Nonlinear Receding Horizon Control of Thickness and Tension in a Tandem Cold Mill with a Variable Rolling Speed

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For the precise control of tension and thickness in a tandem cold mill during acceleration and deceleration, we have developed a nonlinear model including the rolling speed as a time-varying parameter. It is demonstrated from simulation results that nonlinear receding horizon control by real-time optimization results in a satisfactory performance for the nonlinear model.

KEY WORDS: receding horizon control; tandem cold mill; nonlinear model; real-time optimization.

1. Introduction

In recent years, quality improvement through the production of high-tension steel sheets and further improvement of the yield have been required for the cold-rolling process. Therefore, in a tandem cold mill, highly precise control of the thickness at the exit of each stand and the tension between the stands is necessary including precise control during acceleration and deceleration.

Conventionally, in steady-state high-speed rolling, the tandem cold mill is controlled using an approximate linear model of rolling.^{1,2)} However, when the rolling speed changes rapidly, such as during periods of acceleration and deceleration, the effect of nonlinear dynamics, which has an undesirable influence on thickness and tension control, is not negligible, and the linearized model is often inaccurate, resulting in variations in quality. Although a method of determining gains based on a transfer function model depending on the rolling speed has been proposed,³⁾ the transfer function used was essentially a linear model under the assumption of a constant rolling speed, and a rapid change in the rolling speed inevitably deteriorated the control performance. Hence, in this paper, a rolling model is developed to express the dependence of the nonlinear dynamics on the rolling speed. Moreover, a control method is proposed for precise control of the thickness and tension even during acceleration and deceleration.

Although it is generally difficult to analytically design a control system for a nonlinear system, receding horizon control (RHC)⁴⁾ has drawn considerable attention as a control method that is applicable to a wide variety of problems. RHC is also known as model predictive control (MPC). In RHC, the control response is optimized over a finite future at each sampling time, which results in state feedback and a high com-

putational load for nonlinear systems. A real-time algorithm, C/GMRES,^{5,6)} has been developed as a fast computation method of nonlinear RHC (NRHC) for mechanical systems with a sampling period of millisecond order and has been successfully implemented in some applications.^{7–9)} In this paper, NRHC is applied to control the thickness and tension in a tandem cold mill with the rolling subjected to acceleration and deceleration. The control performance and computational time of NRHC are evaluated in numerical simulations.

2. C/GMRES Algorithm for NRHC

2.1. RHC Problem

Every function is assumed to be differentiable as many times as necessary hereafter. Let $x(t) \in \mathbb{R}^n$ be the state vector, let $u(t) \in \mathbb{R}^{m_u}$ be the input vector of a general nonlinear system, and let $p(t) \in \mathbb{R}^{m_p}$ be the vector of time-varying parameters given as functions of time. The state equation of a controlled system is given by

$$\dot{x} = f(x(t), u(t), p(t)), \dots \dots \dots (1)$$

where f denotes an n -dimensional vector-valued function. Moreover, constraints on the state and input are expressed as the following equality constraint:

$$C(x(t), u(t), p(t)) = 0, \dots \dots \dots (2)$$

where C denotes an m_c -dimensional vector-valued function. An inequality constraint can be converted into an equality constraint by introducing a dummy variable.⁷⁾ In RHC, an optimal control problem is solved at each time t to minimize the following performance index:

$$J = \phi(x(t+T), p(t+T)) + \int_t^{t+T} L(x(t'), u(t'), p(t')) dt', \dots (3)$$

where ϕ and L are scalar-valued functions, and the performance index is defined so that its minimization results in the desired control performance. The horizon of the performance index is given by $[t, t + T]$, an interval from the current time t to T seconds later, which is a feature of RHC. That is, the initial state for the optimal control problem is given by the state $x(t)$ at the current time t . The optimal control input $u^*(t'; x(t), T)$ minimizing J is determined for $t' \in [t, t + T]$, and only its initial value $u^*(t; x(t), T)$, corresponding to the current time t , is used as the actual control input $u(t)$ to the system. Since the optimal control u^* depends on the state $x(t)$ at each time t , RHC defines a state feedback control law given by the following equation.

$$u^{\text{RH}}(x(t)) = u^*(t; x(t), T) \quad (4)$$

2.2. Necessary Conditions for Optimality

Let $\tau = t' - t$ be a fictitious time on the horizon. The problem to be solved at each time t in RHC is an optimal control problem over the fictitious time τ with the initial state at $\tau = 0$ given by $x(t)$. The necessary conditions for optimality in this problem are obtained by the calculus of variations.¹⁰⁾ Then, the necessary conditions for optimality are discretized for numerical solution by dividing the horizon $[t, t + T]$ into N steps. The discretization step of the horizon is denoted by $\Delta\tau := T/N$, and the state vector, the costate vector (adjoint variables), the time-varying parameter, the control input, and the Lagrange multiplier at the i th step on the discretized horizon are denoted by $x_i^*(t) \in \mathbb{R}^n$, $\lambda_i^*(t) \in \mathbb{R}^n$, $p_i^*(t) \in \mathbb{R}^{m_p}$ ($i = 0, \dots, N$), $u_i^*(t) \in \mathbb{R}^{m_u}$, and $\mu_i^*(t) \in \mathbb{R}^{m_c}$ ($i = 0, \dots, N-1$), respectively. The costate corresponds to the Lagrange multiplier for the state equation regarded as a constraint. Then, the discretized necessary conditions for optimality are given by the following equations:

$$x_{i+1}^*(t) = x_i^*(t) + f(x_i^*(t), u_i^*(t), p_i^*(t))\Delta\tau \quad (5)$$

$$x_0^*(t) = x(t) \quad (6)$$

$$H_u(x_i^*(t), \lambda_{i+1}^*(t), u_i^*(t), \mu_i^*(t), p_i^*(t)) = 0 \quad (7)$$

$$C(x_i^*(t), u_i^*(t), p_i^*(t)) = 0 \quad (8)$$

$$\lambda_i^*(t) = \lambda_{i+1}^*(t) + H_x(x_i^*(t), \lambda_{i+1}^*(t), u_i^*(t), \mu_i^*(t), p_i^*(t))\Delta\tau \quad (9)$$

$$\lambda_N^*(t) = \phi_x^T(x_N^*(t), p_N^*(t)), \quad (10)$$

where H denotes the Hamiltonian defined as

$$H(x, \lambda, u, \mu, p) := L(x, u, p) + \lambda^T f(x, u, p) + \mu^T C(x, u, p), \quad (11)$$

and suffixes x and u represent partial derivatives.

Equations (5)–(10) define a two-point boundary-value problem (TPBVP) for unknowns $x_i^*(t)$, $\lambda_i^*(t)$, $u_i^*(t)$, and $\mu_i^*(t)$. At each time t , the TPBVP is solved for the measured state $x_0^*(t) = x(t)$. Note that the TPBVP depends on time t and is not discretized with respect to t at this stage of problem formulation.

2.3. C/GMRES Algorithm

A vector $U(t) \in \mathbb{R}^{mN}$ consisting of the control input $u_i^*(t)$ and the Lagrange multiplier $\mu_i^*(t)$ is defined as

$$U(t) := [u_0^{*T}(t) \mu_0^{*T}(t) \cdots u_{N-1}^{*T}(t) \mu_{N-1}^{*T}(t)]^T, \quad (12)$$

where $m := m_u + m_c$. Note that $x_i^*(t)$ and $\lambda_i^*(t)$ are determined as functions of $U(t)$ and $x(t)$ by Eqs. (5), (6), (9), and (10). That is, for a given $U(t)$ and $x(t)$, $x_i^*(t)$ is determined by Eqs. (6) and (5), and $\lambda_i^*(t)$ is determined by Eqs. (10) and (9). Therefore, $U(t)$ is the unknown quantity to be determined by solving the following mN -dimensional equation, which consists of Eqs. (7) and (8).

$$F(U(t), x(t), t) := \begin{bmatrix} H_u^T(x_0^*(t), \lambda_1^*(t), u_0^*(t), \mu_0^*(t), p_0^*(t)) \\ C(x_0^*(t), u_0^*(t), p_0^*(t)) \\ \vdots \\ H_u^T(x_{N-1}^*(t), \lambda_N^*(t), u_{N-1}^*(t), \mu_{N-1}^*(t), p_{N-1}^*(t)) \\ C(x_{N-1}^*(t), u_{N-1}^*(t), p_{N-1}^*(t)) \end{bmatrix} = 0 \quad (13)$$

If $U(t)$ is determined by solving this equation, the actual control input $u(t)$ is obtained by the projection $P_0: \mathbb{R}^{mN} \rightarrow \mathbb{R}^{m_u}$

$$u(t) = P_0(U(t)) := u_0^*(t) \quad (14)$$

An iterative method such as the Newton method is inefficient and requires a long computational time to obtain $U(t)$ by solving the mN -dimensional nonlinear equation given by Eq. (13). Hence, we consider updating $U(t)$ so as to stabilize $F = 0$ instead of solving $F = 0$ by an iterative method at each time. That is, the solution at the initial time, $U(0)$, is determined so that

$$F(U(0), x(0), 0) = 0 \quad (15)$$

is satisfied, and $U(t)$ is updated so that

$$\dot{F}(U(t), x(t), t) = -\zeta F(U(t), x(t), t) \quad (16)$$

is satisfied, where $\zeta > 0$ denotes a damping factor used to stabilize $F = 0$. An equation for $\dot{U}(t)$ is obtained from Eq. (16) as follows.

$$F_U \dot{U} = -\zeta F - F_x \dot{x} - F_t \quad (17)$$

By solving this linear equation for $\dot{U}(t)$ and integrating $\dot{U}(t)$ in real time, $U(t)$ can be updated without using an iterative method. In particular, the linear equation for $\dot{U}(t)$ can be solved efficiently by one of the Krylov subspace methods such as GMRES (Generalized Minimum RESidual method).¹¹⁾ The overall algorithm is called C/GMRES⁶⁾ because it combines the continuation method and GMRES.

3. Nonlinear Model of a Tandem Cold Mill

In a tandem cold mill, nonlinearities in the dynamics of the thickness and tension are not negligible in the case of large variations of the rolling speed because the rolling load and forward slip are complicated nonlinear functions of the rolling speed, as shown later. In this case, the achievable control accuracy is limited if an approximate linear model is used for control design. Therefore, the rolling model of a tandem cold mill should take those nonlinearities into account.

A tandem cold mill consists of several stands as shown in Fig. 1. At stand i , the strip enters and exits the stand with thicknesses h_{i-1} and h_i , respectively. The roll gap is denoted by S_i and the rolling load is denoted by P_i . While the strip passes through stand i , the thickness h_i and tension $\sigma_{f,i-1}$ are controlled by adjusting the roll gap S_i and rolling speed $V_{R,i-1}$.

Fig. 1. Schematic diagram of tandem cold mill.

Table 1. Physical parameters in model of tandem cold mill.

K_g [–]	AGC gain	10
T_g [s]	Time constant of hydraulic servo	0.01
k [–]	Tuning ratio	0.4
M_0 [N = mm]	Mill modulus	4.9×10^6
T_m [s]	Time constant of mill motor	0.003
E [MPa]	Young's modulus	2.1×10^5
L [mm]	Distance between stands	4660
b [mm]	Width	930
C_{eq} [%]	Carbon equivalent	4.0×10^{-1}
F_r [°C]	Finishing temperature	900
C_r [°C]	Coiling temperature	600
H_1 [mm]	Original thickness	4.0
R [mm]	Roll radius	273
C [%]	Sliding ratio	5
W [kN]	Total coil weight	7845
Q_l [l = m2]	Supplied oil	45

3.1. Governing Equations

The physical parameters¹⁾ in the model are summarized in Table 1. The rolling model at each stand consists of the dynamics of the roll gap and rolling direction, which are described below.

First, the roll gap S_i is assumed to be controlled by a local feedback loop of a hydraulic servo and is governed by the following equation:

$$T_g \ddot{S}_i + \dot{S}_i + K_g (h_g(S_i, h_i) - u_{S_i}) = 0, \quad (18)$$

where u_{S_i} denotes the command input, and T_g and K_g are constants specified by the local feedback loop. The local feedback adjusts the roll gap S_i so that $h_g(S_i, h_i)$ coincides with the command input u_{S_i} . The function $h_g(S_i, h_i)$ is defined in terms of the thickness h_i and roll gap S_i as

$$h_g = kh_i + (1-k)S_i, \quad (19)$$

which represents the thickness modified by the tuning parameter k ($0 < k < 1$). The relationship between the thickness h_i and the roll gap S_i is given by

$$h_i = S_i + \frac{1}{M_0} P_i, \quad (20)$$

where M_0 denotes the mill modulus and P_i denotes the rolling load. P_i is a nonlinear function of the tension, thickness, and rolling speed, as described in detail later. Then, the

dynamics of the roll gap S_i is governed by

$$\ddot{S}_i + \frac{1}{T_g} \dot{S}_i + \frac{K_g}{T_g} S_i = \frac{K_g}{T_g} \left(u_{S_i} - \frac{k}{M_0} P_i \right). \quad (21)$$

Next, let us consider the behavior of the strip in the rolling direction. The rolling speed $V_{R_{i-1}}$ is assumed to be controlled by a local feedback loop, and its closed-loop dynamics is described by the following first-order differential equation.

$$\dot{V}_{R_{i-1}} = -\frac{1}{T_m} V_{R_{i-1}} + \frac{1}{T_m} u_{V_{R_{i-1}}} \quad (22)$$

Here the parameter T_m denotes the time constant and the variable $u_{V_{R_{i-1}}}$ denotes the input to the local feedback control system, which is the reference signal of the rolling speed $V_{R_{i-1}}$. Owing to the physical nature of tension, the time derivative of the tension $\sigma_{f_{i-1}}$ is proportional to the difference between the rolling speeds $V_{R_{i-1}}$ and V_{R_i} as

$$\dot{\sigma}_{f_{i-1}} = \frac{E}{L} \{ (1-g_i) V_{R_i} - (1+f_{i-1}) V_{R_{i-1}} \}, \quad (23)$$

where the parameters E and L denote Young's modulus and the distance between two adjacent stands, respectively. The parameters f_{i-1} and g_i are respectively called the forward slip and backward slip, which denote the rates of change in the velocity of the strip near the point of contact with the roll. Moreover, the following equation describing the conservation of the mass flow holds:

$$h_{i-1} (1-g_i) V_{R_{i-1}} = h_i (1+f_i) V_{R_i}, \quad (24)$$

which causes Eq. (23) to be replaced to

$$\dot{\sigma}_{f_{i-1}} = \frac{E}{L} \left\{ \frac{h_i}{h_{i-1}} (1+f_i) V_{R_i} - (1+f_{i-1}) V_{R_{i-1}} \right\}. \quad (25)$$

Note that the rolling load P_i and the forward slip f_i are nonlinear functions of the tension, the thickness of the strip, and the rolling speed as follows:

$$P_i := P_i(\sigma_{f_{i-1}}, \sigma_{f_i}, h_i, h_{i-1}, V_{R_i}) \quad (26)$$

$$f_i := f_i(\sigma_{f_{i-1}}, \sigma_{f_i}, h_i, h_{i-1}, V_{R_i}). \quad (27)$$

3.2. Nonlinear Models of Rolling Load and Forward Slip

The rolling load P_i and forward slip f_i are nonlinear functions of their arguments in Eqs. (26) and (27). In this paper, we employ the functions obtained in the literature,^{12,13)} which explain the experimental data precisely. These functions are given as

$$P_i = \gamma b (K - (0.7\sigma_{f_{i-1}} + 0.3\sigma_{f_i})) \times \sqrt{R'(h_{i-1} - h_i) D_P} \quad (28)$$

$$f_i = \tan^2 \left(\sqrt{\frac{h}{R'}} \frac{H_n}{2} \right), \quad (29)$$

where the parameter K [MPa] denotes the deformation resistance, R' [mm] denotes the deformed roll radius, μ and μ^* denote friction coefficients, and D_P and H_n denote dimensionless quantities. These parameters are also functions of other variables as shown in Eqs. (30)–(35):

$$K = \alpha_1 C_{eq} + \alpha_2 F_r + \alpha_3 C_r + \alpha_4 \epsilon + \alpha_5 V_{R_i} + \alpha_6 \dots \quad (30)$$

$$R' = R \left(1 + c \frac{P}{b(h_{i-1} - h_i)} \right) \cong 1.2R \dots \dots \dots (31)$$

$$D_p = 1.08 + 1.79r\sqrt{1-r}\mu\sqrt{\frac{R'}{h_i}} - 1.02r \dots \dots \dots (32)$$

$$\mu = \frac{\tan^{-1}\sqrt{(h_{i-1}/h_i)-1} - 2\tan^{-1}\sqrt{f_i}}{\tan^{-1}\sqrt{(h_{i-1}/h_i)-1} - 2\tan^{-1}\sqrt{f_s-C}} \mu^* \dots \dots (33)$$

($f_s = f_i$ ($f_i \geq C$), C ($f_i < C$))

$$\begin{aligned} \mu^* = & A_V \cdot e^{B_V V_{R_i}} + A_W \cdot e^{B_W W} + A_Q \cdot e^{B_Q Q_i} \\ & + A_r r + A_H h_{i-1} + A_h h_i + A_b \sigma_{f_{i-1}} \\ & + A_f \sigma_{f_i} + A_K K + A_0 \dots \dots \dots (34) \end{aligned}$$

$$\begin{aligned} H_n = & \sqrt{\frac{R'}{h_i}} \tan^{-1} \sqrt{\frac{h_{i-1}}{h_i} - 1} \\ & - \frac{1}{2\mu^*} \ln \frac{h_{i-1}(K - \sigma_{f_i})}{h_i(K - \sigma_{f_{i-1}})} \dots \dots \dots (35) \end{aligned}$$

Here the logarithmic strain ϵ and the reduction in thickness, r , are defined as

$$\epsilon = -\ln \frac{h_i}{H_1} \dots \dots \dots (36)$$

$$r = \frac{h_{i-1} - h_i}{h_{i-1}} \dots \dots \dots (37)$$

Note that Eqs. (30) and (34) involve the rolling speed V_{R_i} . Although the exact value of R' is obtained by solving Eqs. (28) and (31), an approximate solution $R' \cong 1.2R$ is adopted here owing to the difficulty of finding the exact solution. The parameters in the above equations are summarized in **Table 2**.

Table 2. Parameters in nonlinear model of P and f .

α_1 [MPa/%]	9.752×10^{-3}
α_2 [MPa/°C]	-4.206×10^{-1}
α_3 [MPa/°C]	2.227×10^{-2}
α_4 [MPa]	2.226×10^{-2}
α_5 [MPa·s/mm]	9.864×10^{-4}
α_6 [MPa]	8.385×10^{-2}
A_V [-]	-1.69×10^{-3}
A_W [-]	1.443×10^{-2}
A_Q [-]	6.88×10^{-3}
A_r [-]	-6.2×10^{-3}
A_H [mm]	-4.112×10^{-2}
A_h [mm]	5.148×10^{-2}
A_b [MPa]	2.43×10^{-5}
A_f [MPa]	-6.94×10^{-6}
A_K [MPa]	-6.04×10^{-5}
A_0 [-]	4.049×10^{-2}
B_V [s/mm]	-4.637×10^{-5}
B_W [KN]	-4.24×10^{-5}
B_Q [m ² /l]	-2.990×10^{-2}

3.3. State Equation

The state equation for stand i is now constructed in accordance with the previous subsections. The state vector x and input vector u are defined as

$$x = [x_1, x_2, x_3, x_4]^T := [S_i, \dot{S}_i, \sigma_{f_{i-1}}, V_{R_{i-1}}]^T \dots \dots \dots (38)$$

$$u = [u_1, u_2]^T := [u_{S_i}, u_{V_{R_{i-1}}}]^T \dots \dots \dots (39)$$

Then, the state equation of this system is expressed as

$$\dot{x} = \begin{bmatrix} x_2 \\ f_2 \\ f_3 \\ -(1/T_m)x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ K_g/T_g & 0 \\ 0 & 0 \\ 0 & 1/T_m \end{bmatrix} \begin{bmatrix} u_{S_i} \\ u_{V_{R_{i-1}}} \end{bmatrix} \dots \dots (40)$$

$$f_2 = -\frac{K_g}{T_g}x_1 - \frac{1}{T_g}x_2 - \frac{K_g}{T_g}\frac{k}{M_0}P_i \dots \dots \dots (41)$$

$$f_3 = \frac{E}{L} \left\{ \frac{h_i}{h_{i-1}}(1+f_i)V_{R_i} - (1+f_{i-1})x_4 \right\} \dots \dots \dots (42)$$

3.4. Expression of Thickness

The thickness h_i is a function of the state as shown in Eq. (20). However, it is difficult to express the thickness explicitly from Eq. (20) because the rolling load P_i is also a function of the thickness h_i as in Eq. (26). In this study, we propose an approximate expression for h_i . Assuming that h_i is always controlled within the neighborhood of its desired value h_i^r , Eq. (20) can be approximated by

$$h_i \approx h_i^r + \frac{P_i(x_3, \sigma_{f_i}, h_i^r, h_{i-1}, V_{R_i}) - M_0(h_i^r - S_i)}{M_0 - (\partial P_i / \partial h_i)|_{h_i=h_i^r}} \dots \dots (43)$$

where $h_i = h_i^r + \Delta h_i$ and Δh_i denotes a small deviation of the thickness from its desired value. This approximation gives an explicit expression for h_i that depends on x and h^r .

4. Application of RHC to a Tandem Cold Mill

4.1. RHC for Nonlinear Model

We design RHC for the nonlinear model developed in the previous section. A single stand ($i = 1$) is considered, and the control objective is to keep the thickness and tension constant during changes in the rolling speed. Deviations of the thickness and tension from their reference values are penalized in the performance index for RHC.

In the case of a single stand, the state vector is four-dimensional, $x = [S_1, \dot{S}_1, \sigma_{f_0}, V_{R_0}]^T$, and the input vector is two-dimensional, $u = [u_{S_1}, u_{V_{R_0}}]^T$. The entry thickness and entry tension are denoted by h_0 and σ_{f_0} , respectively, and are assumed to be constant. The strip is accelerated or decelerated in accordance with a given profile for the rolling speed V_{R_i} .

4.1.1. Performance Index

First, the thickness h_1 is obtained explicitly from the approximation given by Eq. (43) as a function of the state, i.e., $h_1 = h_1(x)$. Moreover, the desired state $x^r(t)$ and corresponding desired input $u^r(t)$ are assumed to be obtained appropriately. Then, the performance index is defined to penalize deviations of the state and input from their desired values as

$$J := \bar{x}^T(t+T)S_f\bar{x}(t+T) + \int_t^{t+T} (\bar{x}^T(t')Q\bar{x}(t') + \bar{u}^T(t')^T R \bar{u}(t')) dt', \dots (44)$$

where $\bar{x} := x - x^r$ and $\bar{u} := u - u^r$ denote the deviation of the state from the desired state and the deviation of the input from the desired input, respectively, and $S_f \in \mathbb{R}^{4 \times 4}$, $Q \in \mathbb{R}^{4 \times 4}$, and $R \in \mathbb{R}^{2 \times 2}$ are positive-definite weighting matrices. In this paper, the thickness h_1 itself is not included in the performance index to avoid a complicated performance index. Instead, the desired state is defined so as to achieve the desired thickness and desired tension, and the performance index is defined for the actual state to track the desired state. As a result, the tracking of the desired values of the thickness and tension is achieved indirectly by minimizing the performance index given by Eq. (44).

4.1.2. Desired State

The desired state and corresponding desired input are required in order to obtain the performance index given by Eq. (44). They depend on the rolling speed V_{R_1} and, therefore, are functions of time. Since $\sigma_{f_0} = x_3$ and h_1 are to be kept constant, the desired state x^r and desired input u^r must satisfy

$$\dot{x}^r = f(x^r, u^r) \dots (45)$$

for constant $\sigma_{f_0} = x_3$ and h_1 . The desired constant values of x_3^r and h_1^r and V_{R_1} as a function of time are assumed to be given, and σ_{f_1} and h_0 are also assumed to be given constants. Therefore, $x_1^r, x_2^r, x_4^r, u_1^r$, and u_2^r are quantities to be determined at each time.

First, $V_{R_0}^r = x_4^r$ is determined from the third element of Eq. (45). That is,

$$\dot{x}_3^r = \frac{E}{L} \left\{ \frac{h_1^r}{h_0} (1 + f_1) V_{R_1} - (1 + f_0) V_{R_0}^r \right\} \dots (46)$$

Since h_1^r is constant, and $\dot{x}_3^r = 0$ and $f_0 = 0$ hold identically, we have

$$\frac{h_1}{h_0} (1 + f_1) V_{R_1} - x_4 = 0, \dots (47)$$

which gives $V_{R_0}^r$ as

$$V_{R_0}^r = \frac{h_1}{h_0} (1 + f_1) V_{R_1} \dots (48)$$

Then, $\dot{x}_4^r$ is obtained by differentiating Eq. (48) with respect to time as

$$\dot{V}_{R_0}^r = \frac{h_1}{h_0} \left\{ \frac{\partial f_1}{\partial V_{R_1}} \dot{V}_{R_1} V_{R_1} + (1 + f_1) \dot{V}_{R_1} \right\} \dots (49)$$

Next, $S_1^r = x_1^r$ is determined as follows. Although the thickness h_i is determined approximately by Eq. (43), Eq. (43) is equivalent to Eq. (20) when $h_i = h_i^r$ holds. Therefore, Eq. (20) gives S_1^r as

$$S_1^r = h_1^r - \frac{P_1(\sigma_{f_0}^r, \sigma_{f_1}^r, h_1^r, h_0, V_{R_1})}{M_0} \dots (50)$$

The time derivative $\dot{S}_1^r = x_2^r$ is obtained by differentiating Eq. (50) with respect to time as

$$\dot{S}_1^r = -\frac{1}{M_0} \dot{P}_1 = -\frac{1}{M_0} \frac{\partial P_1}{\partial V_{R_1}} \dot{V}_{R_1} \dots (51)$$

Note that only V_{R_1} is time-varying among the arguments of P_1 . Furthermore, $\dot{S}_1^r = \dot{x}_2^r$ is obtained by differentiating Eq. (51) with respect to time as

$$\begin{aligned} \ddot{S}_1^r &= -\frac{1}{M_0} \frac{d}{dt} \left(\frac{\partial P_1}{\partial V_{R_1}} \dot{V}_{R_1} \right) \\ &= -\frac{1}{M_0} \left(\frac{\partial^2 P_1}{\partial V_{R_1}^2} \dot{V}_{R_1}^2 + \frac{\partial P_1}{\partial V_{R_1}} \ddot{V}_{R_1} \right) \dots (52) \end{aligned}$$

Finally, the desired input to achieve the desired state is determined from the second and fourth elements of Eq. (45) as follows.

$$u_{S_1}^r = \frac{T_g}{K_g} (\ddot{S}_1^r - f_2(x^r, h^r, V_{R_1})) \dots (53)$$

$$u_{V_{R_0}}^r = T_m \dot{V}_{R_0}^r + V_{R_0}^r \dots (54)$$

As shown by Eqs. (52) and (53), it is necessary to specify the values of V_{R_1} and its derivatives $\dot{V}_{R_1}$ and $\ddot{V}_{R_1}$ to determine the desired values of the state and input.

4.1.3. Rolling Speed

In the simulations in this paper, the rolling speed V_{R_1} is first decreased from its maximal value $V_{R_1}^{r2}$ to its minimal value $V_{R_1}^{r1}$ and then is increased to its maximal value $V_{R_1}^{r2}$ again. The deceleration and acceleration are performed at constant rates over time intervals $[t_1, t_2]$ and $[t_3, t_4]$, respectively. The basic pattern of the rolling speed is given by the profile in Fig. 2, in which the rolling speed is given simply as ramp functions connecting the maximal value and minimal value. However, the time derivative $\dot{V}_{R_1}$ is discontinuous at t_1, t_2, t_3 , and t_4 , which causes discontinuities in the desired state. Therefore, these corners are replaced with smooth functions to remove the discontinuities.

The profile of the rolling speed interpolated with cubic curves is shown in Fig. 3. The corners in the previous profile are replaced with cubic curves within intervals $[t_i^p, t_i^f]$ ($i = 1, \dots, 4$), where t_i^p and t_i^f are set to be less than and greater than t_i , respectively. Two different cubic curves are used in $[t_i^p, t_i]$ and $[t_i, t_i^f]$, and are defined so that $\dot{V}_{R_1}$ is continuous at end points t_i^p, t_i , and t_i^f . By using the cubic curves, both the desired state and desired input are continuous, and each $[t_i^p, t_i^f]$ is chosen so that it is bisected by t_i .

Fig. 2. Time history of V_{R_1} (ramp only).

Fig. 3. Time history of V_{R_1} (ramp with cubic curves).

4.1.4. Horizon

RHC involves optimization over a finite future, and the longer the horizon length T , the further in the future the dynamics is optimized. Hence we consider improving the control accuracy by using a horizon with a fixed length, whereas an increasing horizon length is used for initialization in conventional C/GMRES.

If the initial state is identical to the desired state and the exit rolling speed is constant for an interval after $t = 0$, i.e., $T_f < t_1$ in Figs. 2 and 3, then the state is kept constant at the desired state, and the desired input is also constant. That is, $u(t) = u(0)$ holds in the neighborhood of $t = 0$. Similarly, $u^*(\tau, 0) = u(0)$ holds over the horizon $0 \leq \tau \leq T_f$.

Therefore, $u_i^*(0) = u(0)$ ($i = 0, \dots, N-1$) is assumed to be optimal, with $u(0)$ obtained by solving $H_u^T(x(0), \phi_x^T(x(0)), u(0)) = 0$ at $t = 0$. Then, a constant horizon length of $T = T_f$ can be used for optimization over $[t, t + T_f]$ to determine the control input for $t > 0$. It should be noted that the constraint as in Eq. (2) is not imposed and, therefore, the Lagrange multiplier μ is not introduced in the present problem.

4.2. RHC for Linear Model

RHC for our nonlinear model has been formulated in the previous subsection. If the nonlinear model without approximation is used in RHC, RHC will require a significant amount of computation, although precise control can be expected. On the other hand, if an approximate linear model is used in RHC, the computation time will be significantly reduced, although the control accuracy may deteriorate, and various types of linear control theory can also be applied.

Therefore, the control performance in the case of an approximate linear model is investigated in this paper to access the necessity of nonlinear control for the nonlinear model. C/GMRES is used for the real-time optimization of the control input for the linear model.

4.2.1. Linearization of the Model

Since the state and input are assumed to be controlled within the neighborhood of their desired values in the present formulation, the nonlinear model can be linearized around the desired values x^r and u^r . The state equation of the tandem cold mill, Eq. (40), is affine in the input u and can be represented as

$$\dot{x} = f(x) + gu, \quad (55)$$

Table 3. Desired state variables.

Exit-side (Front) thickness	h_1^r	3.4 [mm]
Entry-side (Back) tension	σ_{f0}^r	20 [MPa]
Exit-side (Front) tension	σ_{f1}^r	120 [MPa]
Entry-side (Back) thickness	h_0	4.0 [mm]
Rolling speed	V_{R1}^{r1}	1 050 [mm/s]
	V_{R1}^{r2}	6 316 [mm/s]

where $f(x)$ denotes a vector-valued function of the state and g is a constant matrix. A Taylor expansion of this model up to the first order around the desired values x^r and u^r leads to the following linear system as an approximation of the dynamics of the deviations $\bar{x} := x - x^r$ and $\bar{u} := u - u^r$:

$$\dot{\bar{x}} = A(t)\bar{x} + g\bar{u}, \quad (56)$$

where the higher-order terms of $\bar{x}$ and $\bar{u}$ are ignored and the matrix $A(t)$ is given by

$$A(t) := \frac{\partial f}{\partial x}(x^r(t)). \quad (57)$$

The matrix A in Eq. (56) is time-varying depending on the variation in x_r if V_{R_1} is either increased or decreased. On the other hand, if V_{R_1} is constant, the system given by Eq. (56) can be regarded as a linear time-invariant system.

4.2.2. Use of Linear Model in RHC

In RHC using the approximate linear model, the state equation is given by Eq. (56) and the optimal control problem is solved using the performance index given in terms of $\bar{x}$ and $\bar{u}$ as

$$J = \bar{x}^T(t+T)S_f\bar{x}(t+T) + \int_t^{t+T} (\bar{x}^T Q \bar{x} + \bar{u}^T R \bar{u}) dt'. \quad (58)$$

That is, the control input is determined so that the deviations from the desired values are zero at each time in the horizon. The state equation and the partial derivatives of the Hamiltonian H , H_x and H_{u_x} are linear functions of the state $\bar{x}$ and input $\bar{u}$, which markedly simplifies the computation of optimal control at time t .

5. Numerical Examples

5.1. RHC for Nonlinear Model

The desired values for the thickness and tension of a strip, described in 4.1.2, are given in Table 3. Simulation results in this setting are presented for the two profiles of exit rolling speed V_{R_1} described in 4.1.3. The sampling period is given as $\Delta t = 0.001$ [s], the number of discretization steps on the horizon is $N = 50$, and the horizon length is fixed as $T = 0.1$ [s]. The sampling period Δt in the simulation corresponds to the time step for numerical integration of the state equation and is not necessarily identical to the discretization step of the horizon $\Delta \tau = T/N$. The damping factor in Eq. (16) is chosen as $\zeta = 1/\Delta t = 1\,000$ according to the error analysis of the algorithm.⁶⁾ The weighting matrices in Eq. (44) are chosen as

$$Q = \text{diag}(200, 2\,000, 500, 20) \quad (59)$$

$$R = \text{diag}(10\,000, 1\,000) \quad (60)$$

$$S_f = \text{diag}(50, 500, 100, 1). \dots\dots\dots (61)$$

The parameters for the two profiles of V_{R_i} in 4.1.3 are given as follows:

1. Ramp

$$[t_1, t_2, t_3, t_4] = [11, 19, 21, 29][s].$$

2. Ramp and cubic curves

$[t_1, t_2, t_3, t_4] = [10, 18, 22, 30][s]$, and cubic curves are used for 2 [s].

Time histories of the variables in the simulations with the above settings are shown in **Figs. 4** and **5**. The ramp profile for V_{R_i} is used in Fig. 4 and the ramp profile modified with cubic curves is used in Fig. 5. In these figures, *Error* shows the norm $\|F\|$ of the nonlinear function F , which indicates the error in the optimality condition $F = 0$ at each time in RHC. The smaller the error, the more accurate the computation.

In the case of the ramp only (Fig. 4), u_{S_1} and $\dot{S}_1$ exhibit discontinuities at changes in the slope of V_{R_i} . The maximal deviations in the thickness h_1 and tension σ_{f_0} are approximately $\pm 2.0 \times 10^{-4}$ [mm] and ± 0.2 [MPa], respectively. In contrast, if cubic curves are used, the deviations in the thickness h_1 and tension σ_{f_0} are below $\pm 1.0 \times 10^{-4}$ [mm] and ± 0.15 [MPa], respectively, even at changes in the slope of V_{R_i} . It is clear that the use of cubic curves smooths $\dot{S}_1$ and σ_{f_0} .

In both cases, deviations in the thickness h_1 and tension σ_{f_0} are large when the rolling speed varies at a low speed. The deviations increase during deceleration, decrease during periods of constant speed, and increase again during accel-

eration. Symmetry in the time histories of deviations is also observed between deceleration and acceleration.

Most of the computational time is required for iterations in GMRES. The number of iterations is almost zero at a constant rolling speed but increases to about 10 during acceleration and deceleration.

Simulations are performed in C language on a personal computer (CPU: Pentium 4, 3.0 GHz). One iteration in GMRES takes about 1.1 [ms] for $N = 50$ and, therefore, each update of the control input takes about 11 [ms]. This computation time per input update is larger than the assumed sampling period; thus, the present NRHC cannot be implemented in real time, although the computation time is rather short considering the complexity of the model and algorithm. However, the present simulation results demonstrate the reasonable possibility of real-time implementation of NRHC with the use of a more powerful computer, appropriate simplification of the model, and the development of a numerical technique to deal with a larger sampling period.

5.2. Control Based on Approximate Linear Model

Next, simulation results are presented for the control of the tandem cold mill based on the approximate linear model in subsection 4.2. The parameters and V_{R_i} are the same as in the previous subsection.

Time histories of the variables in the simulations are shown in **Figs. 6** and **7**. The maximal deviations of the thickness h_1 for the two profiles of V_{R_i} in Figs. 2 and 3 are approximately 6.0×10^{-4} [mm] and 2.0×10^{-4} [mm], respec-

Fig. 4. Time histories of variables (nonlinear model, ramp).

Fig. 5. Time histories of variables (nonlinear model, ramp with cubic curves).

Fig. 6. Time histories of variables (linear model, ramp).

Fig. 7. Time histories of variables (linear model, ramp with cubic curves).

tively. Although they are larger than the values in the previous subsection, they are still smaller than 1.0×10^{-3} [mm]. On the other hand, the corresponding maximal deviations of the tension σ_{f_0} are approximately 0.6 [MPa] and 0.5 [MPa]. Therefore, the deviation deteriorates significantly for the tension rather than for the thickness. The peaks of the deviations in the thickness and tension are observed before and after the rolling speed attains its minimum, respectively, and the overall patterns of the deviations during acceleration and deceleration are similar to those in control based on the nonlinear model. Nevertheless, we have clearly shown the effectiveness of control based on the nonlinear model in reducing the deviations of the thickness and tension, compared with control based on the approximate linear model.

6. Conclusion

In this paper, a nonlinear model has been developed for control of the thickness and tension in a tandem cold mill during acceleration and deceleration. NRHC was applied to the nonlinear model, and numerical simulations demonstrated that the accuracy of the thickness and tension is improved by employing NRHC compared with control based on the approximate linear model.

In spite of acceleration and deceleration by more than 5000 [mm/s] within 10 s, precise control is achieved with the nonlinear model, suppressing the maximal deviation in the thickness to below 1.0×10^{-4} [mm]. In particular, smoothing of the profile of the rolling speed results in improved accuracy. Moreover, the deviation in the tension is also successfully suppressed to below 0.2 [MPa]. On the other hand, if the approximate linear model is used, the deviations in the thickness and tension are larger than those in control using the nonlinear model, although the computation time is significantly reduced. Therefore, it is concluded that NRHC is valid for improving the accuracy of thickness and tension in a tandem cold mill.

Future work includes the simplification of the model. The present model is complicated owing to it fully taking nonlinearities in the forward slip and rolling load into account, which also makes computation of the control input complicated, consequently, increasing the computation time required to update the input. Therefore, it is important to simplify the model using suitable approximations not only to reduce the computation time but also to preserve nonlinear characteristics while maintaining a reasonable amount of computation.

It is also desirable to guarantee the robustness of RHC. In the present design of RHC, the thickness to be controlled is not directly fed back. Instead, the desired state in the performance index is obtained from the desired thickness via the model. Therefore, an error in the model can cause a steady-state deviation in the thickness. It is desirable to develop a control method that directly penalizes the deviation in the thickness.

Finally, future work also includes the extension of the present control method to achieve the coordinated control of thickness and tension in multiple stands to optimize the operation of an entire tandem cold mill.

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