since the complex dielectric constant is almost always much larger (typically 10 times) than the complex permeability, the small electric field causes frequency shifts and Q changes that must be taken into account. Thus the magnetic measurements require subtractions that depend upon reliable values at each temperature of the complex permittivity. Only after these corrections have been made, can the complex magnetic susceptibility χ_m , be determined by using Eq. (24):

$$\frac{\Delta f}{f_m} + j \left(\frac{1}{2Q_{L,S}} - \frac{1}{2Q_{L,E}} \right) = \frac{-\chi_m}{1 + F_{sh}\chi_m} \left(\frac{V_s}{V_c} \right) A \dots (24)$$

where f_m is the cavity mode frequency in permeability test, and the Δf and Q are only from the magnetic field induced contribution. Even after these corrections, other problems exist in the interpretation of the apparent measured magnetic susceptibility. If the sample has a moderately large high-frequency electrical conductivity (which is determined by the permittivity measurements), the time-varying microwave magnetic field can induce electrical fields in the sample that produce azimuthal induced currents in the sample, completely analogous to low frequency induction heating. This represents another mechanism of power loss in the sample, and produces power loss and a reduction in Q . However, these losses are mainly due to the electrical conductivity of the sample, which is part of the imaginary permittivity. These “induced electric field” losses are not significant in the permittivity measurements because of the very low magnetic field in the central region of the TM_{0n0} cavity, relative to the electric field.

Another problem occurs if the sample conductivity is very high, as with an almost metallic sample. If the conductivity is such that the skin depth is small relative to the diameter of the sample, this implies that the induced currents produce a “bucking” magnetic field in the sample interior that exactly counters the cavity magnetic field. In this way, all magnetic fields are excluded from the interior of the sample. This is the typical Lenz Law situation, and implies an “apparent” magnetic susceptibility, $\chi_m = -1$, or a value $\mu_r = 0$ if interpreted incorrectly using Eq. (24). These problems can only be avoided by a detailed knowledge of the physical and solid state properties of the sample at all temperatures.

5. Results and Discussion

A typical ferromagnetic material, magnetite concentrate, obtained from the Tilden Mine in Michigan, was used to demonstrate the difference between the values of penetration depth calculated by Eq. (5) and Eq. (20). The phase compositions were determined using a Scintag XDS2000 powder X-Ray diffractometer with a graphite monochromator and $\text{Cu K}\alpha$ radiation and the X-Ray diffraction pattern is shown in **Fig. 2**. The analysis shows the sample contains 3 phases, mainly magnetite (Fe_3O_4 , JCPDS card: 79-0419) with minor amount of quartz (SiO_2 , JCPDS card: 88-2302) and siderite (FeCO_3 , JCPDS card: 29-0696).

In the permittivity measurement, magnetite concentrate

powders with particle size less than 0.075 mm were uniaxially pressed at about 172 350 kPa in a die lined with tungsten carbide to form 3 pellets with a diameter of about 3.63 mm and a total, stacked length (height) of 13.75 mm. The bulk density (room temperature) of the pellets was 2.77 g/cm³. During the measurements, the pellets were step-heated in a resistance furnace to the designated temperatures in 0.01 L/min flowing argon. The XRD (not shown in the paper) after the measurement confirmed that no main phase changed during the heating. In the permeability experiment, the same punch/die unit was used to form pellets with a diameter of about 3.65 mm and length of 14.6 mm. The bulk density of pellets (room temperature) was 2.84 g/cm³. The permeability measurements started at room temperature (24°C) and used ~50°C steps to 850°C. The argon gas flow was the same as that for permittivity test.

The measured values of the real and imaginary parts of complex relative permittivities and permeabilities of magnetite concentrate under microwave irradiation are shown in Fig. 3 and Fig. 4, respectively.

In order to characterize the temperature dependence of the permittivity and permeability of magnetite, the curves in Fig. 3 and Fig. 4 were fitted to a polynomial equation of high degree (6), given as

$$f(T) = a + bT + cT^2 + dT^3 + eT^4 + fT^5 + gT^6 \dots (25)$$

where $f(T)$ represents the temperature (T , °C) dependent per-

mittivity (ϵ_r' and ϵ_r'') or permeability (μ_r' and μ_r''), and the coefficients, a, b, c, d, e, f, g , are constants. The variations of permittivity and permeability with increasing temperature were characterized through the determination of the function. The fitted parameters are listed in Table 1.

The experimental data of relative permittivities and permeabilities were used to calculate the power penetration

Fig. 3. Real and imaginary parts of complex relative permittivities of magnetite concentrate pellets (2.77 g/cm³ density) as a function of temperature.

Fig. 2. X-Ray diffraction pattern of magnetite concentrate.

Fig. 4. Real and imaginary parts of complex relative permeabilities of magnetite concentrate pellets (2.84 g/cm³ density). Both decrease dramatically around 575°C. The Curie point of magnetite is 585°C.³⁵⁾

Table 1. Values of constants in the polynomial function (for ϵ_r and μ_r) represented by Eq. (25).

$f(T)$	a	b	$c/\times 10^{-4}$	$d/\times 10^{-7}$	$e/\times 10^{-9}$	$f/\times 10^{-12}$	$g/\times 10^{-16}$	R^2
ϵ_r' (915MHz)	3.762	0.382	-40.600	169.204	-31.676	27.479	-89.103	0.978
ϵ_r' (2450MHz)	12.482	0.005	-1.838	10.562	-2.016	1.998	-7.651	0.985
ϵ_r'' (915MHz)	-1.086	0.085	-8.021	27.600	-3.966	3.343	-12.819	0.995
ϵ_r'' (2450MHz)	-3.307	0.187	-19.900	84.768	-16.551	15.358	-53.557	0.994
μ_r' (915MHz)	1.504	0.0113	-1.229	5.758	-1.273	1.293	-4.887	0.890
μ_r' (2450MHz)	1.461	0.00234	-0.225	1.139	-0.312	0.390	-1.751	0.828
μ_r'' (915MHz)	0.00938	0.0114	-1.289	5.920	-1.238	1.185	-4.230	0.467
μ_r'' (2450MHz)	0.0478	0.0109	-1.191	5.559	-1.215	1.231	-4.681	0.473

Table 2. Values of constants in the polynomial function (for D_p) represented by Eq. (25).

D_p	a	$b/\times 10^{-5}$	$c/\times 10^{-7}$	$d/\times 10^{-9}$	$e/\times 10^{-11}$	$f/\times 10^{-14}$	$g/\times 10^{-18}$	R^2
915MHz	0.0471	-0.453	4.184	-4.845	1.298	-1.366	5.099	0.921
2450MHz	0.0161	-1.863	2.182	-1.603	0.395	-0.405	1.502	0.901

Fig. 5. Calculated microwave power penetration depth (meter) in the magnetite concentrate at density of about 2.8 g/cm³.

depths as a function of temperature at 915 MHz and 2450 MHz (**Fig. 5**). Below 600°C, the penetration depths determined by Eq. (20) are much smaller than those calculated by Eq. (5) as expected when the magnetic losses are included. Above 600°C, the difference between penetration depths determined by the two equations becomes negligible, reflecting the fact that the magnetic absorption is very small (in principle, zero) above the Curie point of magnetite (585°C). For example, at 780°C and 915 MHz, the penetration depths determined by Eq. (5) and Eq. (20) are 0.00505 m and 0.00491 m, respectively. This shows that the permeability should be considered in the determination of penetration depth of magnetic dielectrics, especially at temperatures below the dielectric's Curie point.

It is useful to characterize the penetration depth with increasing temperature by curve-fitting the values calculated by Eq. (20). The temperature (T , °C) dependent penetration depth (D_p , meters) can be expressed in the same function represented by Eq. (25). **Table 2** lists the values of constants yielding the polynomial function of penetration depth in meters.

As can be seen from Fig. 5, the optimum dimension for uniform heat deposition in a sample being irradiated from both sides in a 2450 MHz microwave field is about 0.03 m (approximately two power penetration depths). The deposited microwave energy can be relatively uniformly distributed by double-sided irradiation in this dimension, and rapid relatively uniform temperature increases can be achieved. This is important since, with a larger sample, there will be obvious temperature gradients. In a smaller sample, a central hot spot may be produced by surface cooling. The accurate determination of penetration depth helps to optimize the load size in the microwave applicator.

6. Conclusions

A simplified equation was derived, including the effects of permeability, with which to calculate the penetration depth of microwaves in materials with non-zero susceptibility. Using measured values of the permittivity and permeability of magnetite concentrate, the difference between the penetration depth values determined by with and without consideration of the permeability were demonstrated. The difference is large at low temperature (particularly below the Curie point of the magnetic dielectrics). The equation can be used to easily determine the penetration depth of magnetic dielectrics, and help guarantee the volumetric heating of dielectrics under microwave irradiation.

Acknowledgements

This work was supported by Michigan Public Service Commission, U.P. Steel, and the United States Department of Energy (DOE). Microwave Properties North (www.mpn.ca) is acknowledged for performing the permittivity and permeability measurements.

REFERENCES

- 1) D. Adam: *Nature*, **421** (2003), 571.
- 2) Y. V. Bykov, K. I. Rybakov and V. E. Semenov: *J. Phys. D: Appl. Phys.*, **34** (2001), R55.
- 3) Y. Fang, J. Cheng and D. K. Agrawal: *Mater. Lett.*, **58** (2004), 498.
- 4) P. Mishra, G. Sethi and A. Upadhyaya: *Metall. Mater. Trans. B*, **37** (2006), 839.
- 5) M. Huang, J. Peng, J. Yang and J. Wang: *J. Phys. D: Appl. Phys.*, **39** (2006), 2255.
- 6) N. Yoshikawa, E. Ishizuka, K. Mashiko and S. Taniguchi: *Metall. Mater. Trans. B*, **38** (2007), 863.
- 7) D. Malmberg, P. Hahlin and E. Nilsson: *ISIJ Int.*, **47** (2007), 533.
- 8) Y. Makino: *ISIJ Int.*, **47** (2007), 539.
- 9) T. Kuroki, Y. Uchida, H. Takizawa and K. Morita: *ISIJ Int.*, **47** (2007), 592.
- 10) Q. Yang, H. Zhang, Y. Liu, Q. Wen and L. Jia: *Mater. Lett.*, **62** (2008), 2647.
- 11) W. Madhuri, M. P. Reddy, N. R. Reddy, K. V. S. Kumar and V. R. K. Murthy: *J. Phys. D: Appl. Phys.*, **42** (2009), 165007.
- 12) F. Guo, G. Ji, X. Chang, M. Zheng, Y. Shi and Y. Zheng: *Nanotechnology*, **21** (2010), 035606.
- 13) U. S. Inan and A. S. Inan: *Electromagnetic Waves*, Prentice Hall, Upper Saddle River, NJ, (1999), 48.
- 14) W. H. Hayt and H. A. Buck: *Engineering Electromagnetics*, McGraw-Hill, New York, (2001), 371.
- 15) M. A. Harahsheh and S. W. Kingman: *Hydrometallurgy*, **73** (2004), 189.
- 16) D. M. Mingos and D. R. Baghurst: *Chem. Soc. Rev.*, **20** (1991), 1.
- 17) E. T. Thostenson and T. W. Chou: *Appl. Sci. Manuf.*, **30** (1999), 1055.
- 18) Y. P. Wu, C. K. Ong, G. Q. Lin and Z. W. Li: *J. Phys. D: Appl. Phys.*, **39** (2006), 2915.
- 19) C. A. Pickles: *Miner. Eng.*, **29** (2009), 1102.
- 20) J. D. Ford and D. C. Pei: *J. Microwave. Power.*, **2** (1967), 61.
- 21) T. T. Chen, J. E. Dutrizac, K. E. Haque, W. Wyslouzil and S. Kashyap: *Can. Metall. Q.*, **23** (1984), 349.
- 22) Y. Hua, Z. Lin and Z. Yan: *Miner. Eng.*, **15** (2002), 451.
- 23) E. B. Kulumbaev, V. E. Semenov and K. I. Rybakov: *J. Phys. D:*

- Appl. Phys.*, **40** (2007), 6809.
- 24) D. K. Xia and C. A. Pickles: *CIM Bull.*, **90** (1997), 96.
 - 25) G. Mu, N. Chen, X. Pan, H. Shen and M. Mu: *Mater. Lett.*, **62** (2008), 840.
 - 26) Y. Kang, M. Cao, J. Yuan and X. Shi: *Mater. Lett.*, **63** (2009), 1344.
 - 27) J. Y. Hwang, X. Huang, S. Qu, Y. Wang, S. Shi and G. Caneba: EPD Cong. Proc., TMS Publications, Warrendale, PA, (2006), 219.
 - 28) Y. Wang: PhD Dissertation, Michigan Technological University, Michigan, (2005), 110.
 - 29) D. E. Clark, D. C. Folz and J. K. West: *Mater. Sci. Eng A: Struct.*, **287** (2000), 153, 287.
 - 30) A. C. Metaxas and R. J. Meredith: Industrial Microwave Heating, Peter Peregrinus, London, (1983), 80.
 - 31) C. A. Pickles, J. Mouris and R. M. Hutcheon: *J. Mater. Res.*, **20** (2005), 18.
 - 32) R. M. Hutcheon, M. de Jong and F. P. Adams: *J. Microwave Power E.*, **27** (1992), 87.
 - 33) P. Adams, M. de Jong and R. M. Hutcheon: *J. Microwave Power E.*, **27** (1992), 131.
 - 34) R. M. Hutcheon, G. F. Morin, B. H. Smith, M. S. de Jong and F. P. Adams: *Ceram. Trans.*, **36** (1993), 469.
 - 35) P. C. Morais, E. C. Lima, D. Rabelo, A. C. Reis and F. Pelegrini: *IEEE Trans. Magn.*, **36** (2000), 3038.