Theoretical Characterization of Steady-state Heat Wave Propagating in Iron Ore Sintering Bed

Masanori NAKANO, Kazuaki KATAYAMA and Shunji KASAMA


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Knowledge of heat wave is a key to control the iron ore sintering process. To enhance the knowledge, numerical integration of differential equations for gas and solid is prevailing; this report, otherwise, used an analytical method and introduced several solutions by the assumption that the heat wave descended in steady state and that gas and solid temperatures coincided. The solutions correlated characters of heat wave with operational factors in a simple and explicit manner like:

1) Heat behind speed \( (U_B) \) was a function of gas velocity \( (u_g) \) as \( U_B = \frac{C_g}{C_s} u_g \);  

2) Solid maximum temperature \( (T_P) \) comprised an adiabatic temperature increase due to coke combustion \( (\theta c) \) and a heat convection due to downdraft necessary for solid temperature to heat up to coke ignition \( (\theta ig) \) as \( T_P = \theta c + \theta ig \);  

3) Heat front speed \( (U_F) \) correlated with \( U_B \) \( (U_B \cdot n) \) and the terms for coke combustion heat generation \( (\theta c) \) and ignition temperature \( (\theta ig) \) as \( U_F = U_B \cdot n (1 + \theta c/\theta ig) \).

KEY WORDS: agglomeration; flame front speed; heat behind speed; heat wave; iron ore sintering; maximum temperature; steady state.

Table 1. Basic differential equations describing heat wave propagation in sintering bed.\(^6\)

<table>
<thead>
<tr>
<th>Equation</th>
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<tbody>
<tr>
<td>([\text{Heat balance for solid}]) ( c_r \rho \frac{\partial T}{\partial t} = q \ A(T_g-T_s) + q ) ( \frac{\partial T}{\partial x} ) ( \ldots(i) )</td>
</tr>
<tr>
<td>([\text{Heat balance for gas}]) ( c_g \rho_g \frac{\partial T}{\partial t} = u_g c_g \frac{\partial T_g}{\partial x} - q_c (T_g-T_s) A(T_g-T_s) \ldots(ii) )</td>
</tr>
<tr>
<td>([\text{Heat generation of coke}]) ( q^* = 4\pi r_n k \ A \ \text{CO}_2 (k^* k^* k^* k^*) \ldots(iii) )</td>
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<tr>
<td>([\text{Simplified Equation}]) ( c_r \rho \frac{\partial T}{\partial t} = -u_g c_g \frac{\partial T_g}{\partial x} + q \ldots(iv) )</td>
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1) and gas (Eq. (ii)) with a rate equation for coke combustion (Eq. (iii)): Muchi \textit{et al.}\(^4\) applied a numerical method with the aid of computer and realized heat wave analysis. Suffering with unstable excursion, maybe caused by exponential behavior of coke combustion, the method prevailed over Japan in 1970’s in success to explain the experimental facts and to estimate the effect of operational factors. Nevertheless, it might have failed to increase our principal understanding of the phenomena.

Analytical approaches stand on the assumption that gas and solid temperatures agree, which gives a unique equation (Eq. (iv)): Block\(^5\) obtained an analytical solution for the equation; his solution still has an integration term that needs numerical calculation. Wajima \textit{et al.}\(^6\) showed a fast calculation method by assuming an exponential type curve for heat behind and allowing overall heat balance to decide \( T_P \); their method still needed an empirical relationship between gas velocity and cooling speed.

Simple and explicit equations linking sintering operational factors to heat wave’s characters will be available for direct understanding of the phenomena as well as for design and practice of sintering operation. Horio \textit{et al.}\(^7\) showed the existence of steady-state propagation for well-developed heat wave and lead equations for \( U_F \), \( U_B \) and \( T_P \) under the condition; unfortunately, their equations needed some computer calculation, for they treated gas and solid temperature separately.

The steady-state assumption for heat wave seems potent; then, adding the assumption of gas–solid temperature coincidence and applying heat balance for each zone of steady-
state heat wave, we will show simple, directly understandable, if not strict, equations describing heat wave’s characters.

2. Equations for $U_p$, $U_b$ and $T_F$

2.1. Triangle Heat Wave

We will start with a heat wave of simple shape that the third clause defines below. Assumptions are:

1) Heat exchange is quick enough for gas and solid temperatures to coincide.
2) There exists steady-state propagation of heat wave.
3) As shown in Fig. 1, where the horizontal axis, $z$, means the distance from surface and the vertical axis, $T$, means the solid (bed) temperature defined based on the ambient one ($T_0 = 0$), the heat wave has triangle shape: In heat front zone (HF) the temperature increases to the maximum ($T_F$) straightly; in heat behind zone (HB) the temperature decreases straightly to the ambient temperature at the surface. The HF moves at a constant speed ($U_p$), keeping the maximum temperature at a constant value ($T_F$).

Comparing the heat waves before and after moving for an incremental distance ($\Delta z$) (Fig. 1) tells that the increase of heat accumulation in HB is $c_s \rho_s \frac{A \Delta z T_F}{2}$ with a geometric consideration. The corresponding heat generation for the moment is $A \Delta z Q$. This is equal, which gives Eq. (1) with use of a parameter, $\theta$, for $Q/(c_s \rho_s)$:

$$T_F = 2 \theta c \quad \text{...............(1)}$$

where $T_F$: the maximum temperature of solid on ambient basis (K)
$c_s$: heat capacity of solid (J/kg/K)
$\rho_s$: bulk density of solid (kg/m$^3$)
$Q$: calorific value per unit volume (J/m$^3$)
$\theta c$: $= Q/(c_s \rho_s)$ (K)
$A$: area (m$^2$)
$\Delta z$: an incremental distance (m).

Wendeborn$^{16}$ has already reached to the same result with another method, which Shimomura$^{18}$ introduced to Japan in his review as the first theoretical approach in the field.

Figure 2 shows a heat balance through HF with use of the coordinate moving at the speed of $U_p$ along $z$-axis in synchronozation with HF. In terms of heat flux (J/m$^2$), solid brings 0 and takes $c_s \rho_s U_p T_F$: gas brings $c_g u_g T_F$ and takes 0; coke generates $QU_p$. The sum of them has to be zero, which makes:

$$c_s \rho_s U_p T_F = c_g u_g T_F + QU_p$$

where
$c_s$: heat capacity of gas in volume basis (J/m$^3$/K)
$u_g$: gas velocity (m/s)
$U_p$: heat front speed (m/s).

Substituting $2Q/(c_s \rho_s)$ for $T_F$ ($= 2 \theta c$) and using a parameter, $\nu$, for $(c_g u_g)/(c_s \rho_s)$, then

$$U_p = 2 \nu \quad \text{...............(2)}$$

where
$\nu$: $=(c_g u_g)/(c_s \rho_s)$ (m/s)

2.2. Trapezoidal Heat Wave with Infinite Coke Combustion Rate

To obtain equations including some parameters that characterize coke combustion properties and an equation for $U_{ib}$, we will restart with assuming another shape for the heat wave (Fig. 3).

1) Heat wave has a trapezoidal shape consisting of HF, TOP and HB. HF corresponds to preheating zone of solid. The solid holds at the maximum temperature ($T_F$) straightly; in heat behind zone (HB) the temperature decreases straightly to the ambient temperature at the surface. The HF moves at a constant speed ($U_p$), keeping the maximum temperature at a constant value ($T_F$).

Making another heat balance through HB shows that solid takes $c_s \rho_s U_p T_F$; gas brings $c_g u_g T_F$; no coke combustion. Equaling them and substituting ($\theta c + \theta bg$) for $T_F$,

$$U_p = \nu (1 + \theta c/\theta bg) \quad \text{...............(4)}$$

Making another heat balance through HB shows that solid takes $c_s \rho_s U_p T_F$; gas brings $c_g u_g T_F$; no coke combustion. Thus,

$$U_b = \nu \quad \text{...............(5)}$$

2.3. Trapezoidal Heat Wave with a Finite Coke Combustion Rate

When coke burns at a finite rate (Fig. 4), HF will divide into two zones: drying and combustion zones. Heat balance through the drying zone gives: $U_i = \nu$. Thus,

$$
\begin{align*}
H_F & \quad \text{TOP} & \quad \text{HF} \\
\theta = \frac{Q}{c_s \rho_s} & \quad \theta c & \quad \theta bg
\end{align*}
$$

2.3. Triangle Heat Wave Having Temperature Difference between Solid and Gas

For obtaining $U_f$ faster than $v$, we will shift closer to the fact, namely discard the first assumption that $T_s=T_g$ and assume $T_g>T_s$ in HF.

1) Start with triangle heat wave like Sec. 2.1 for simplicity.

2) Coke burns from an ignition temperature ($\theta_{ig}$) at a finite rate, emitting heat $Q$.

3) $T_g=T_s$ in HB as well; $T_g>T_s$ in HF.

The triangle heat wave gives: $T_p=2\theta_c$, as is shown for Eq. (1).

Letting $T_{s,ig}$ and $T_{g,ig}$ designate solid and gas temperature at the interface between drying and combustion zones, respectively, then solid gains $c_p(T_p-T_{s,ig})U_f$; gas loses $c_gu_g(T_{g,ig}-T_{g,ig})$ as heat flux through drying zone. Thus,

$$U_f=\frac{v(T_{g,ig}/T_{s,ig})}{...} \tag{6}$$

Through combustion zone, solid gains $c_p(T_p-T_{s,ig})U_f$; gas loses $c_gu_g(T_p-T_{g,ig})$, coke generates $QU_f$. Taking heat balance, and putting $2\theta_c$ for $T_p$ and $v(T_{g,ig}/T_{s,ig})$ for $U_f$,

$$T_{g,ig}=2T_{s,ig} \tag{7}$$

Combining Eqs. (6) and (7) gives $U_f=2v$, suggesting that $U_f$ is independent on whether gas and solid temperatures are same or not. The assumption of triangle heat wave may give strong restriction to the solution for $U_f$.

Assuming more complicated shapes for heat wave, we tried to build a finite coke combustion rate ($q$) and a combustion duration ($\tau$) in the expression for $U_f$. However any trial failed so far with the result that they appeared in the form of $v\times\theta (=Q)$ anytime.

2.4. Interpretation of the Equations Describing $T_p$, $U_f$ and $U_g$

2.4.1. Maximum Temperature ($T_p$)

The parameter $\theta_c$ means an adiabatic temperature increase due to coke combustion. It partly contributes to make up $T_p$. The other contributor is downward heat transfer by airflow.

Equation (1) shows rough contributions of the two factors; they are half and half.

Equation (3) adds more information that the magnitude of heat transfer contribution meets the heat necessary for solid temperature to elevate up to the coke ignition temperature, suggesting:

- Increase in coke content, use of low CW ores and heat generating materials, such $\theta_c$-increasing actions allow the $T_p$ to increase;
- Increase in oxygen content, use of high reactivity coke, size down of coke diameter; such $\theta_{ig}$-decreasing actions decrease the $T_p$.

2.4.2. Heat Behind Speed ($U_g$)

Many researchers have already proven that $v$ coincides to heat behind speed, or the heat wave speed without coke combustion.$^{10}$ We have confirmed it with Eq. (5).

2.4.3. Heat Front Speed ($U_f$)

Equation (2) predicts without explanation that the HF moves twice as quickly as HB.

Equation (4) gives a more reasonable explanation on the phenomena: the speed gain of HF from that of HB is governed by the ratio of heat generation, most of which coke generates, to coke’s ignition temperature. The Eq. (4) suggests:

- Increase in gas velocity and gas heat capacity raise HF speed in conjunction with $v$;
- The $\theta_{ig}$-decreasing actions result in the increase in HF speed;
- The $\theta_c$-increasing actions lead to the increase in HF speed.

Equation (6) highlights another aspect of the phenomena. The other driving force of heat wave, except for gas velocity, is the temperature difference between solid and gas. The ratio of gas’s temperature to solid’s one determines the displacement of HF speed from $v$ according to the equation. As a matter of fact, the difference won’t be detective; otherwise the equation can be available to estimate the difference based on measured $U_f$.

3. Validation of Equations

3.1. Heat Front Speed ($U_f$)

To validate the equations derived for $U_f$, we will use the typical values for the constants: $c_p=1.3$ kJ/Nm$^3$/K, $u_g=0.3$ Nm/s, $c_g=0.83$ kJ/kg/K, $\rho_s=2000$ kg/m$^3$, and refer to Kawaguchi et al.$^{11}$ correlations of $U_f$ to $u_g$:

[Calculation] $U_f$ (mm/s) = 1.05$u_g$ (Nm/s) + 0.1

[Calculation] $U_f$ (mm/s) = 1.4 – 1.5$u_g$ (Nm/s)

Order estimation of $U_f$ with Eq. (2) follows:

$U_f$ (m/s) = $2((c_gu_g)/(c_p\rho_s)) \times (1.3/0.83/2000)$ $u_g$

$U_f$ (mm/s) = 1.08$u_g$ (Nm/s);

Equation (4) offers another estimate:

$U_f$ (m/s) = $((c_gu_g)/(c_p\rho_s))(1+\theta_c/\theta_{ig})$ = $((1.3/0.83/2000)(1+576/600)) \times u_g$

$U_f$ (mm/s) = 1.06$u_g$ (Nm/s)

Both estimates have slight difference and well meet the pot test result. Here has been proven the validity of the equations for $U_f$. 

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3.2. Maximum Temperature ($T_p$)

For $T_p$, validation we investigated the effect of coke content on $T_p$, by pot test, varying coke content from 4.0 to 7.0 mass% and measuring bed temperatures at the depth of 100 mm, 150 mm and 450 mm with a standard method using a 300 mm-dia and 600 mm-height pot, a 12 kPa-constant suction and a typical ore blend. According to the result (Fig. 5), the maximum temperature at any point increased with increasing coke content; the steady state propagation attained at least at the depth of 450 mm, where the ratio of temperature increase by coke content was between 60 and 200 K/mass%; for 300 mm depth, the state also seemed to establish over 4.5 mass% coke content with the ratio of increase being approx. 50 K/mass%.

$Q$ should reflect not only coke’s combustion but also other relevant heat sources’ reactions. According to Table 2, the sum of them gives 1205 MJ/m$^3$ to $Q$ at 4 mass% of coke content. Based on Eq. (3), an estimated maximum temperature at 4 mass% coke content becomes:

$$T_p = \theta_{bg} + Q/(c_s \rho_s)$$

$$= 600 + 1205000/(0.83 \times 2000)$$

$$= 1326 \text{ (K)}$$

which happens to be close to the observed value.

The differential form of the Eq. (3), can also tell quantitatively a gain in maximum temperature per one mass% increase of coke content:

$$dT_p/d(\% C) = (1/(c_s \rho_s))dQ/d(\% C)$$

$$= (1/1660) \times 602000$$

$$= 363 \text{ (K/mass%)}$$


3.3. Comparison with the Conventional Muchi Method

The conventional Muchi method is logically strict, but uneasy to use because of needing time-consuming calculation and yielding results with uncertainty due to unknown parameter and numerical calculation itself, but difficult to understand the cause-effect relation from the dumped results.

The present method has given simple and explicit correlations between cause factors and effects on heat wave, which will ease the understanding of the phenomena. The equations involve coke’s ignition temperature ($\theta_{bg}$) needing individual measurement for coke by coke; contrarily, the Muchi method uses unique coke’s property as a combination of chemical reaction rate ($k$) and activation energy ($E$). The Muchi’s description familiar with reaction engineering seems universal; however, the advantage is superficial because the method pushed the complexity into another parameter of surface area, the evaluation of which requires individual measurement.

Since the measurement for coke ignition temperature is more productive than that for the surface area, the present method offers an alternative industrially preferable to estimate on sintering heat wave. Note that the application is restricted to the well-developed heat wave. The phenomena are out of scope during the ignition and the transient sintering in upper layer.

4. Concluding Remarks

Assuming steady-state propagation of heat wave and coincidence of gas–solid temperature and neglecting melting behavior, a set of simple and explicit equations has been introduced, which correlates characters of heat wave with operational factors like:

1. Heat behind speed ($U_{bg}$) was a function of gas velocity ($u_g$) as $U_{bg} = (c \rho_s)/u_g$.
2. Bed maximum temperature ($T_p$) comprises an adiabatic temperature increase due to coke combustion ($\theta_c$) and a heat convection due to downdraft necessary for solid temperature to elevate to coke ignition ($\theta_{bg}$), as $T_p = \theta_c + \theta_{bg}$.
3. Heat front speed ($U_f$) correlates to $U_{bg}$, the heat generation ($\theta_c$), ignition temperature of coke ($\theta_{bg}$) as $U_f = \nu (1 + \theta_c/\theta_{bg})$.

These equations give good perspective of the operating factors’ effect on heat wave characteristics. As for the heat front speed the theoretical and the experimental results
shows good accordance; but not for the maximum temperature owing to lacking knowledge about melting.

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