1. Introduction

Japanese iron and steelmaking industry has to reduce CO₂ emission by 10.5% in 2010 relative to the level of emissions in 1990. Stable blast furnace operation is required to reduce energy consumption and CO₂ emission in iron and steelmaking industry. For the stable blast furnace operation, precise controlled drainage is one of the important factors. Many researchers attempted to solve the issues by using experimental and mathematical models. However, there are many problems to understand drainage behavior in a blast furnace hearth because several important hearth values such as coke diameter, void fraction, temperature of iron and slag in the hearth, and levels of iron and slag cannot be measured during a tap. Because of these issues, there are many unrevealed phenomena in the blast furnace hearth.

In a previous paper, we developed a three-dimensional mathematical model, which can predict gas–slag and slag–iron interfaces shapes, total drainage rate, iron and slag drainage rates during a tap affected by various hearth conditions. The effect of various in-furnace conditions on drainage rates, gas–slag and slag–iron interfaces shapes and maximum gas–slag interfaces height were examined in another previous paper. In addition, the effect of iron and slag dripping pattern, FeO concentration in the dripping slag on the iron and slag surfaces, thermal properties of refractory and brick on drainage temperature, temperature distribution in the hearth, temporal variation of iron and slag drainage rates and interfaces shapes were investigated by using the tree-dimensional mathematical model. However, interactions between dripping iron, molten slag and coke in the packed bed were not implemented in the mathematical model because there were not enough reliable experimental data.

Therefore in this work, cold model experiments were conducted to investigate the dripping behavior of molten iron in the coke packed bed filled with molten iron in packed coke beds filled with molten slag. The dripping behaviors were described by three comparatively simple mathematical models.

2. Dripping Behavior of Pseudo-iron Droplet at Small Supply Rate

2.1. Experimental Setups for Relatively Small Supply Rate

Figure 1 shows a schematic diagram of an experimental apparatus to investigate the dripping behavior of molten iron in the coke packed bed filled with molten slag. The dripping behavior or dripping rate of pseudo-iron droplets in packed beds filled with immiscible liquid was described well by three comparatively simple mathematical models.

KEY WORDS: iron and slag flow; immiscible liquid; blast furnace hearth; ironmaking; packed coke bed.
2.2. Dripping Behavior of Droplet

At first, dripping behavior of droplets in the immiscible fluid in the absence of the packed particles was observed, and the dripping rates of the droplets were measured by changing the droplet diameter. Then, dripping behavior of droplets in the packed bed filled with the immiscible fluid was observed, and the dripping rates of the droplets were measured by changing the droplet diameter and the packed particles diameter.

2.2.1. Dripping Behavior of Droplet in Immiscible Fluid

When the diameter of a droplet is small enough, the shape of the droplet can be approximated as a sphere. The dripping behavior of the droplet is very similar to that of a solid particle dripping in a fluid.

When a particle falls in a fluid, the gravitational force, buoyancy force, viscous resistance will act on the particle. Therefore, equation of motion is described by Eq. (1).

\[
\rho Au = (\rho - \rho')Vg - R = 0. \quad (1)
\]

where, \( \rho \) is the density of the particle (kg/m\(^3\)), \( \rho' \) is the density of the fluid (kg/m\(^3\)), \( V \) is the volume of the particle (m\(^3\)), \( a \) is the acceleration of the particle (m/s\(^2\)), \( g \) is acceleration of gravity (m/s\(^2\)), \( R \) is the resistance (kg/s\(^2\)). The resistance, \( R \), is a function of the dripping rate of the particle. The equation of motion can be rewritten by Eq. (2) in steady state.

\[
(\rho - \rho')Vg - R = 0 \quad (2)
\]

where, \( R \) in Eq. (2) is a function of Re number, and \( R \) can be estimated by the following 3 equations according to Re number.

\[
Re < 2; \quad R = 3\pi \mu uD \quad \text{(Stokes' s equation)}
\]

\[
2 < Re < 500; \quad R = \frac{5}{4} \pi (uD)^{5/2} \sqrt{\mu \rho'} \quad \text{(Allen’s equation)}
\]

\[
500 < Re < 10^5; \quad R = 0.055 \pi \rho' (uD)^{3/2} \quad \text{(Newton’s equation)}
\]

where \( u \) (m/s) is the dripping rate of the droplet.

Figure 2 shows comparison of measured and calculated dripping rate of droplet (HCFC) in the immiscible fluid (Paraffin). The dripping rate of the droplets can be estimated by the Stokes’ (Eq. (3)) and Allen’s (Eq. (4)) equation according to the Re number. In the region where the Allen’s equation can be applied, the measured dripping rates show slightly lower values than the rates estimated by Eqs. (2) and (4) because of the change in the droplet shape due to the large viscous resistance.

2.2.2. Dripping Behavior of Droplets in Packed Beds Filled with Immiscible Fluid

Figure 3 shows examples of the observations with particles (glass beads) of 4 mm and 1 mm in diameter, respectively. In the case of 4 mm in particle diameter, the HCFC droplets dripped along the particles in the packed bed in the shape of droplet, and the diameters of the droplets were slightly smaller than the space formed by the particles. On the other hand, in the case of 1 mm in particle diameter, the shape of the HCFC was not a droplet. The HCFC moved downward with filling a gap between the particles rather than dripping in the packed bed. The shape of dripping molten iron in coke packed beds filled with molten slag supposed to be affected by iron droplet diameters and coke diameters in the packed beds. In other words, the dripping rate of molten iron in the coke packed beds strongly depends on the iron droplet diameters and the coke diameters. In both cases, the HCFC drops relatively just below the initial dripping position.

2.3. Modeling of Dripping Behavior in Packed Beds Filled with Immiscible Fluid

2.3.1. Dripping Behavior of Droplet

According to the decrease of the packed particles diameter, increase of the dripping rate, shape of the dripping droplet changed from “spherical droplet” to “string shape” or “moving downward with filling a gap between the packed particles rather than dripping in the packed beds”. When the shape of the droplets is spherical, the droplets drip in the following manner.

We consider regularly-arranged closest packing structure consists of iterative 3 layers (ABCABC···) which is shown in Fig. 4. When a droplet drips into the aperture consists of three particles, the droplet goes through the aperture, and then, falls on the top of a particle just below the aperture. The possible path of the droplet is limited to 3 directions at
this time (Fig. 5), and one of the directions will be selected stochastically. When the diameter of the droplet is smaller than the size of the aperture, the droplet which passes through the aperture will fall on just before the center of the particle because the packed bed is filled with fluid having very large viscosity. Then, the possible path of the droplet is limited to 2 directions. When the droplet contacts with the next particle, the possible path of the droplet is limited to 1 direction because the droplet falls on just before the center of the particle. At the result, the droplet drips in the packed bed in a spiral manner.

We also consider regularly-arranged closest packing structure consists of iterative 2 layers (ABAB···) which is shown in Fig. 6. A droplet drips in the packed bed in the same manner as in the ABCABC type packed bed for the first and/or second layers, after that, the droplet reaches to the apertures which consist of 3 particles, and selectively-drips into the apertures.

2.3.2. Modeling of Dripping Behavior

The dripping behaviors of the droplet in the packed beds were divided into 2 ways when the supply rate is less than a certain rate. Therefore, we consider 2 models to predict the dripping rate of the droplet in the packed bed.

(1) Lost Time Estimation Due to Collision

We defined the lost time, $\Delta t$ (s), which is the time difference required to drip a specific distance with and without the packed particles. When the number of collision to drip the specific distance is $n$ (—), and an additional time due to 1 collision is $t'$ (s), the lost time $\Delta t$ is described by $nt'$ as shown in Fig. 7. When the dripping rate in the fluid without packed particles is $u$, and the dripping rate in the fluid with packed particles is $u'$ (m/s), the dripping distance is $L$ (m), the lost time $\Delta t$ is described by Eq. (6).

$$\Delta t = L/u' - L/u$$

(6)

When the closest packed structure is considered, the number of collisions is given by Eq. (7) as shown in Fig. 8.

$$n = \frac{L}{\sqrt{2/3}D}$$

(7)

where, $D$ (m) is the diameter of packed particles.

The average dripping rate of the droplet in the packed bed $u'$ (m/s) is given by Eq. (8) by considering an actual void fraction $\epsilon'$ (—) and an ideal void fraction of the closest packed structure $\epsilon$ (—).

$$\frac{1}{u'} = \frac{1}{u} + \frac{t'}{\sqrt{2/3}D\epsilon'}$$

(8)

Thus, the average dripping rate in the packed bed $u'$ will be obtained by estimating an additional time due to 1 collision which depends on the ratio of droplet diameter and packed particles diameter.
(2) Droplet Diameter Smaller Than Apertures (Model A)

The schematic view of Model A when the droplet diameter is smaller than the size of aperture which consists of 3 particles is shown in Fig. 9. This model supposes the droplet slips on the packed particles. This dripping behavior was observed, even the droplet diameter is relatively large, when the size of the apertures is large enough compared to the droplet.

Figure 10 shows the schematic view of the forces act on the droplet. The equation of motion of the droplet is given by Eq. (9). The effect of interfacial forces was neglected because the forces ought to be relatively small compared to the other terms.

\[ \rho V a = (\rho - \rho') V g \sin \theta - R \] ..........................(9)

If the droplet is small enough, the dripping rate reaches the rate at steady-state. When \( \theta (\text{rad}) \) is equal to \( \pi/2 \), the droplet departs from the particle, and the dripping rate reaches the terminal velocity in the fluid without packed particles. Therefore, the additional time \( t' \) corresponds to the time from the droplet falls on the particle to the droplet departs from the particle. The additional time is described by Eq. (10) from dripping rate without packed particles \( u \), and the initial contact angle \( \theta_0 \) (rad).

\[
t' = \int_{\theta_0}^{\pi/2} \left( \frac{D/2}{u \sin \theta} \right) d\theta - \frac{D \cos \theta_0}{2u} \]

\[
= \frac{D}{2u} \left\{ - \ln \left( \tan \left( \frac{\theta_0}{2} \right) \right) - \cos \theta_0 \right\} ............(10)
\]

When the droplet diameter is smaller than the size of the apertures, the droplet does not fall on the top of the particle but falls on the position where the angle is \( \theta_0 \) as shown in Fig. 11. The initial angle \( \theta_0 \) is calculated by Eq. (11).

\[
\sin \theta_0 = \frac{D(1/\sqrt{3} - 1/2) - d/2}{D/2} ............(11)
\]

The average dripping rate \( u' \) is affected by both the droplet diameter and packed particles diameter.

(3) Droplet Diameter Larger Than Apertures (Model B)

The schematic view of Model B when the droplet diameter is larger than the size of aperture, which consists of 3 particles, is shown in Fig. 12. This model supposes the droplet drips in the aperture by changing the shape. This dripping behavior was observed when the diameter of the packed particles was small and the droplet diameter was relatively large. Especially, when the diameter of the packed particles was less than 2 mm, the droplet showed this behavior.

Figure 13 shows the schematic illustration of Model B. When the aperture, which consist of 3 particles, is approximated by a fine tube, the required time \( t \) (s) for the droplet to pass through the tube, is given by Eq. (12).

\[
\mu = \frac{\pi g \rho h r^4 t}{8 V} ............(12)
\]
where, $\mu$ (Pa·s) is the viscosity of the fluid, $h$ (m) is the height of the droplet, $r$ (m) is the radius of the fine tube, $V$ (m$^3$) is the volume of the droplet, $l$ (m) is the length of the tube. The radius and length of the tube, $r$ and $l$, are calculated by the following procedure. The $l$ is the double of the distance between the center of the particle and the first contact position of the droplet and particles shown in Fig. 14.

$$l = 2 \cdot (D/2) \sin \theta \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots}
eter of both the packed particles and droplet strongly affects on the dripping rate of the droplet.

3. Dripping Behavior of Pseudo-iron at Large Supply Rate

3.1. Experimental Setups for Large Supply Rate

Figure 18 shows the schematic diagram of the experimental apparatus for large supply rate of the fluid. The dimensions of the vessel were 90 mm in width, 90 mm in depth, 180 mm in height. Liquid paraffin was used as pseudo-slag, and fluorinated inert liquid (HCFC) was used as pseudo-iron. Specific diameters of glass beads (2.0, 3.0, 4.0, 5.0, 10 mm) were used to form pseudo-coke bed. At first, the vessel was filled with liquid paraffin, and then the HCFC was flown into the vessel from the top of the vessel. The HCFC separates from liquid paraffin due to the difference in specific gravity, and piled up in the bottom of the vessel. The piled HCFC were drained from the drain located at the bottom of the vessel, pumped up to the tank, and flown from the top of the vessel again to achieve steady state. The dripping behaviors of HCFC were observed by changing the supply rate of HCFC and diameter of the packed particles.

3.2. Dripping Behavior of Fluid in Packed Beds

Figure 19 shows examples of experimental results. The dispersion width of the liquid widen by increasing the supply rate or decreasing the diameter of the packed particles. When the supply rate of the liquid increases, at a certain supply rate, the supply rate of the fluid exceeds the maximum dripping rate, \( u \cdot \pi r^2 \), in Eq. (16). In other words, the supply rate exceeds the dischargeable rate for 1 channel. Therefore, the fluid has to find other channels to satisfy the material balance. The dischargeable rate for 1 channel is a function of the packed particle diameter, \( D \), as shown in Eq. (16), thus, the final dispersion width of the liquid widen by decreasing the particle diameter. On the other hand, after the dispersion width reaches the maximum width, the dispersion width tends to keep the same width.

3.3. Modeling of Dripping Behavior in the Packed Bed

The number of apertures in the packed bed per unit area is given by Eq. (17) as shown in Fig. 15.
When we assume the aperture as a fine tube, and use the average cross-sectional area and maximum dripping rate \( u_c \), the dischargeable rate of the liquid per 1 aperture \( q \) (m\(^3\)/s) is given by Eq. (18).

\[
q = \frac{1}{4} \sqrt{3} \frac{D^2 l - \pi}{l} \int_0^{1/2} \left( \frac{D}{2} - x^2 \right) dx
\]  

When total dischargeable rate balances with the supply rate, the dispersion of the liquid reaches steady state. The cross-sectional area filled with the supplying liquid \( S \) (m\(^2\)) at a certain supply rate \( Q \) (m\(^3\)/s) is given by Eq. (19).

\[
S = \frac{Q}{nq}
\]

The dispersion rate of the fluid is determined by the packed structure because the rate depends on the inclination angle of the apertures to the vertical axis. The maximum dispersion width, \( W_{\text{max}} \) (m), and the distance when the dispersion reaches steady state, \( L_{\text{max}} \) (m), are given by Eq. (20).

\[
W_{\text{max}} = 2 \sqrt{\frac{S}{\pi}}, \quad L_{\text{max}} = \frac{9 \cdot W_{\text{max}}}{4 \sqrt{2}}
\]

### 3.4. Results and Discussions

Figures 20 and 21 show comparison of the calculated result with measured one. Slight fluctuations can be seen in the measured results; however, the calculated results reproduce the dispersion behavior of the fluid well. Regarding the maximum dispersion width, \( W_{\text{max}} \), and the distance when the dispersion reaches steady state, \( L_{\text{max}} \), the calculation results agree well with the measured results. From these results, when the supply rate is relatively large compared to the dischargeable rate from one aperture, the dripping behavior of a liquid in a packed bed filled with an immiscible liquid are strongly affected by the balance of supply and dischargeable rate, and packed bed structure. The dispersion of the liquid is not determined by probability.

### 4. Conclusions

In this work, cold model experiments and mathematical modeling were conducted to investigate the dripping behavior of molten iron in the coke packed bed filled with molten slag, and following results were obtained.

1. The dripping rate of a droplet in packed beds filled with an immiscible fluid is determined by the relative size of the droplet diameter to that of the packed particles. When the supply rate of the fluid is relatively small, the dripping rate of the droplet can be estimated by 2 models which depend on the relative size of the droplet to that of the apertures consist of 3 particles.

2. When the supply rate is relatively large, in other words, the supply rate exceeds the dischargeable rate, the dispersion behavior of the dripping liquid depends on the balance of the supply rate and the dischargeable rate which depends on the packed bed structure, such as, the diameter of the packed particles, arrangement of the particles. The final dispersion width is determined by the balance of dischargeable rate (maximum dripping rate) and supply rate.

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### REFERENCES