1. Introduction

As already reported\(^1\) we have proposed the Utility of Stock hypothesis which assumes that an in-use stock of constructional material is a function of GDP. And we have found a clear correlation between the world steel stock, which was computed in the simplest method for estimation, and the world GDP. It has become clear that the relationship between the world GDP and the steel stock can be approximated as linear. It has led to the estimation that the world demand for iron ore (primary iron) depends not on the volume of GDP but on the variation of GDP, as already reported. In this study, the world steel stock in use is computed. Sensitivity analyses are conducted to show the effect of lower reliable data such as the usage period (lifetime) of iron-containing final products. Clear correlation is found between the in-use steel stock and the steel stock. Hence, the Utility of Stock hypothesis is verified.

KEY WORDS: iron source; steel; demand; stock; outlook; in-use; economy; simulation; modeling; lifetime; acceleration principle.

2. Material Flow to Compute In-use Steel Stock

Figure 1 shows the material flow to compute the world steel stock in use, \(S_o\), which represents the stock of obsolete products, was added to the material flow in the previous article\(^1\). Concerning the material flow, whose unit is Fe equivalent ton, Eqs. (1)–(10) were assumed.

\[ F' = F'_2 + F'_3 + F'_4 + F'_5 \]  \hspace{1cm} (1)
\[ F' = F'_2 + F'_3 + F'_4 + F'_5 \]  \hspace{1cm} (2)
\[ F'_2 = F'_1 - F'_2 - F'_4 - F'_5 \]  \hspace{1cm} (3)
\[ F'_2 = F'_2 + F'_3 + F'_4 \]  \hspace{1cm} (4)
\[ F'_2 = F'_1 + F'_2 + F'_3 \]  \hspace{1cm} (5)

\(^1\)Another remaining issue, which is to compute the consumption of scrap (secondary iron) and analyze the relationship with the production of iron ore (primary iron) and crude steel, is reported separately.\(^2\)
3. Method to Compute In-use Steel Stock

3.1. Method to Compute In-use Steel Stock

In this study, the top-down approach, which is based on the data of iron and steel production and has advantage to fit analysis of time-series, was taken to compute the in-use steel stock \( S(u) \), in the same way as the computation of steel stock \( S(t) \) in the previous article.\(^1\) The production of crude steel \( F \), which is long-term reliable datum, was core datum to compute the in-use steel stock. As shown in Sec. 3.2–3.7, \( F \) \(_1\), which represents amount of iron contained in final products such as buildings and automobiles, was estimated by using yield ratio of steelmaking process, scrap ratio and others. And \( F \) \(_2\), which represents amount of iron contained in obsolete products, was estimated by assuming lifetime of iron-containing final products. Then the in-use steel stock \( S(u) \) was computed by Eq. (8).

\[
S(u) = \sum_{t=1870}^{t} (F_2(t) - F_3(t) - F_4(t))
\]

3.2. Iron Input into Steel Cycle in Use and Production of Crude Steel

\( F_1 \), which represents iron input into the steel cycle in use, or primary iron, and \( X \), which represents GDP, are identical to those of the previous article.\(^1\) The data sources of \( F_2 \), which represents the production of crude steel, are the same as those of the world production of pig iron in \( F_1 \); Statistisches Bundesamt Deutschland over the period 1870–1996 and IISI (International Iron and Steel Institute) over the period 1967–2005.

3.3. Steelmaking Process \((P_2)\)

3.3.1. Data for Material Flow of Steelmaking Process

\( F^{n}_{i} \) \(_{45B} \), which represents amount of iron which goes into steelmaking process, is equal to the sum of primary iron into steelmaking process \( F_{1} - F_{1}\) and secondary iron into steelmaking process \( F_{2} - F_{2}\), as expressed by Eq. (3). The data of primary iron are available as \( F_1 \). However, the breakdown into \( F_{1}\) and \( F_{2}\), which means primary iron used to produce crude steel, and \( F_{3}\), which means primary iron used in foundry process, is not available before 1983. Concerning secondary iron, decisive statistical data on total scrap consumption to produce crude steel and iron casting are not available.\(^{31}\)

Shimomura et al.\(^{51}\) have advocated that consumption of pig iron and scrap in foundry process can be neglected, because production of iron casting in foundry process is smaller than that of crude steel in steelmaking process. They have also neglected scrap and uncollected amount of iron (amount of iron which goes out from the steel cycle in use) generated in steelmaking process, and estimated scrap consumption for the production of crude steel by calculating difference between production of crude steel and consumption of iron ore. Neelis et al.\(^{4}\) and Ohji et al.\(^{6}\) have estimated amount of iron which goes into steelmaking process based on the production of crude steel and an available yield ratio of a certain country in a certain year. Collected amount of iron (scrap) and uncollected amount of iron (amount of iron which goes out from the steel cycle in use) have been estimated based on the production of crude steel with a recovery ratio and an unrecoverable ratio of a certain country in a certain year.

Table 1 shows existing studies on yield ratio of steel-making process including those mentioned above.\(^{6,12}\) It is inferred that different definitions of crude steel contribute to considerable variation in input–output ratio in Table 1. For example, the notes *7 and *8 in Table 1 indicate the following. With respect to ingot casting (top-poured ingot-making method), the input–output ratio of “liquid steel/crude steel” can be interpreted two ways: “liquid steel/steel ingot” and “liquid steel/slab”, because its process is “liquid steel→steel ingot→slab→steel plate”. With respect to continuous casting process, however, the input–output ratio of “liquid steel/crude steel” is determined as “liquid steel/slab”, because its process is “liquid steel→slab→steel plate”. When “crude steel” is defined as “steel ingot,” the input–output ratio of “liquid steel/crude steel” of Germany ingot-making method in 1964\(^{49}\) is 1.04 which is close to the value of the World Steel Dynamics (WSD).\(^{47}\) As commented in the note *7, however, when “crude steel” is defined as “slab”, which is consistent with continuous casting process, the input–output ratio is 1.20, which is close to the value of Ohji et al.\(^{6}\) and the value of US Steel in 1968.\(^{9}\)

Thus, the definition of crude steel seems to vary depending on the country and the year of statistics. Neelis et al.\(^{4}\) point out, “crude steel production in the statistics of the International Iron and Steel Institute used to be defined as the total output of usable ingots, continuously cast semi-finished products and liquid steel for casting (IISI, 1990), although no definition is given anymore in the most recent version of the statistical year”. While crude steel production in Japanese statistics is defined as usable ingots, crude steel production in statistics of some countries seems to be defined as ingots including nonusable ingots or semi-finished products such as slabs.

3.3.2. Method to Compute Material Flow of Steelmaking Process in this Study (Initial Assumption)

Long-term data of \( F_2 \), which represents the world production of crude steel, are available as mentioned in Sec. 3.2.

3.3.2.1. \( F^{n}_{3} \) (Amount of Iron Which Goes into Steelmaking Process)

In this study, \( F^{n}_{3} \) (amount of iron which goes into steel-making process) was obtained from \( F_3 \) (production of crude steel) by assuming yield ratios of steelmaking process and casting process based on Table 1.

\[ F^{n}_{3} = F_3 \]

\[^{12}\] Steel stock of obsolete products is accumulation of iron contained in obsolete products that have exited use but not entered scrap processing and waste management, such as end-of-life vehicles, illegal dumps, or temporary storage sites.\(^6\) Those obsolete products have not yet been decided to belong to \( F_2 \) or \( F_3 \) for over a year (the observation period).
The values of World Steel Dynamics (WSD) in 2003 were used for yield ratios of steelmaking (open-hearth furnace (OHF), basic oxygen furnace (BOF), electric arc furnace (EAF)) and yield ratios of casting process (ingot casting, continuous casting). We assumed that input–output ratios of Bessemer process, Thomas process and crucible process were the same as that of OHF (1.200). Time series variation of yield ratio was not considered, as Table 1 or upgrading of secondary refining does not indicate improvement of yield ratio of steelmaking process over time.

We assumed that pig iron contained about 96% iron and DRI (direct reduced iron) contained about 91% iron. Concerning scrap, we understood that return scrap contained about 98% iron and purchased scrap contained 90–95% iron. In this study, we simplified that input of iron source contained 96% iron.

Therefore, the values of WSD were multiplied by 0.96 to compute amount of iron for respective steelmaking process. As mentioned in Sec. 3.3.1, in this study, crude steel of ingot-making method was considered as steel ingot, while crude steel of continuous casting process was considered as semi-finished products such as slabs.

The values of World Steel Dynamics (WSD) in 2003 were used for yield ratios of steelmaking process. As mentioned in Sec. 3.3.1, in this study, crude steel of ingot-making method was considered as steel ingot, while crude steel of continuous casting process was considered as semi-finished products such as slabs.

3.4. Rolling Process ($P_3$)

### 3.4.1. Data for Material Flow of Rolling Process

Rolling process was defined as process to produce finished steel from crude steel. Long-range data of world production of finished steel are not available.

According to the German data in the report of the Iron and Institute of Japan (1996), the input–output ratio of rolling process is as follows. By the ingot-making method, "steel ingot/steel plate" is 1.390 and "slab/steel plate" is 1.200. By the continuous casting method "slab/steel plate" is 1.185. (Table 3) Neelis et al. used the yield ratio of rolling process in the IISI statistical yearbooks by which the import and export of

### Table 1. Existing studies on yield ratio of steelmaking process.

<table>
<thead>
<tr>
<th>Researchers (source)</th>
<th>WDS</th>
<th>Moli</th>
<th>Oji</th>
<th>ISIJ (1998)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country covered</td>
<td>U.S.A.</td>
<td>EU-15</td>
<td>World</td>
<td>Japan</td>
</tr>
<tr>
<td>Yield ratio of steelmaking process</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input of iron source / crude steel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input of iron source / liquid steel (OHF)</td>
<td>1.200</td>
<td>1.200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input of iron source / liquid steel (BOF)</td>
<td>1.170</td>
<td>1.140</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input of iron source / liquid steel (EAF)</td>
<td>1.111</td>
<td>1.060</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liquid steel / crude steel (ingot casting)</td>
<td>1.045</td>
<td>1.045</td>
<td></td>
<td>1.20</td>
</tr>
<tr>
<td>Liquid steel / crude steel (continuous casting)</td>
<td>1.030</td>
<td>1.030</td>
<td></td>
<td>1.08</td>
</tr>
<tr>
<td>Scrap ratio, Non recovery ratio</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1)</td>
<td>0.96</td>
<td>0.96</td>
<td></td>
<td>0.96</td>
</tr>
</tbody>
</table>

* The unit of "input of iron source" given in this table is gross ton but not Fe equivalent ton. When the iron content of "input of iron source" is 96%, input of iron source = (Ft × 1.00 × 1.00) + (Ft × 0.96 × 1.00) = Fp1.
*2 No recovery is assumed for the input of iron source not embodied in the production of liquid steel. Of the liquid steel not embodied in the crude steel, 60% is assumed to be recovered as scrap.
*3 In a study by the American Iron and Steel Institute, the total iron contained in by-products such as dust, sludge, slag etc. is estimated at 3.24 Mt on a total crude steel production of 59.26 Mt.
*4 A total input of 92.4 Mt of pig iron and 17.0 Mt of scrap is mentioned in BOF for a crude steel production of 98.3 Mt.
*5 Of the liquid steel not embodied in the crude steel, 100% is assumed to be recovered as scrap.
*6 The figures (1.20; 1.00) are obtained by "liquid steel / slab.
*7 This figure (1.04) is obtained by "liquid steel / steel ingot." 1.20 is calculated by "liquid steel / slab.
*8 This figure (1.06) is obtained by "liquid steel / slab.
*9 Of the liquid steel not embodied in the crude steel and finished products, 100% is assumed to be recovered as scrap.
*10 Acid Bassemer converter; 11 Bottom-blown basic converter; 12 OHF; 13 LD converter
*14 These figures (1.12-1.15) for OHF and (1.10-1.14) are obtained by "input of iron source / steel ingot." Therefore, they should be regarded as "input of iron source / crude steel.

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finished steel products are converted into crude steel equivalents by multiplication with the following factor:

\[
F_2(t) = \frac{1.3}{1 + 0.175c(t)} F_1(t) \quad \text{......(15)}
\]

From Eq. (15), “crude steel/finished steel” is 1.3 by the ingot-making method \((c(t)=0\%\)), 1.11 by the continuous casting process \((c(t)=100\%)\).

The IISI statistician points out that Eq. (15) can probably not be used anymore because it is from 1970’s and it is based on yield from old technology (less continuous casting at this time).13

3.4.2. Method to Compute Material Flow of Rolling Process in This Study (Initial Assumption)

Long-range data of production of hot-rolled steel in the world’s leading economies are available. Therefore, in this study, the production of finished steel \((F_3)\) was obtained from “crude steel/hot-rolled steel” \((F_2/F_{2–3})\) and “hot-rolled steel/finished steel” \((F_{2–3}/F_3)\).

3.4.2.1. “Crude Steel/Hot-rolled Steel” \((F_2/F_{2–3})\)

“Crude steel/hot-rolled steel” \((F_2/F_{2–3})\) for 1990–2005 was calculated from the IISI statistics of production of crude steel and hot-rolled steel for 10 main countries and 1 region (EU15). In the United States and China, “crude steel/hot-rolled steel” \((F_2/F_{2–3})\) was set at 1.1 since 1970’s and it is based on yield from old technology (less continuous casting at this time).13

3.4.2.2. “Hot-rolled Steel/Finished Steel” \((F_{2–3}/F_3)\)

Concerning Japanese production of steel for fiscal 1980–2006, “crude steel/hot-rolled steel” \((F_2/F_{2–3})\) varies between 1.05 and 1.12 (i.e. in the range of 7.6%), but “hot-rolled steel/finished steel” \((F_{2–3}/F_3)\) varies between 1.02 and 1.03 (i.e. in the range of 1.0%). In this study, “hot-rolled steel/finished steel” \((F_{2–3}/F_3)\) is set to 1.02, which was obtained by the cumulative total value in Japan for fiscal 1980–2006, and the time series variation was not taken into account.

3.4.2.3. Finished Steel \((F_3)\)

Finished steel \((F_3)\) is calculated according to Eq. (16).

\[
F_3(t) = \frac{F_2(t)}{1.02(F_2/F_{2–3})(t)} \quad \text{......(16)}
\]

Amount of iron losses during rolling process \((F'_3)\) was assumed to be negligible, as Neelis et al.4) assumed.

\[
F'_3(t) = 0 \quad \text{......(17)}
\]

\[
F'_3(t) = F_3(t) - F_3(t) \quad \text{......(18)}
\]
3.5. Foundry Process (P_F)

3.5.1. Data for Material Flow of Foundry Process

Although the data of world production of iron casting from foundries for 1970–2005 are available on the statistics of Modern Casting, the previous data are not available.

As mentioned in Sec. 3.3.1, Shimomura et al.\(^4\) assume that consumption of pig iron and scrap for foundry process can be neglected, because production of iron casting from foundry process is smaller than that of crude steel from steelmaking process.

In contrast, Neelis et al.\(^4\) state that foundries are important consumers of scrap and can therefore not be neglected in the material flow analysis. They assume the foundry production to be a certain percentage of the crude steel consumption. They use the ratio of foundry iron production compared to apparent crude steel consumption as derived for the US for all regions. For the year 1900, they use a ratio of 0.4. For the year 1938, they use a ratio of 0.17, and they use linear interpolations between 1900 and 1938. For the projection until 2100, they use the ratio of 2003 (0.08) as constant ratio for all years.

3.5.2. Method to Compute Material Flow of Foundry Process in this Study (Initial Assumption)

In this study, the data of \(F_F\), which represents the world foundry production, are from the statistics of Modern Casting for 1970–2005, and the previous data (i.e. for 1870–1969) were calculated based on “foundry iron/crude steel” (“\(F_F/F_{Cr}\)”), in the same way as in Neelis et al.\(^4\). For the year 1970, we used a ratio of 0.124, which is calculated from the above statistics. For the year 1938 and the year 1900, we used the same ratio as that in Neelis et al. (i.e. 1938 (0.17) and 1900 (0.4)), and we used linear interpolations for other years. Figure 3 shows changes of “foundry production” (“\(F_F^t\)” and “foundry iron/crude steel” (“\(F_F/F_{Cr}\)”)).

Amount of iron losses during foundry process (“\(F_F^t\)”) was assumed to be negligible, as Neelis et al.\(^4\):

\[
F_F^t(t) = 0 \quad \text{..................}(19)
\]

3.6. Manufacturing Process of Iron-containing Final Products (P_F)


As there are no statistics of world total amount of iron in iron-containing final products such as buildings and automobiles, it is necessary to estimate steel stock.\(^1\)

In Japan, the Japan Ferrous Raw Material Association has conducted interview surveys on amount of scrap generation and shipping and amount of steel consumption from steel consumers.\(^1\) Neelis et al.\(^4\) conducted literature research on prompt scrap ratios, which represent the ratio of amount of prompt scrap relative to amount of finished steel consumption or foundry consumption. Table 4 shows these existing studies.

3.6.2. Method to Compute Material Flow of Manufacturing Process of Iron-containing Final Products in this Study (Initial Assumption)

\(F_F\), which represents the world total amount of iron in iron-containing final products, was obtained by assuming the ratio of scrap generation in manufacturing process of iron-containing final products (“\(F_F^t\)” compared to production of finished steel (“\(F_{Fr}\)” or “\(F_{Fr}\)”)). From Table 4, we used ratios as follows: “\(F_F^t\) from \(F_{Fr}\)” : 1900 (0.3), 1975 (0.2), 2003 (0.13), “\(F_F^t\) from \(F_{Fr}\)” : 1975 (0.15), 2003 (0.14). We used linear interpolations for other years, assuming that the ratio declines with each passing year, as the prompt scrap is a key factor for cost in manufacturing. The prompt scrap (“\(F_F^t\)” was calculated according to Eq. (20).

\[
F_F^t(t) = \frac{F_F^t}{F_{Fr}}(t)F_{Cr}(t) + \frac{F_F^t}{F_{Cr}}(t)F_{Fr}(t) \quad \text{...(20)}
\]

Amount of iron losses during manufacturing process of iron-containing final products (“\(F_F^t\)”) was assumed to be negligible, as Neelis et al.\(^4\):

\[
F_F^t(t) = 0 \quad \text{..................}(21)
\]

Amount of iron in iron-containing final products (“\(F_F\)” was calculated by Eq. (7) and Eq. (21).

\[
F_F(t) = F_F^t(t) + F_F^t(t) - F_F^t(t) \quad \text{...(22)}
\]

3.7. Generating Process of Obsolete Products

3.7.1. Data for Material Flow of Process of Obsolete Products

Unlike the material flows of manufacturing process described from Sec. 3.3 to 3.6, \(F_S\), which represents amount of iron contained in obsolete products, does not depend on the production for current year, but on the consumption of iron-containing final products in past years and the lifetime of these products.

From in-use steel stock (“\(S_u(t)\)”), \(F_S(t)\) is generated after the lifetimes of iron-containing final products (i.e. that have existed use). It is divided into three groups: (i) \(F_S(t)\), which represents obsolete scrap that is recycled within the steel cycle in use for the year, (ii) \(F_S(t)\), which represents iron losses that goes out from the steel cycle in use for the year, (iii) \(S_d(t)\), which represents stock of obsolete products that...
have not yet been decided to belong to $F(t)$ or $F(t)$ over the year (observation period).

Existing studies have calculated the amount of obsolete products ($F_t$) by assuming the product mix made from steel and the lifetime of these products.

Regarding the amount in Japan, there are estimations by the Japan Ferrous Raw Material Association (JFRMA) and by Daigo et al. The Japan Ferrous Raw Material Association categorizes consumers of finished steel into 11 sectors. The consumption of finished steel for each sector is obtained by statistics. The lifetime in 9 sectors is estimated by expected life (top-down approach), while the lifetime in the 2 sectors of buildings and automobiles is estimated through a bottom-up approach. Average lifetime of sectors is assumed from their statutory useful life. At first, no lifetime distributions were assumed, that is 100% generation of obsolete products in just the year of average lifetime. As obsolete products are generated in the vicinity of the year, in fact, normal lifetime distributions have been assumed recently. The amount of obsolete products in the 2 sectors of buildings and automobiles is calculated by summing in-use stock of iron-containing final products in the end of the previous year and production of iron-containing final products in the current year, from which deducting in-use stock of iron-containing final products in the end of the current year, as there are statistics of product stocks for buildings and automobiles. Then, amount of iron contained in obsolete products is calculated by multiplying each specific iron content by each obsolete product. The JFRMA analyzes that the average expected life of 17 years, which is obtained by weighted average of all sectors with the share, is consistent with the gap of 17 years between the trend line of recovered amount of obsolete scraps and the trend line of steel stock in 3-year moving average.

Daigo et al. categorizes consumers of finished steel into 7 sectors and estimates the amount of obsolete products by assuming the lifetime of these products. They use nonparametric lifetime distributions from statistics for 2 sectors of cars and trucks and parametric lifetime distributions of Weibull distribution function (cumulative density function) for the other sectors.

Muller et al. and Neelis et al. estimate the amount of obsolete products by assuming the normal lifetime distributions. Dahlstrom et al. estimate the amount of obsolete products by using three types of lifetime distributions (no distribution, log-normal distribution and Weibull distribution). Table 5 shows the lifetime assumptions used by existing studies.
3.7.2. Method to Compute Material Flow of Process of Obsolete Products in this Study (Initial Assumption)

Table 6 shows the initial assumption of the share of iron amount, the average lifetime and the lifetime distribution in this study. The share of iron amount and the average lifetime are data by JFRMA. Normal lifetime distribution was used by assuming the standard deviation with reference to the data of Daigo et al.,16 Neelis et al.,8 and Muller et al.9,10 Amount of iron in generation of iron-containing obsolete products ($F_t$) was calculated by Eq. (23), Eq. (24) and Eq. (25).

\[
F_{ts}(t) = F_t(t)S_{ts}..........................(23)
\]

\[
F_{ts}(t) = \sum_{t=1970}^{t} \frac{1}{\sigma_s \sqrt{2\pi}} \int_{-1}^{1} e^{-\left(\frac{(x-\tau_s)^2}{2\sigma_s^2}\right)} dx \times F_{ts}(\tau)...............................(24)
\]

\[
S_{ts} = \frac{1}{\sigma_s} \int_{-1}^{1} e^{-\left(\frac{(x-\tau_s)^2}{2\sigma_s^2}\right)} dx 
\]

\[
F_{ts}(t) = \sum_{t=1970}^{t} F_{ts}(t)..........................(25)
\]

Figure 4 shows in-use steel stock ($S_u(t)$) computed in this study (initial assumption). Figure 5 shows the relationship between World GDP (PPP) and World in-use steel stock, 1870–2005.
4.2.2. Evaluation of Method to Compute In-use Steel Stock by Comparison of Steel Stock

The steel stock $S(t)$ can be an approximate quantity of the in-use steel stock $S_u(t)$, provided that $F_{1}$, which is a flow going out from the steel cycle in use, is negligible small. As the steel stock can be computed with relative accuracy, it can be evaluated that uncertainty of data to compute the in-use steel stock is small if the steel stock can be approximate on the in-use steel stock.

Figure 8 shows the world steel stock $(S(t))$ obtained in our previous article and the in-use steel stock $(S_u(t))$ is around 10.9 billion tons, which means that $S_u/S$ is 39.2%. This figure (39.2%) is too small to justify the presumption in our previous article that the steel stock $S(t)$ can be an approximate quantity of the in-use steel stock $S_u(t)$. And also, this figure seems small from the view of IISI “When steel products reach the end of their useful life, at least 85% of the material is recycled”, although $S_u/S$ is not identical with recycling rate.

4.2.3. Sensitivity Analyses of Method to Compute In-use Steel Stock

The computation of in-use steel stock is uncertain, as we made a rough estimation of yield ratio, scrap ratio, foundry production, lifetime of iron-containing final products and others with values in a country and in a year. As mentioned above, in addition, $S_u/S$ is smaller than we expected initially.

With respect to items obtained from uncertain data, sensitivity analyses were conducted in the direction of increase of $S_u/S$ from 39.2% (initial assumption). We considered that the data of $F_1$, which represents iron input into the steel cycle in use, the data of $F_2$, which represents the production of crude steel, and the data of $F_3$, which represents the production of iron casting from calculated by subtracting $S_u(t)$ from $S(t)$ in Fig. 8 is demarcated as the amount of iron which goes out from the steel cycle in use during manufacturing process $(\sum F_2 + F_3 + F_4)$ and the unused obsolete products $(S_o + \sum F_5)$.

In 2005, the steel stock $(S(t))$ is around 27.8 billion tons (Fe equivalent ton), and the in-use steel stock $(S_u(t))$ is around 10.9 billion tons, which means that $S_u/S$ is 39.2%. This figure (39.2%) is too small to justify the presumption in our previous article that the steel stock $S(t)$ can be an approximate quantity of the in-use steel stock $S_u(t)$. And also, this figure seems small from the view of IISI “When steel products reach the end of their useful life, at least 85% of the material is recycled”, although $S_u/S$ is not identical with recycling rate.
foundries were reliable.

4.2.3.1. Sensitivity Analysis with Respect to Item of Steelmaking Process
When varying input data for material flow of steelmaking process except \(F_2\), which we regarded as reliable, \(F_2^a\), \(F_2\) and \(F_2^a\) change but in-use steel stock \(S_u(t)\) does not change. Therefore, no sensitivity analysis with respect to item of steelmaking process was conducted.

4.2.3.2. Sensitivity Analysis with respect to Item of Foundry Process (Sensitivity Assumption I)
As we considered that the data of world production of iron casting from foundries \((F_F)\) for 1970–2005 were reliable, a sensitivity analysis with respect to item of foundry process before 1970 was conducted. The initial assumption employed linear interpolation of \(F_F^a/F_F\) with two straight lines for 1870–1938 and 1938–1970. The sensitivity analysis with respect to item of foundry process (Sensitivity Assumption I) employed linear interpolation of \(F_F/F_F^a\) with one straight line for 1870–1970, maintaining two points of \(F_F/F_F^a: 0.4\) in 1900, and 0.124 in 1970.

4.2.3.3. Sensitivity Analysis with Respect to Item of Rolling Process and Manufacturing Process of Iron-containing Final Products (Sensitivity Assumption II)
The sensitivity analysis with respect to item of rolling process and manufacturing process of iron-containing final products (Sensitivity Assumption II) employed yielding ratios for 1870–2005 of rolling process and manufacturing process of iron-containing final products as 100%, that is, \(F_3^a=0\) and \(F_4^a=0\).

4.2.3.4. Sensitivity Analysis with Respect to Item of Generating Process of Obsolete Products (Sensitivity Assumption III)
The sensitivity analysis with respect to item of generating process of obsolete products (Sensitivity Assumption III) doubled the average lifetime of iron-containing final products to calculate generation of obsolete products, while maintaining the share of iron amount and the lifetime distribution in Table 6.

4.2.3.5. Result of Sensitivity Analyses
Table 8 shows the results of initial assumption and sensitivity analyses according to the above mentioned (2)–(4). While \(S_u/S\) vary from 39.2 to 55.2%, correlation coefficient \(R\) of \(S_u\) and \(S\) are from 0.990 to 0.999, which mean clear correlation. The uncertainty of the exogenous data in this study makes it difficult to estimate the in-use steel stock, but we suppose that the uncertainty does not significantly affect the conclusion that there is a clear correlation between \(S_u\) and \(S\). The presumption in our previous article \(^1\) which is that the steel stock \(S(t)\) can be an approximate quantity of the in-use steel stock \(S_u(t)\), can not be verified. However, it does not affect the conclusion in our previous article that there is a clear correlation between the world steel stock \((S)\) and GDP, which is based on the utility of stock hypothesis that the in-use steel stock \((S_u)\) is a function of GDP.

4.2.3.6. Additional Sensitivity Analyses
Table 8 suggests that the only way to bring \(S_u/S\) close to 100% is lifetime extension of iron-containing final products to calculate generation of obsolete products. Hence, we conducted additional sensitivity analyses to observe the influence of lifetime extension. The analyses were conducted with a simple lifetime distribution function, which is no distribution and generates obsolete products in bursts after a common lifetime for all kinds of iron-containing final products (Table 9). \(S_u/S\) in 2005 was 78.6% when the lifetime \((\tau)\) is 40 years, and 87.2% when the lifetime \((\tau)\) is 50 years. While \(S_u/S\) increases according to lifetime extension, it does not significantly affect the conclusion that there is a clear correlation between \(S_u\) and \(S\). The above assumptions of lifetime such as 40 years and 50 years are longer than those in existing studies. Having based on the premise that the data of \(F_F\), which represents iron input into the steel cycle in use, the data of \(F_F\), which represents the production of crude steel, and the data of world production of iron casting from foundries \((F_F)\) for 1970–2005 are reliable, there is a possibility that world recycling rate of iron-containing final products after lifetime is not so much high. However, there is also a possibility that the presumption that stocks in process of manufacturing and distribution were negligible makes \(S_u/S\) in this study lower than the true recycling rate. Furthermore, it should be taken into account that some of the unused obsolete prod-

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<tbody>
<tr>
<td>S: World steel stock</td>
<td>27.795</td>
<td>0.996</td>
<td>1982</td>
<td>1.000</td>
</tr>
<tr>
<td>Su: World in-use steel stock</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Initial assumption</td>
<td>10.904</td>
<td>0.991</td>
<td>2012</td>
<td>0.997</td>
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<tr>
<td>Sensitivity analysis</td>
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<tr>
<td>Sensitivity assumption I *</td>
<td>10.910</td>
<td>0.990</td>
<td>2012</td>
<td>0.997</td>
</tr>
<tr>
<td>(increase of foundry production)</td>
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<tr>
<td>Sensitivity assumption II **</td>
<td>14.988</td>
<td>0.979</td>
<td>2012</td>
<td>0.990</td>
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<tr>
<td>(increase of process yield)</td>
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<tr>
<td>Sensitivity assumption III ***</td>
<td>15.340</td>
<td>0.995</td>
<td>2012</td>
<td>0.999</td>
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<tr>
<td>(lifetime extension)</td>
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* Linear interpolation between 1870 year and 1970 year for the ratio of foundry iron production compared to crude steel production is assumed, while we assume two linear interpolations on the initial assumption (1870–1938, 1938–2000).
** Perfect process yield is assumed after steelmaking process, namely \(F_F^a=0\) and \(F_F=0\).
*** Doubling process yield is assumed after steelmaking process, namely \(F_F^a=0\) and \(F_F=0\).
ucts ($S_u + \sum F_j$) will be recycled, and there are semipermanent iron-containing final products ($F_j$) in civil engineering and construction such as dam, tunnel and foundation pile.

5. Conclusion

In this study, the world steel stock in use ($S_u$) was computed. $S_u$ in 2005, which was obtained on the initial assumption as a result of our effort to fit reality from available data, is around 27.8 billion tons, and $S_u/S$, which is the ratio of the in-use steel stock against the steel stock, is 39.2%. Although the presumption in our previous article, which is that the steel stock can be an approximate quantity of the in-use steel stock ($S_u$), could not be verified, there is a clear correlation between $S_u$ and $S$ in all sensitivity analyses. Hence, the conclusion that there is a clear correlation between the world steel stock ($S$) and GDP, which is based on the utility of stock hypothesis that the in-use steel stock ($S_u$) is a function of GDP, was verified.

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REFERENCES

9) Iron and Steel Institute of Japan: History of Continuous Cast Technology for Steel in Japan (Wagakuni niokeru Hagane no Rezoku-tyouzoujutsushi), Tokyo, (1996), 64, 90, 328.
12) T. Mitsu: Economic Efficiency of Large Electric Furnace and Basic Oxygen Furnace (Ogatadenryo oyobi Sansouwabukitenero no Keizai-sei), Tekkokai, Japan Iron and Steel Federation, Tokyo, July issue (1955), 14.