The Fractal Multiscale Trend Decomposition of Silicon Content in Blast Furnace Hot Metal

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With the data of silicon content in the hot metal collected from the three blast furnaces as a sample space, the multiscale trend decomposition was employed to analyze the local trend of time series. The singularity of fluctuation at different scales was quantified by the Hurst index. The Horton-Strahler topological classification of ramified pattern was constructed to identify the difference between Hurst index computed on different time series. Simulation results demonstrate that time series of silicon content in the hot metal from the three different blast furnaces exhibits significant local singularity. The duration of increasing and decreasing trends as well as the fluctuation of silicon content series are the main cause of differences among the Hurst index computed.

KEY WORDS: Hurst index; fractal dimension; multiscale trend decomposition; silicon content in hot metal.

1. Introduction

The steel industry is a key economic item for most of countries. As an important upstream procedure of steelmaking, the ironmaking process has received broad interest during the past decades for modeling and control purposes,1,2) especially for the furnace temperature modeling and control. Because of the difficult measurement conditions, the silicon content in hot metal ([Si]) is often taken as the chief indicator of the in-furnace thermal state, and has positive correlation with the furnace temperature. It is very important to control the silicon content at a certain level to facilitate the control of blast furnace ironmaking process. The control of [Si] sequence is always an important issue in the iron and steel industry. During the last decade, many methods have been applied to analysis [Si] series.3–9) To better control the blast furnace ironmaking process, it is necessary to examine the characteristics of silicon content series. Recently, the chaotic and fractal characteristics of silicon content series have attracted many research interests.10–12) It was found that the process changes often associate with time and equipment conditions. The silicon content series exhibit different kinds of characteristics at different levels and local conditions. In view of this, the current study tries to further explore the fractal characteristics, and thus to find more efficient approaches to control the blast furnace ironmaking process. Fractal phenomena can be observed in the real world, such as the coastline, trees with bifurcation, veins in the body etc.13–15) During the recent years, the fractal theory has developed as an important branch in mathematics.16) Many methods have been designed to estimate the fractal dimension, for example, the R/S analysis, the Whittle method and the wavelet analysis. Among these, the R/S analysis is the most popular. However, most of these methods cannot give the reliable estimation in the case of small sample problems.17) For the blast furnace ironmaking process, the silicon content data can only be collected every two hours, therefore collecting hundreds of data would take several months. For this kind of small sample data, the multiscale trend decomposition has proved to be a viable method.18–20) This paper utilizes multiscale trend decomposition to analyze the [Si] sequence and discusses the causes of divergence for the fractal dimension.

The multiscale trend decomposition was first proposed by Ilya Zliapin, and has been successfully applied to the field of the biomedicine, the geological survey etc. The multiscale trend decomposition analyzes the local trend of the time series X(t). It computes the Hurst index21) of X(t) at every scale, then determine the main cause of the Hurst index divergence and thus the cause of the fluctuation of silicon content series.

This article utilizes silicon content data collected from three blast furnaces for the multiscale trend decomposition. The Hurst index in every scale is firstly computed, and the Horton-Strahler topological method is used to classify the ramified patterns for every time series. The results are then analyzed in the framework of blast furnace ironmaking process. The remainder of this article is as follows. Section 2 describes the data used; Section 3 gives a brief introduction to the multiscale trend decomposition. Experimental results and analysis are given in Section 4 and Section 5 states our conclusions.

2. Data Description

The silicon content data used for analysis in this paper were collected from a large-size blast furnace (4700 m³, denoted as BF1), a medium-sized blast furnace (2500 m³, BF2) and a pint-size blast furnace (750 m³, BF3). The data is shown in the Fig. 1.

It can be seen from the Fig. 1 that the distribution of [Si]
series of the large-size blast furnace is relatively steady, while the series from the other 2 blast furnaces are more volatile. The statistics of the three [Si] series are shown in the Table 1.

As can be seen from the Table 1, [Si] series of BF3 demonstrates the biggest standard deviation, and a clear peak and fat tail can be observed, which are strong indications of fractal characteristics. Mean and standard deviation of BF1 [Si] sequence are smaller than that of BF2 and BF3, and its kurtosis is close to 3, which is close to a normal distribution. Therefore, compared with the small and medium size blast furnaces, the large-size blast furnace are more stable and the distribution of [Si] series is more similar to normal distribution.

### 3. Multiscale Trend Decomposition

The multiscale trend decomposition is a method used to approximate time series $X(t)$ by the piecewise linear least square function in every scale. The decomposition stops as a certain accuracy has been reached, then more detailed analysis can be done on the piecewise linear fitting data. The multiscale trend decomposition then constructs a tree structure called a hierarchical tree $T_X$ from a large scale to a small scale by linear fitting functions and fitting intervals. Hierarchical trees can be applied directly on the time series $X(t)$ to identify local trends.

#### 3.1. Method of Multiscale Trend Decomposition

First, the time series $X(t)$, $t \in [0,1]$ is approximated by the least square fitting $L_{0,t}^X(t)$ which describes the global trend of the series $X(t)$. Here, the superscript 1 means the numbers for dividing the area $[0,1]$ and the subscript 0 indicates being at level 0. The above fitting process may produce the vertex $v_0$ at level 0 of the hierarchical tree $T_X$. Then, the area $[0,1]$ needs to be partitioned further, which is thought to be an important issue to perform the subsequent multiscale trend decomposition. Generally, the more the partition subintervals are, the lower the fitting error is while the computational complexity will increase. A tradeoff should be made between the fitting accuracy and the computational complexity. To this task, the following function is defined

$$C_i^q(N_i^q, E_i^q) = \frac{\log(E_i^q / E_0)}{N_i^q - 1} \quad (1)$$

where $N_i^q$ is the number of intervals at the $i^{th}$ level corresponding to the $q^{th}$ division method, and $E_0$, $E_i^q$ are the fitting error of $X(t)$ by the linear fitting functions $L_i^q(t)$ at the level 0 and $L_i^q(t)$ at the $i^{th}$ level corresponding to the $q^{th}$ division method, respectively. The fitting error $E_i^q$ can be computed by

$$E_i^q = \sum_{t \in [0,1]} (X(t) - L_i^q(t))^2 \quad (2)$$

Finding the optimal points and the fitting error are a global optimization process. First, traverse $N$ from 1 to $T(T \geq 1)$, where $T$ is the maximum decomposition interval for each fixed $N$, list all the possible decomposition intervals in $i^{th}$ level by

$$\{A^q_i\}^{N}_{i=1} = [1, T], A_i^q \cap A_j^q = \emptyset, i = 1, ..., N, j = 1, ..., N$$

$$\{A^q_i\}^{N}_{i=1} \cup \{A^q_j\}^{N}_{j=1} = \Phi$$

Approximating $X(t)$ by the linear least square function and calculating the fitting error in each interval $A_i^q$, then we can get the optimal interval division method which makes the function $C(N, E)$ maximum. That is to say, the optimal $N_i^q, E_i^q$ satisfy the following relation

$$C_i^q(N_i^q, E_i^q) = \max_{N_i^q, E_i^q} C_i^q(N_i^q, E_i^q) \quad (4)$$

where $N_i^q$ and $E_i^q$ are optimal interval number and the fitting errors respectively.

Take the data of BF2 as an example to illustrate how to find the optimal intervals number $N^*$ and the error $E^*$ in the first level.

Traverse $N$ from 1 to 400. For each fixed $N$, list all possible decomposition interval ranges: $\{A^q_i\}^{N}_{i=1} = [1, 400], A_i^q \cap A_j^q = \emptyset, i = 1, ..., N, j = 1, ..., N$. Then approximate $X(t)$ by the linear least square function in each interval $A_i^q$, find interval number $N$ and the fitting error $E$ which maximizes $C(N, E)$. Define $N^*$ and $E^*$ as the first level’s interval number and the fitting error respectively. The results are shown in the Fig. 2.

In Fig. 2, graph (a) is the result of $X(t)$’s global linear fit, $L_0(t)$ is the global fitting curve; graph (b) is the result of the multiscale trend decomposition in the first level based on function $C(N, E)$, the optimal intervals number $N^* = 3$ and the corresponding points are 89 and 171 respectively, so the best division in the first level can be expressed as

$$\{A^q_i\}^{N}_{i=1} = [1, 400], A_i^q \cap A_j^q = \emptyset, i = 1, ..., N, j = 1, ..., N$$

$$\{A^q_i\}^{N}_{i=1} \cup \{A^q_j\}^{N}_{j=1} = \Phi$$

The hierarchical tree $T_X$ corresponding to the two-level decomposition of BF2 blast furnace [Si] sequence can be shown in Fig. 3.

Let $v_i^q$ represent the $i^{th}$ interval of the $q^{th}$ level in this decomposition and $v_i^0$ denote level 0. It can be seen that there
is only one interval in level 0, and the fitting linear function is monotonically decreasing. The trend is downward, so we describe this trend with a white circle. The linear fitting function in is monotonically increasing; and intervals in level 1, which are defined as , , , the linear fitting function in is monotonically decreasing. The trend is downward, so we describe this trend with a gray circle. Again, there are 3 intervals on the internal of these parent vertices. The interval lengths of the children vertices are multiplied by the interval lengths of the parents vertices, .

Repeat the above interval division and the least square approximation process to every , then we can get the piecewise linear fitting function in level 2. Denote as piecewise linear fitting function of time series in the th level, and divide the interval [0,1] into subintervals, denoted as

\[ I_l^i = \{ I_{l,i}^i, i = 1, ..., N_l \} \] .................. (6)

In every interval, the least square fitting function and the fitting error can be expressed as

\[ I_l^i(t), t \in I_{l,i}^i = \{ I_{l,i}^{i_1}, I_{l,i}^{i_2}, ..., I_{l,i}^{i_N} \}, i = 1, ..., N_l \] .................. (7)

\[ e_l^i = \sum_{t \in I_{l,i}^i} (X(t) - \hat{X}_l^i(t))^2 \] .................. (8)

For any \( l \)

\[ L_l(t) = \bigcup_{i=1}^{N_l} I_{l,i}^i(t), t \in I_l \] .................. (9)

And the global fitting error in the th level can be defined as

\[ E_l^2 = \sum_{i=1}^{N_l} (e_l^i)^2 = \sum_{t \in [0,1]} (X(t) - \hat{X}_l(t))^2 \] ........................ (10)

### 3.2. Computation of Hurst Index

Computation of the Hurst index, \( H \), of time series \( X(t) \) is one of the most important application of the multiscale trend decomposition. The Hurst index is a measurement of statistical correlation of a time series, it is an important indicator of the fractal dimension, \( D \). The relation between the fractal dimension and the Hurst index is \( H = 2-D \).

According to the self-affine properties of time series \( X(t) \), we can calculate Hurst index of \( X(t) \) through the multiscale trend decomposition. When \( X(t) \) satisfies the following equations, we refer to \( X(t) \) as a self-affine time series

\[ t' = rt \] ........................... (11)

we can use self-affine properties of time series to get the corresponding Hurst index. In the multiscale trend decomposition process, constructing a hierarchical tree is the application of the above mechanism. In hierarchical trees, child vertices are selected on the internal of these parent vertices. The interval lengths of the children vertices are multiplied by the interval lengths of the parents vertices, .

The Horton-Strahler topological structure define vertices without descendants as order 1. The internal vertices are those with descendants. If the internal vertex has several distinct descendants, then its order is the maximal order of all its descendants. On the other hand, if the order of all its descendants are equal, the order of this internal vertex is set as the order of its descendants plus 1. Take data from BF2 as an example, the Horton-Strahler topological classification of ramified patterns are constructed in the Fig. 4, assuming that in the level 2, the decomposition has reached a certain precision and stops. There are a total of 6 intervals \( v_i^2 (i = 1, ..., 6) \) in level 2, the order of these intervals in the

![Fig. 3. Hierarchical tree of multiscale trend decomposition](image)

![Fig. 4. Horton-Strahler topological classification of ramified patterns.](image)
Horton-Strahler topological classification is 1. As Fig. 4 shows, \( v_2^i, v_3^i \) are the descendants of \( v_1^i \) and they have the same order which is 2. Similarly, \( v_i \) has 3 descendants \( v_j^i (i = 1, 2, 3) \), and the order of the 3 intervals is different, so the order of \( v_i \) would be 2.

In the Horton-Strahler topological structure, different orders of intervals may distribute in different level of the hierarchical tree and the maximum order of this classification is less than the level of decomposition. The cause of fractal dimension divergence for different time series can be found according to the Horton-Strahler topological classification of ramified patterns.

In the Horton-Strahler topological structure, every vertex \( U_i^k \) in the \( i^{th} \) level of the hierarchical tree \( T_N \) corresponds to the order \( k \), the interval length \( r_i \), and the fitting error \( e_i \). Let \( N(k) \) denote the vertex number in the level \( k \); \( R(k) \) is the average of total interval length of the order \( k \); \( E(k) \) is the average fitting error of the order \( k \); \( B_N, B_R \) and \( B_E \) satisfy the following equations

\[
N(k) \sim 10^{-B_N k}; \quad R(k) \sim 10^{B_R k}; \quad E(k) \sim 10^{B_E k} \quad \ldots \quad (13)
\]

\( B_N \) describes its topological structure, \( B_R \) and \( B_E \) relate to the metric structure based on properties of the interval partition, which can be denoted as the \( r \)-metric and the \( e \)-metric. \( B_R \) characterizes intervals length of fluctuation trend; \( B_E \) reflects the fluctuation condition of time series which is related to the local trend. The vertex number \( N(k) \) and the average length \( R(k) \) of order \( k \) determine the fractal dimension of the hierarchical tree \( T_N \), and the formula is

\[
N(k) = R(k)^d \quad \ldots \quad (14)
\]

Where \( d \) is

\[
d = \frac{B_N}{B_R} \quad \ldots \quad (15)
\]

4. Experimental Results and Analysis

Apply the multiscale trend decomposition on the three [Si] sequences collected from the 3 different blast furnaces. First select a large decomposition scale in order to achieve sufficient decomposition, here we choose 40 levels. The results show that when decomposition conducts to 14 levels, the outcome remains unchanged, indicating that the time series are decomposed sufficiently, so the proper level for this process is 14.

The Hurst index for the 3 time series in every level is computed according to the equation (12), as shown in the Table 2.

From this table, we can see that all the Hurst index of three blast furnace [Si] series are less than 0.5, so the [Si] series are negatively correlated, i.e., when the trend of the [Si] in the previous interval tends to increase (decrease), the trend in the next interval is likely to decrease (increase). The possible reason is that as an aim level to be controlled, the silicon content is stationary in the long-term sense. With the multiscale trend decomposition going on, the Hurst index of BF1 increases from 0.041 to 0.311, for BF2 it increases from 0.034 to 0.154, while it for BF3 it increases from –0.071 to 0.116. Thus we can come to the following conclusions.

(1) In the Table 2, the Hurst index in the first level is called global Hurst index. The global Hurst index of BF3 blast furnace [Si] series is the smallest among the three [Si] series. For BF3 whose size is 750 m³, its temperature is less stable, and exhibits larger singularity.

(2) In the Table 2, when decomposition comes to the 14th level, the average decomposition interval lengths are 1.39, 2.04 and 3.09 respectively, and all Hurst indexes are less than 0.5. From this we can see that all the three [Si] sequence has the strong local singularity. Among the Hurst indexes of the three [Si] series, BF1 is the largest and BF3 is the smallest. The blast furnace iron-making process is a very complex physical and chemical process, in the short run, changing of material conditions and operating strategy leads to the dynamics change. For the medium and pint sized furnaces, they are easily influenced by the inside and outside fluctuation. Therefore the stronger local singularity may exist, while for a large size blast furnace, and it is more obvious, and they are less sensitive to small disturbances. The production for large-size furnaces is more stable, thus its local fractal dimension is less than the other two furnaces.

(3) As the decomposition goes on, the Hurst index of three [Si] series basically show an increasing trend, thus the fractal dimension are decreasing. Therefore the singularity is weakening. When constructing predictive models for the silicon content, we can consider constructing the model on this scale where [Si] series has lower singularity.

We can unify the hierarchical tree by Horton-Strahler topological classification of ramified patterns in order to analyze the causes of the Hurst index divergence for three [Si] series. According to the formula (10), we can obtain the fractal dimension of different orders, the results are shown in the Table 3.

As is shown in this table, after unified by the Horton-Strahler topological classification of ramified patterns of the three [Si] sequences, all the series of different orders show significant multifractral characteristics. Figure 5 is a comparison of the topology structure, the \( r \)-metric and the \( e \)-metrics for three [Si] series.

It is shown that for the three [Si] series, \( B_N \) is close to each other, while \( B_R \) and \( B_E \) are much different. From the Table 3 and Fig. 5 we can come to the following conclusions.

(1) As shown in the Fig. 5(a), the three [Si] sequences have

| Table 2. | Hurst index of [Si] series of BF1,BF2,BF3 at different scales. |
|-----------|------------------|------------------|------------------|
| data      | BF1      | BF2      | BF3      |
| level     | 1        | 1        | 1        | 1        | 1        | 1        |
| Hurst index | 0.041   | 0.311   | 0.034  | 0.154   | –0.071  | 0.116   |

| Table 3. | Fractal dimension of different orders in BF1, BF2, BF3 blast furnace [Si] series. |
|-----------|------------------|------------------|------------------|
| BF1       | order  | 1    | 2    | 3    | 4    | 5    |
| fractal dimension | 1.370  | 1.224 | 1.263 | 1.267 | 1.963 |
| BF2       | order  | 1    | 2    | 3    | 4    | –    |
| fractal dimension | 1.355  | 1.515 | 1.555 | 1.789 | –    |
| BF3       | order  | 1    | 2    | 3    | 4    | –    |
| fractal dimension | 1.308  | 1.093 | 1.422 | 2.242 | –    |
similar $B_N$ values, which represent the topology structure of hierarchical tree. Thus we can say that the three hierarchical trees have the similar topology structure. Topology structure of hierarchical tree denotes the blast furnace internal chemical reaction mechanism. Owing to the fact that all blast furnace have the similar chemical reaction mechanism, the topology structures of three hierarchical trees are similar to each other.

(2) As shown in Fig. 5(b), $B_E$ values are almost the same when order $k=1$ and $k=2$; however, the divergence becomes wider when $k>2$. The $B_E$ index represent the length of trend intervals of hierarchical tree, reflecting the duration of furnace temperature increasing or decreasing trend. Among the three [Si] series, the operation of BF1 blast furnace is more smooth. The average length of order $k=2$ interval, which is the average number of data points in the interval as 6.48, 6.78, 8.41 respectively. In other words, the duration of BF1 blast furnace temperature increasing or decreasing trend is 7 taps. We can consider 7 taps as a duration for BF2 and BF3 real-time stability control, thereby reducing the fractal dimension as well as the singularity inside the blast furnace. We take the 7 [Si] data points for a sample set, when a new data point arrives, if the temperature still has the trend of fluctuations increase (decrease), some measures should be used to reduce (increase) the temperature, for example, reducing (increasing) coke rate, reducing (increasing) the oxygen-enrichment. Through these measures, the fractal dimension and the probability of mutation can be decreased, thus stabilize the blast furnace operation.

(3) From the Fig. 5(c) we can see, $B_E$ represent the data condition relative to the local trend. When $k=1$, $2$, $3$, three [Si] series have quite different $B_E$ values. When $k>3$, $B_E$ of the three series approach. This results indicate that the three [Si] series are different in short-term fluctuations, so it is possible to make BF2 and BF3 as stable as BF1 by controlling their short-term fluctuations. In the blast furnace ironmaking process, the temperature fluctuation is sensitive to the feed rate, the permeability changes, the blast temperature and the pulverized coal injection. These variables have great effects on the short-term fluctuations of the blast furnace ironmaking process, and thus can be controlled to attain the above purpose.

(4) In the Horton-Strahler topological structure, fractal dimensions are different for different orders, so there is a significant difference between the singularity of the three series, which is the cause of the overall fractal characteristics. Therefore, we can model these sequence according to the fractal dimension to study the overall fractal characteristics, so that better results may be achieved.

5. Conclusions

In this paper, we use three [Si] series collected from the large-size, medium size and pint-sized blast furnaces from three steel companies to execute the multiscale trend decomposition process. The corresponding hierarchical tree shows that the three [Si] series exhibit obvious the multifractal characteristics as well as strong the local singularity. In the meantime, the hierarchical trees can be unified by Horton-Strahler topological classification of ramified patterns, thus the causes of the Hurst index divergence for [Si] series in different scales can be further analyzed. Comparing the topological structure, r-metric, e-metric of the three [Si] series, we come to the conclusion that the three hierarchical trees have the similar topological structure, while their r-metric and e-metric are obviously different. The duration of increasing and decreasing of silicon content in hot metal as well as the fluctuation of silicon content in hot metal are the main cause of differences among the Hurst index computed. Based on the analysis, we propose some possible operation strategy to guarantee the stability control for the blast furnace ironmaking process.

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