CFD Analysis of Sag Line Formation on the Zinc-coated Steel Strip after the Gas-jet Wiping in the Continuous Hot-dip Galvanizing Process

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One of the surface defects on the steel strip surface after the gas-jet wiping employing an air-knife system in the continuous hot-dip galvanizing process is called sag lines or snaky coating. The sag line defect is the oblique patterns such as “W”, “V” or “X” on the coated surface. The present paper presents an analysis of the sag line formation and a numerical simulation of sag lines by using the numerical data produced by Large Eddy Simulation (LES) of the three-dimensional compressible turbulent flow field around the air-knife system.

In order to simulate the sag line formation, a novel perturbation model has been developed to predict the variation of coating thickness along the transverse direction. The thickness obtained by the proposed model yields very similar results with those obtained by the conventional equation. It is observed that the coating thickness along the transverse direction is affected more by the pressure gradient than the surface shear stress in the stagnation region, while in the far field the shear stress becomes the major factor to determine the thickness.

Finally, the three-dimensional coating surface was obtained by the present perturbation model. It was found that the sag line formation is determined by the combination of the instantaneous coating thickness distribution along the transverse direction near the stagnation line and the feed speed of the steel strip. The computed mean distance between the crests and the shape of the simulated sag show relatively good resemblance with the real sag lines on the strip surface.

KEY WORDS: air-knife; coating thickness; jet wiping; LES (large eddy simulation); perturbation method; sag line.

1. Introduction

A number of surface coating techniques are used in a variety of industrial fields such as paper, photographic film manufacturing, automobile, shipbuilding and home appliance industries. Among them, the galvanized steel surface has good corrosion resistance and it is produced by a continuous hot-dip galvanizing process in which the fluid is molten zinc. In that process, a metallurgically clean and heat-treated steel strip is continuously passed through a molten zinc bath and is drawn up with the molten zinc adhered on both sides of the strip as schematically depicted in Fig. 1. In this case the thickness of the adhered zinc film is usually about 10 times thicker than desired thickness of around 20 μm. Such excessive zinc mass must be removed by mechanical, electromagnetic, or hydrodynamic operations. Gas-jet wiping process is a hydrodynamic method to remove the excessive molten zinc adhered on the strip. In this process, a pair of opposing horizontal plane gas jets is located just above the bath, and the strip is vertically drawn up between the pair of opposing jets as shown in Fig. 1. The working gas is either air or nitrogen gas. This system is often referred to as “air-knife system” in the continuous hot-dip galvanizing industry. The gas wiping mechanism in the air-knife system has been well investigated theoretically and experimentally.

Thornton and Graff¹ assumed that the final coating thickness is affected only by the longitudinal pressure gradient induced by the impinging jet on the strip. Tuck² have proposed a similar approach and investigated the stability

Fig. 1. Schematic diagram of a hot-dip galvanizing process with air knife.
of the coating thickness for long wavelength perturbations. Ellen and Tu\textsuperscript{10} have suggested a non-dimensional model that takes into account the surface shear stress and their model gives significantly more accurate prediction of the final coating thickness. Also, Tu and Wood\textsuperscript{9} have experimentally demonstrated that the final coating thickness depends on both the pressure and shear stress distributions on the strip surface. Although Tuck and Vanden-Broeck\textsuperscript{10} and Yoneda\textsuperscript{8} suggested that the final thickness depends on the surface tension and the shear stress on the strip, recently Gosset and Buchlin\textsuperscript{7} have shown that the surface tension does not play any role in determining the final coating thickness.

There have appeared several numerical simulations of the gas wiping process to predict the final coating thickness by using CFD. One of the recent studies is that by Lacanette \textit{et al.}\textsuperscript{1,9} who numerically simulated the final coating thickness by using volume of fluid-large eddy simulation (VOF-LES) modeling. Their results were satisfactorily compared with their experimental data. They found that the coating thickness is independent of the nozzle-to-plate distance when it is smaller than 8 times of the nozzle slot width. Myrillas \textit{et al.}\textsuperscript{10} tried to validate a model for the gas jet–liquid film interaction in the gas wiping process. In their study, they found that two-phase numerical simulations using LES is more suited to model of the complex interaction than \textit{k–\varepsilon} turbulence model. Through these previous studies it can be concluded that the dominant factors affecting the final coating thickness are pressure gradient and shear stress on the strip.

So far, most of the previous studies were focused on the average final coating thickness along longitudinal direction and their numerical and mathematical models are made in two-dimensional domain. In spite of its good productivity and easy control of the zinc coating thickness, however, the coated film surface after gas wiping has frequently three dimensional surface defects such as dents, blow lines, peculiar features and sag-lines.\textsuperscript{11} Therefore, in order to obtain a uniform coating on the strip, it is necessary to investigate in details the three dimensional character of the coated surface after the gas wiping process.

Among these surface defects, the sag lines (or snaky coating) cause many problems such as irregularity in the electrical and thermal characteristics and the diffused reflection on the coated surface. The snaky coating is that with an oblique patterns appearing on the coated film surface after the gas wiping process. Depending on its seriousness, the snaky coating is usually classified into five grades as shown in Fig. 2. The arrows in Fig. 2 indicate the moving direction of steel strip. The first grade indicates that no sag line is observed on the surface. In the surface pattern of the 2nd grade, rather short sag lines appear irregularly, thus a pattern of sag lines is vaguely discernible. On the other hand, the 3rd–5th grade snaky coatings reveal oblique checkmark patterns such as "W", "V" or "X" on the coated surface. Figure 3 demonstrates the micro-scale defects with 3rd grade sag lines on the surface. The height from a valley to the nearby crest is about 2–3 mm and the distance between nearby crests is usually 3–4 mm. A principal cause of sag line defect was firstly found by Yoon \textit{et al.}\textsuperscript{11} They numerically simulated the turbulent flow field in the gas wiping region using LES technique. And by scrutinizing their simulated flow field in details, they found that high and low pressure points appear almost periodically along the impingement stagnation line across the strip surface. They revealed that such a pattern of periodical pressure distributions moves alternatively sideways and together with the longitudinal feed of the strip, it forms the sag lines of checkmark pattern on the surface. In a recent study,\textsuperscript{13} they further showed that the snaky coating defect can be avoided by suppressing the periodic variation of the surface pressure by utilizing a secondary parallel gas jet adjacent to the main jet. On the other hand, it was demonstrated that other surface defects can be improved by controlling the aluminum content and temperature of the bath\textsuperscript{14} or stirring the molten zinc in the bath.\textsuperscript{15}

The main objective of the present study is to predict the sag line formation along the transverse direction by analyzing numerical data obtained from Large Eddy Simulation for the 3-D unsteady compressible flow filed around the airknife system. Another objective is to suggest a new perturbation model to predict the variation of coating thickness along the transverse direction and to clarify the effects of the pressure gradient and surface shear stress for inducing the sag line formation.

2. Numerical Analysis

In the present study, LES technique with the Smagorinsky–Lilly sub-grid scale model is used to simulate the instantaneous flow field of the 3D unsteady compressible impinging jet.\textsuperscript{16} Boundary conditions and the computational domain are shown in Fig. 4. The transverse (\textit{i.e.}, horizontal)
width of the strip and the air-knife jet is 100 mm which is taken as the transverse length of the calculation domain. The opposing jet regions outside of the strip edge are not considered because the sag lines appear mostly in the interior region. It is assumed that the mean flow field is homogeneous in the transverse z-direction and Neumann condition is given at both sides of the calculation domain. The homogeneity in the transverse direction has been tested with a number of different transverse lengths, and it was found the 100 mm selected in the present study is wide enough to preserve the z-direction homogeneity of the flow field.

In Fig. 4, the strip moves vertically upward in y-direction and the calculation domain in the vertical (i.e., longitudinal) direction is taken to be 60 mm. The atmospheric pressure condition is used at the top and bottom boundaries. The symbols d and L in Fig. 4 represent the jet slot width and the distance from the nozzle exit to the moving plate, respectively. The origin of x-coordinate is located at the strip surface, and that of y-coordinate is taken to be at the same level of the lower lip of the jet slot (see Fig. 4). And the origin of z-coordinate is put at the right most end of the computational domain.

Working fluid is nitrogen gas used in the actual wiping process. In a previous experimental study, 3rd grade snaky coating was observed when an experimental air-knife system with d=1.5 mm and L=100 mm was operated at the stagnation pressure of 25 kPa and the stagnation temperature of 340 K. These experimental conditions are used as a bench-mark case to validate the present analysis model for predictions of severe sag lines and the final coating thickness. Tu and Wood\cite{4} have experimentally demonstrated that the final coating thickness depends on both the pressure and shear stress distribution. Tuck\cite{2} proposed a similar approach and studied the stability of their solutions of the coating thickness for long wavelength perturbations. Ellen and Tu\cite{3} suggested a non-dimensional model which for the first time takes into account the surface shear stress and their model significantly improved the prediction accuracy of the final coating thickness. Tu and Wood\cite{5} have experimentally demonstrated that the final coating thickness depends on both the pressure and shear stress distribution.

The smallest grid size is used near the nozzle exit and in the jet-plate impinging region. It is determined to be about the same size as the Taylor’s micro-length scale, \( \lambda \). In order to estimate the Taylor’s micro-length scale, the energy dissipation rate is calculated by using the inviscid estimate of Kolmogorov given by \( \varepsilon=\frac{u'^{2}}{l} \), where \( u' \) is the rms of the stream-wise velocity fluctuations and \( l \) is the integral length scale. Assuming that the turbulent velocity fluctuations is about 10% of the jet exit velocity and that the integral length scale is one half of the nozzle slot width, the Taylor’s micro-length scale was estimated by \( \varepsilon=\frac{15\bar{u}^{2}/\lambda^{2}}{5} \). In this way, we obtained \( \lambda \approx 0.1 \) mm. Thus, the smallest grid spacing was taken to be 0.1 mm near the nozzle exit and in the jet-plate impinging region and the grid size was increased at a successive rate in the \( \pm y \)-direction.

Time step of the unsteady solver was fixed to be \( \Delta t=5.0 \times 10^{-7} \) s which was determined from the smallest grid size and the exit velocity of the plane jet. When the exit velocity is 230 m/s, the Courant number \( N_{Cours}=U\Delta t/\Delta x \) is 1.15 for the smallest grid size and it was found that the Courant number varies from about 0.5 to 1.5 in the whole computational domain in the present study. Considering that the appropriate range of the Courant number is generally from 0.3 to 2 in LES simulation, the grid size and time step used in the present study are suitable.\cite{16,17} Meanwhile, PISO algorithm was used as the pressure-velocity coupling and bounded central differencing method was applied for the discretization scheme.

3. Analytical Models for Predicting the Coating Film Thickness

3.1. Model to Calculate the Film Thickness Distribution along the Longitudinal Direction

Several models have been suggested to predict the zinc film thickness produced by the gas-jet wiping process. One of the first analytical models is the one proposed by Thornton and Graff.\cite{1} They assumed that the final coating thickness is affected only by the pressure gradient induced by the impinging jet on the strip. Tuck\cite{5} proposed a similar approach and studied the stability of their solutions of the coating thickness for long wavelength perturbations. Ellen and Tu\cite{3} suggested a non-dimensional model which for the first time takes into account the surface shear stress and their model significantly improved the prediction accuracy of the final coating thickness. Tu and Wood\cite{5} have experimentally demonstrated that the final coating thickness depends on both the pressure and shear stress distribution.

The principal step in predicting the coating thickness is the calculation of the steady state flux of the molten zinc adhered to the steel strip using a simplified form of the Navier–Stokes equation.\cite{12} The flow of molten zinc on the surface is assumed to be steady, incompressible and creep flow. The fluid properties such as viscosity, density and
thermal conductivity are assumed constant. It is also assumed that surface tension and oxidation effects can be neglected. Using the coordinate system shown in Fig. 1, the governing equations and boundary conditions are summarized as follows.

Continuity and momentum equations:
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \] ..............................................(1)
\[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial y} - \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - g \] ..............(2)

Since the film thickness is very thin, it can be assumed that the pressure across the thin coating layer is constant and the velocity component normal to the strip is zero. Then, Eqs. (1) and (2) can be simplified to the following Eqs. (3) and (4), respectively.
\[ \frac{\partial v}{\partial y} = 0 \] .......................................................(3)
\[ \frac{1}{\rho} \frac{\partial p}{\partial y} + g = \frac{\mu}{\rho} \frac{\partial^2 v}{\partial x^2} \] ....................................(4)

Now, Boundary conditions for these equations are
\[ v(x, y)_{y=0} = V_s \] ..............................................(5)
\[ \tau_w(y) = \frac{\mu}{\rho} \frac{\partial v}{\partial x} \] ............................................(6)

Integration of Eq. (4) with the boundary conditions Eqs. (5) and (6) leads to the velocity profile given by the following Eq. (7).

\[ v(x, y) = V_s + \frac{1}{\mu} \left[ \tau_w x + \left( \frac{dp}{dy} + \rho g \left( \frac{1}{2} x^2 - \delta(y)x \right) \right) \right] \] .......(7)

Then, the volumetric liquid flow rate per unit width of the strip becomes
\[ q = \int_0^{\delta(y)} v(x, y) dx = V_s \delta(y) + \frac{1}{2} \frac{\tau_w}{\mu} \delta(y)^2 - \frac{\delta(y)^3}{3\mu} \left( \rho g + \frac{dp}{dy} \right) \] ...........(8)

Equation (8) is expressed with the normalized variables and dimensionless physical group as follows:
\[ (1 + \hat{\nabla} \hat{P}) \hat{\delta}^3 - 1.5 \hat{T} \hat{\delta}^2 - 3 \hat{\delta} + 2 \hat{Q} = 0 \] ..............(9)

The normalized variables and dimensionless physical group in Eq. (9) are defined as follows:
\[ \hat{\delta} = \frac{\delta}{\delta_0}, \quad \hat{x} = \frac{x}{d}, \quad \hat{Q} = \frac{q}{q_0}, \quad \hat{\delta}_0 = \frac{2}{3} V_s \delta_0 \]
\[ \hat{\nabla} \hat{P} = \frac{\nabla P}{\rho_0 g}, \quad \hat{T} = \frac{\tau_w}{\tau_{w0}}, \quad \hat{\delta}_0 = \sqrt{\frac{\mu V_s}{\rho_0 \theta}} \] ..........................(10)

Subscript 0 refers to the dragged film flow without wiping and 1 to the liquid phase. The solution of Eq. (9) for determining the film thickness along the longitudinal (vertical) direction \( \delta(y) \) is obtained by solving locally the cubic equation and using the pressure gradient and shear stress distribution obtained by numerical simulation or measurement of an impinging jet on the surface.

3.2. Model to Calculate the Film Thickness Distribution along the Transverse Direction

Figure 5 demonstrates the distributions of the static wall pressure, its gradient and the wall shear stress along the longitudinal direction computed by the present study.

At a certain transverse distance, it is convenient to represent the local wall pressure gradient and local shear stress denoted by \( \nabla \hat{p} \) and \( \hat{T} \) respectively, as a sum of its mean over the transverse direction and the fluctuation values from its mean. In this way, a perturbation model can be developed as follows:
\[ \nabla \hat{p} = \nabla \bar{p} + \nabla \hat{p'}, \quad \hat{T} = \bar{T} + \hat{T'} \] ..........................(11)

Here, \( \bar{\cdot} \) refers to the mean value over the transverse direction and \( /H11005/H11001/H11002 /H11001 /H11005 \) to the fluctuation. Now, let’s consider the Eq. (9) in the following form.
\[ (1 + \nabla \bar{p}) \hat{\delta}^3 - 1.5(T) \hat{\delta}^2 + 2 \hat{Q} = 0 \] ..........................(12)

Fig. 5. Wall static pressure, pressure gradient and shear stress along the y-direction.
Here, it should be note that \( \hat{\delta}_0 \) is not the mean value of \( \hat{\delta} \) over the transverse direction. Rather, it is thought of as an unperturbed value of the coating thickness corresponding to the mean wall pressure gradient and the mean shear stress. In addition, local coating thickness (\( \hat{\delta} \)) can be expressed as the sum of the unperturbed state, \( \hat{\delta}_0 \) and the deviation part, \( \hat{\delta}' \).

\[
\hat{\delta} = \hat{\delta}_0 + \hat{\delta}' 
\tag{13}
\]

Then, the Eq. (9) is re-expressed by inserting the Eqs. (11) and (13) as follows:

\[
(1 + \nabla \overline{P} + \nabla P')(\hat{\delta}_0 + \hat{\delta}')^3 = -1.5(\hat{T} + \hat{T}')(\hat{\delta}_0 + \hat{\delta}')^2 - 3(\hat{\delta}_0 + \hat{\delta}') + 2Q = 0
\tag{14}
\]

Subtraction of Eq. (12) from Eq. (14) leads to the following equation.

\[
(1 + \nabla \overline{P} + \nabla P')(3\hat{\delta}_0)\hat{\delta} + 3\hat{\delta}_0(\hat{\delta})^2 + (\hat{\delta}')^2 + \nabla \hat{P}(\hat{\delta}_0)^3 = -1.5(\hat{T} + \hat{T}')(2\hat{\delta}_0\hat{\delta} + (\hat{\delta})^2) - 1.5\hat{T}(\hat{\delta}_0)^2 - 3\hat{\delta}' = 0
\tag{15}
\]

Here, if the perturbation magnitude is very small in comparison with the mean value, one may assume that

\[
\hat{\delta}_0 = \hat{\delta}' = \hat{T} = \hat{T}' = \nabla \overline{P} = \nabla \hat{P} \quad \tag{16}
\]

Note that the fluctuation of pressure gradient is significantly larger than the mean of static pressure gradient which value is constant. Finally, the conditions in Eq. (16) simplify the Eq. (15) to the following form:

\[
\hat{\delta}(z) = \frac{\nabla \hat{P}\hat{\delta}_0 - 1.5\hat{T}\hat{\delta}_0^2}{3 - (1 + \nabla \hat{P})\hat{\delta}_0 + \hat{T}\hat{\delta}_0 + 1} \quad \tag{17}
\]

Therefore, the zinc coating thickness along the transverse direction is obtained by adding the average coating thickness to the fluctuation as

\[
\hat{\delta}(z) = \hat{\delta}_0 + \hat{\delta}' \quad \tag{18}
\]

This novel formula is very simple and compact to calculate the film thickness in comparison with solving the third order Eq. (9).

4. Computational Results and Discussions

Figure 5 represents the average static wall pressure, static wall pressure gradient and wall shear stress distribution along the longitudinal direction on the strip. The average static pressure distribution on the strip is well described by a Gaussian law.\(^{4,18}\) The stagnation line corresponds to the place where both the wall pressure gradient and the wall shear stress vanish. While the magnitude of the pressure gradient becomes negative above the stagnation line, the wall shear stress is positive, and vice versa under the stagnation line. It may be interesting to note that the wall shear stress reaches its positive maximum significantly later than the negatively maximum value of the wall pressure gradient. As will be seen later in Fig. 12, it is related to the formation of the uneven coating surface.

Figure 6 shows the average zinc coating thickness distribution along the longitudinal direction. It was obtained by solving Eq. (9) with the numerically computed average wall pressure gradient and surface shear stress. Here, it is worth noting that the coating thickness is minimum where the average wall pressure gradient has its maximum value. In order to compare the sensitivity of the coating thickness to the pressure gradient and to the surface shear stress, the pressure gradient and the shear stress were arbitrarily increased by 50%, and the resulting coating thickness distributions are compared with those under the real pressure gradient and the shear stress in Fig. 7. The solid lines in the figures indicate the results under the original pressure gradient and the shear stress, the dash-dotted lines show the results with 50% increased pressure gradient, and the dotted lines demonstrate the results with 50% increased shear stress. As can be seen in Fig. 7(a), while a 50% increase in pressure gradient results in a reduction of 12% of the film thickness, the same percentage increase in shear stress results in a reduction of only 1% in the stagnation region.

Figure 8 displays the instantaneous wall pressure contours on the \(xz\)-plane at \( y = 1.4 \text{ mm} \). As can be seen in the figure, the wall pressure varies almost periodically between 5 kPa and 18 kPa along the impingement stagnation line on the surface of the steel strip. It is also observed that the static pressure is distributed in a wavy form in the space between the strip and the jet exit plane.

Figure 9 shows an instantaneous distribution of the wall...
pressure in the jet-strip impingement region on the steel strip, and a schematic sketch of the corresponding wavy form of the coating surface. The wall pressure contour evidently reveals the alternating appearance of high and low pressure region along the impinging stagnation line. And it should be noted that the CFD result reveals that such a sinusoidal distribution of the wall pressure at the stagnation line moves almost cyclically right and left sides as a whole.

Figure 10 demonstrates the transverse distributions of instantaneous coating thickness, wall static pressure, wall static pressure gradient and wall shear stress at different y-locations at 0.75 mm, 2.7 mm, 10 mm and 20 mm. The dotted lines in the figures of the coating thickness indicate the results obtained by solving the cubic Eq. (9), and the solid lines show the results from our perturbation model of Eq. (17). The proposed perturbation model yields very similar results with those obtained by the conventional cubic Eq. (9). On comparing them, it is evident that our perturbation model is simpler and more convenient to use than the conventional method using Eq. (9). Once we know the distribu-

Fig. 8. Static pressure contours on xz-plane at y=1.4 mm.

Fig. 9. Wave form of the molten zinc on the surface at the instantaneous time.

Fig. 10. Instantaneous coating thickness, wall static pressure, pressure gradient and wall shear stress along z-direction at different y-locations. (a) y=0.75 mm, (b) y=2.7 mm, (c) y=10 mm, (d) y=20 mm.
tions of the wall pressure gradient and shear stress on the strip from experimental or numerical study, variations of the coating thickness along both the longitudinal and transverse directions are readily obtained by using Eqs. (12), (17) and (18).

Location at $y=0.75$ mm in Fig. 10(a) coincides with the center of the jet slot. The jet collides with the strip surface in the closest distance. Here the wall pressure varies almost periodically in the transverse direction. Comparing the transverse variations of the wall pressure and the coating thickness, one can conclude that the coating thickness is locally thickest where the wall pressure reaches its local minimum: It is physically evident that the molten zinc is removed at a maximum rate where the wall pressure becomes locally strongest. When one compares the magnitude of the wall shear stress at this location with those at other locations of Figs. 10(b)–10(d), the wall shear stress is relatively very small. Therefore, along the impinging stagnation line, the wall shear stress does not have any effect on removing the molten zinc, and the coating thickness is determined mostly by the wall pressure. Here, a notable observation is that the distributions of the coating thickness and the pressure gradient are nearly the same. This implies that the coating thickness is more directly related to the pressure gradient rather than the wall pressure itself.

Now, consider the case of Fig. 10(b) at $y=2.7$ mm that still belongs to the impinging stagnation region. The wall shear stress has significantly large magnitude in comparison that at $y=0.75$ mm. Even with such large magnitude of the wall shear stress, the same relations as in case (a) can be observed, although weakly related for this case, between the coating thickness and the wall pressure, and between the coating thickness and the wall pressure gradient. Therefore, in the impinging zone, the wall pressure and the pressure gradient are dominant variables for the zinc film thickness. This result is similar to the recent observation of Lacanette et al.\textsuperscript{9} in which the sensitivity of their analytical model of coating thickness to pressure gradient and shear stress has been investigated.

On the other hand, when the strip moves up farther than 10 mm, say, the wall pressure becomes vanishingly small, while the magnitude of wall shear stress does not change much as can be seen in Figs. 10(c) and 10(d). Therefore, the flow field condition to determine the coating thickness is obviously different from that in the impinging stagnation region. In contrast to the impinging stagnation region, the main wiping factor in the far field is evidently the wall shear stress. Since, farther downstream from the impingement region after colliding with the strip, the impinged gas moves in the direction parallel to the strip in a manner of wall jet and the wall static pressure asymptotes gradually to the ambient pressure, the molten zinc is subjected to the effect of the surface shear stress only. In fact, Figs. 10(c) and 10(d) reveal that the coating thickness is nearly inversely proportional to the wall shear stress. Figure 7(b) shows the quantitative comparison the contribution to the zinc coating thickness from the pressure gradient and the shear stress in the far field. While a 50% increase in wall shear stress results in a reduction of 6% of the film thickness, the same percentage increase in the pressure gradient does not affect at all the film thickness after wiping. This relation can also be theoretically deduced from Eq. (17): When the pressure gradient is vanishingly small, the only variable to affect the final coating thickness is the wall shear stress.

Figure 11 shows the variation of the transverse distribution of the instantaneous coating thickness along the longitudinal direction. Initially in the impinging stagnation region, the coating thickness is notably very uneven, but as the strip travels upward such unevenness gradually becomes weak. Quantitatively, the rms variation of the coating thickness at $y=2.7$ mm is 1.5 $\mu$m and gradually it decreases to 0.8 $\mu$m at $y=20$ mm.
Figure 12 exemplifies the instantaneous three-dimensional zinc coating surface that was obtained by using the instantaneous distributions of wall static pressure and the wall shear stress in the \(yz\)-plane at a certain time. It was obtained by employing Eqs. (12), (17) and (18). Figure 12(a) shows that the coating thickness varies very widely along the stagnation line in the impinging stagnation region. And Fig. 12(b) reveals that the coating thickness first becomes thinnest where the wall pressure gradient has its maximum value and then it grows a little while until the end of the stagnation region at about \(y = 20\) mm. Such a growth has been shown in previous studies.\(^{3,8–10,19,20}\)

In order to find out such a growth at far downstream, the computational domain in the longitudinal direction is extended up to 90 mm, and the same grid spacing and time step have been used. Table 1 shows the computational results for computed film thickness averaged over the transverse direction at various \(y\)-locations and the standard deviation of the fluctuating film thickness over the transverse direction. As can be seen, the average coating thickness increases gradually up to about \(y = 30\) mm and thereafter it remains more or less the same of about 24.2 \(\mu\)m, and the standard deviation of the coating thickness decreases from about 1.5 to 0.3 \(\mu\)m. Considering that the real coating thickness is about 22–23 \(\mu\)m with standard deviation of about 2 \(\mu\)m, the final sag line formation may be determined in a range \(y < 10\) \(\mu\)m. In order to determine the location where the real sag lines are formed, the figurations of the coated uneven surface with sag lines are numerically produced, and they are shown in Fig. 13, and for comparison purpose it includes a picture of a real coated surface (with 3rd grade sag lines). Note that all of the figurations in Fig. 13 are drawn in the same scale.

Figures 13(a)–13(f) display the simulated sag lines when the figuration of the film surface is determined at \(y = 6, 10, 20, 30, 50\) and 80 mm, respectively. Here, again recall that the numerically simulated surface is generated by a combination of the alternatingly sidewise movement of the uneven coating surface profile at the given \(y\)-location and the upward feed of the steel strip result in the sag line formation on the steel surface.

On comparing the simulated surface figurations with the picture of the real film surface, it is found that numerically simulated surfaces determined at \(y = 6\) mm in the stagnation region, are nearly similar to the picture. From these observations of the average film thickness, its standard deviation and the surface figuration, it may be concluded that the film surface is determined in the stagnation region.

Now, an important question may arise: At \(y = 6\) mm or 10 mm, the film temperature is about 455–460°C, well above the solidification temperature of the molten zinc.

<table>
<thead>
<tr>
<th>(y) (mm)</th>
<th>(\bar{\delta}) ((\mu)m)</th>
<th>(\delta') ((\mu)m)</th>
</tr>
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<tbody>
<tr>
<td>6</td>
<td>21.5</td>
<td>±1.5</td>
</tr>
<tr>
<td>10</td>
<td>23</td>
<td>±0.9</td>
</tr>
<tr>
<td>20</td>
<td>23.8</td>
<td>±0.8</td>
</tr>
<tr>
<td>30</td>
<td>24.2</td>
<td>±0.5</td>
</tr>
<tr>
<td>50</td>
<td>24.2</td>
<td>±0.3</td>
</tr>
<tr>
<td>80</td>
<td>24.2</td>
<td>±0.3</td>
</tr>
</tbody>
</table>

Fig. 13. Comparison of real sag line picture with simulated sag lines determined at different \(y\)-locations. (c) \(y = 6\) mm, (d) \(y = 10\) mm, (e) \(y = 20\) mm, (f) \(y = 30\) mm, (g) \(y = 50\) mm, (h) \(y = 80\) mm.
Then why could the film surface determined at such high temperature be unchanged downstream? In order to theorize the problem, let us consider a local time scale in the molten zinc film. In the stagnation region, major parameters for the relaxation of the molten zinc film, highly strained by the static pressure, are the local wall shear stress and the viscosity of the molten zinc. Therefore, the relaxation time scale is given by the following equation. \[ t_{\text{relax}} = \frac{\mu}{\tau_w} \] (19)

Physically, the relaxation time indicates the time duration within which the very viscous molten zinc film completes to be deformed responding to the local external disturbance such as the static pressure and wall shear stress. As can be confirmed in Fig. 5, since the wall shear stress is very close to zero in the impingement region, the relaxation time is in a range \( t_{\text{relax}} \geq 0.5 \text{s} \). This means that the surface figuration of the zinc film formed in the impingement region maintains its configuration for a while during its upward movement. And after the relaxation time of about 0.5 s, the steel strip has moved upward by about 1.2 m where the film temperature dropped down below the solidification temperature. Following this reason, we may conclude that the uneven film surface figuration is determined in the stagnation region. In addition, the mean distance between the nearby crests is 4 mm±10% which is nearly the same as the experimentally measured distance in Fig. 3.

5. Conclusion

The sag line formation on galvanized strip surface caused by the gas jet-wiping process has been studied by numerical simulation and analytical modeling. In order to simulate the sag line formation after gas wiping in a continuous hot-dip galvanizing process, CFD simulation for the 3-D compressible turbulent flow field around the air knife has been carried out by using the commercial code, FLUENT. LES technique was used to simulate the unstable and complex 3-D flow field. It was confirmed that the periodically alternating sidewise movement of the peak pressure points along the transverse stagnation line with the upward moving steel strip at a constant speed results in the sag line formation on the zinc coating strip surface.

A simple mathematical model has been suggested to predict the coating thickness distribution along the transverse direction. The coating thickness calculated by our new model shows good agreement with that obtained by the existing model. Our proposed model is an explicit formula to predict the film thickness, and it reveals better understanding about the relationship between the unevenness of the zinc film thickness and wiping parameters such as static pressure gradient and wall shear stress.

Finally, the 3-D uneven surface figuration with the sag lines on the zinc coated steel strip was successfully generated by using the perturbation model and wiping factors obtained from our numerical simulation. Relaxation time is introduced to interpret the position where the sag lines are determined.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>Taylor’s micro-length scale (m)</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Turbulent energy dissipation rate (m(^2)/s(^2))</td>
</tr>
<tr>
<td>( u )</td>
<td>Liquid velocity in x-direction (m/s)</td>
</tr>
<tr>
<td>( v )</td>
<td>Liquid velocity in y-direction (m/s)</td>
</tr>
<tr>
<td>( x )</td>
<td>Distance of horizontal direction (m)</td>
</tr>
<tr>
<td>( y )</td>
<td>Distance of vertical direction (m)</td>
</tr>
<tr>
<td>( g )</td>
<td>Acceleration of gravity (m/s(^2))</td>
</tr>
<tr>
<td>( p )</td>
<td>Density of liquid zinc (kg/m(^3))</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Dynamic viscosity of liquid zinc (Pa·s)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Impinging pressure (Pa)</td>
</tr>
<tr>
<td>( V_c )</td>
<td>Moving velocity of the steel strip (m/s)</td>
</tr>
<tr>
<td>( \tau_w )</td>
<td>Wall shear stress (Pa)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Coating thickness (m)</td>
</tr>
<tr>
<td>( q )</td>
<td>Volumetric liquid flow rate per unit width of strip (m(^3)/s)</td>
</tr>
<tr>
<td>( d )</td>
<td>Nozzle slot height (m)</td>
</tr>
<tr>
<td>( \nabla P )</td>
<td>Dimensionless pressure gradient in y-direction</td>
</tr>
<tr>
<td>( \nabla P' )</td>
<td>Mean of dimensionless pressure gradient over the z-direction</td>
</tr>
<tr>
<td>( \hat{T} )</td>
<td>Dimensionless shear stress</td>
</tr>
<tr>
<td>( \hat{\tau} )</td>
<td>Mean of dimensionless shear stress</td>
</tr>
<tr>
<td>( \hat{T}' )</td>
<td>Fluctuation of dimensionless shear stress</td>
</tr>
<tr>
<td>( \hat{\delta} )</td>
<td>Dimensionless coating thickness</td>
</tr>
<tr>
<td>( \delta_0 )</td>
<td>Unperturbed dimensionless coating thickness</td>
</tr>
<tr>
<td>( \delta' )</td>
<td>Deviation value over the unperturbed dimensionless coating thickness</td>
</tr>
</tbody>
</table>

REFERENCES