Simulation of Austenite Flow Curves under Industrial Rolling Conditions Using a Physical Dynamic Recrystallization Model

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Hot compression tests were carried out on three steels: i) a 0.038% Nb-0.11%C microalloyed grade; ii) a Nb-modified TRIP steel; and iii) a Ti-stabilized low carbon steel. The tests were performed at strain rates up to 1 s⁻¹ and over the temperature range 880–1 200°C. The initiation of dynamic recrystallization (DRX) was observed under all testing conditions. Two sets of equations were derived from the experimental curves: i) a work hardening relation pertaining to the grains in which DRX has not yet nucleated; and ii) a separate work hardening expression describing the mean flow stress applicable to the grains in which DRX is taking place. With the aid of the temperature and strain rate dependences determined from the data, and using the law of mixtures, extrapolated flow curves were calculated applicable to strain rates up to 100 s⁻¹, i.e. to those involved in strip mill rolling. The simulations show that, once DRX has been initiated, the flow stress is controlled by the kinetics of the softening mechanisms.

KEY WORDS: austenite flow curve modeling; dynamic recrystallization; work hardening parameters; avrami kinetics; high strain rates.

1. Introduction

The effect of dynamic recrystallization (DRX) on steel processing has been extensively studied in order to predict the fractional softening attributable to this mechanism. One approach involves using Avrami kinetics to describe the net softening with respect to a calculated work hardening curve derived from the initial part of the flow curve. The aim of this technique is to provide a straightforward method of modeling the austenite flow curve, which is required for calculation of the separation force and torque.

The occurrence of DRX during high temperature deformation has also been studied using cellular automata, phase field modeling, and polyphase plasticity. These approaches provide useful insight into the microstructural changes taking place during DRX, but have not yet been applied to the prediction of rolling load or torque for particular grades of steel. Still further methods have been proposed employing irreversible thermodynamics as well as a viscoplastic model. These do provide data for roll force prediction, but require knowledge of the values of a large number of input parameters.

An improvement in the method of Ref. 8) was proposed in a recent study based on employing the mean flow stress of the material undergoing DRX together with the work hardening curve pertaining to the grains in which DRX has not yet nucleated. The innovation in this newer method consisted of the simplifying assumption that the flow curve pertaining to the material that has undergone DRX begins at the critical strain for dynamic recrystallization and follows the kinetics of the overall flow curve. This method has been shown to model austenite flow curves with acceptable accuracy. In the present study, the improved model is applied to three different grades of steel and used to predict flow curves at the strain rates involved in industrial rolling operations, which are not readily attainable in the laboratory.

2. Modeling Method

2.1. General Equation Describing the Flow Curves

In a recent study, it was demonstrated that the austenite flow curve can be modeled using the simple rule of mixtures represented by Eq. (1). This rule of mixtures is based on the current values of the work hardening flow stress pertaining to the unrecrystallized material and the dynamic recrystallization flow stress, which represents the average flow stress of the material that has already undergone dynamic recrystallization.

\[ \sigma = (1 - X)\sigma_{wh} + X\sigma_{rex} \] .......................... (1)

Here \( X \) is the fractional softening due to DRX, which can be defined as \( X = (\sigma_{wh} - \sigma) / (\sigma_{wh} - \sigma_{rex}) \).

2.2. The Work Hardening Model

Accurate descriptions of \( \sigma_{wh} \) and \( \sigma_{rex} \) are necessary for modeling flow curves using Eq. (1). This is accomplished by using a dislocation density model to describe the work hardening behavior. The model, expressed by Eq. (2), states that the evolution of the dislocation density \( \rho \) with strain only depends on the current value of the dislocation density via the work hardening parameters \( r \) and the athermal work hardening rate. Here, the work hardening parameters \( r \) and \( h \) are taken to be strain independent.
\[ \frac{d\rho}{d\varepsilon} = h - r\rho \]  
\[ \sigma = M\mu b\sqrt{\rho} \]

Here \( M \) is the Taylor factor, \( \mu \) the shear modulus, \( b \) the Burgers vector and \( \alpha \) a material constant (taken here as equal to 0.5). The detailed steps involved in this procedure are described in equations A1 to A9 in Ref. 8). Finally the work hardening flow stress \( \sigma_{wh} \) for the unrecrystallized portion of the material is specified by Eq. (4), as demonstrated in our previous paper. 

\[
\sigma_{wh} = \left[ \sigma_{sat}^2 - \sigma_0^2 \right] \exp \left(-r\left(\varepsilon - \varepsilon_0\right)\right)^{1/2} \]  
Eq. (4)

Here \( \sigma_0 \) is the yield stress, at which point \( \rho = \rho_0 \) and \( \varepsilon = \varepsilon_0 \), and \( \sigma_{sat} \) depends on \( h \) and \( r \) (equations A7 to A9 in Ref. 8) according to:

\[
\sigma_{sat} = M\mu b\sqrt{h/r} \]  
Eq. (5)

To describe \( \sigma_{rex} \), the general formalism of Eq. (4) is used and modified to take into account the specificities of the unrecrystallized material; it is given by Eq. (6). The \( \sigma_{rex} \) flow curve begins at the critical strain \( \varepsilon_c \), with an initial value of \( \sigma_0 \). In this relation, the saturation stress \( \sigma_{sat} \) in Eq. (3) is replaced by the critical stress \( \sigma_c \), the stress value for the initiation of DRX, which has previously been demonstrated to be equal to the steady state stress \( \sigma_{ss} \); it can be considered as the average stress when all the grains have undergone at least one cycle of DRX. The work hardening parameter \( r' \), is introduced to take into account the average recovery rate of the crystals that are currently at different stages of DRX in the recrystallized portion of the material.

\[
\sigma_{rex} = \left[ \sigma_c^2 - \sigma_0^2 \right] \exp \left(-r'\left(\varepsilon - \varepsilon_c\right)\right)^{1/2} \]  
Eq. (6)

Here \( r' \) is obtained from \( \varepsilon_c \) and \( h \) using the relation: \( \sigma_c = M\mu b\sqrt{h/r'} \).

A schematic description of this model is presented in Fig. 1(a), where an experimental flow curve \( \sigma_{exp} \) is compared with its associated work hardening flow stress \( \sigma_{wh} \) and dynamic recrystallization flow curve \( \sigma_{rex} \). The critical and steady state stresses are also displayed, where the latter is defined by the equality \( \sigma_{sat} = \sigma_c \).

### 2.3. Calculation of the Work Hardening Parameters

To determine \( r \) and \( h \), Eq. (2) is rewritten in terms of flow stress, using Eq. (3), and then converted into the following form:

\[
\frac{d\rho}{d\varepsilon} = (M\mu b)^2 d\sigma^2 / d\varepsilon = 2(M\mu b)^2 \sigma d\sigma / d\varepsilon \]  
Eq. (7)

By employing Eqs. (3), (5) and (7), Eq. (2) can be rewritten as follows:

\[
2\theta\sigma = r\sigma_{sat}^2 - r\sigma^2 = (M\mu b)^2 h - r\sigma^2 \]  
Eq. (8)

Here \( \theta \) is the current value of the work hardening rate \( d\sigma / d\varepsilon \).

The formalism of Eq. (8) indicates that the work hardening parameters \( r \) and \( h \) can be readily determined by replotting the experimental stress-strain data in the form of \( 2\theta\sigma \) vs. \( \sigma^2 \). As long as these plots are linear, \( r \) can be obtained from the slope while \( h \) can be derived from the vertical intercept \( \sigma_{sat}^2 = (M\mu b)^2 h \). An example of the conversion of such stress-strain data into a \( 2\theta\sigma \) vs. \( \sigma^2 \) plot is displayed in Fig. 1(b).

### 2.4. Determination of the Avrami Kinetics

The last element required to complete Eq. (1) is \( X \), the fractional softening. For this purpose, the Avrami formalism is used here to describe the kinetics of DRX. The fractional softening associated with the occurrence of DRX can be expressed using Eq. (9), where it is represented as a function of the time \( t \), referred to as the dynamic recrystallization time, the Avrami constant \( k \), and the Avrami time exponent \( n \). The coefficients \( k \) and \( n \) are evaluated in turn using Eq. (10).

\[
X = 1 - \exp \left(-kt^n\right) \]  
Eq. (9)

\[
\log\ln \left[ 1 / (1 - X) \right] = \log k + nt \]  
Eq. (10)

### 2.5. Modeling Procedure

In order to model flow curves using Eq. (1), the equations for \( X \), \( \sigma_{wh} \) and \( \sigma_{rex} \) must be expressed as functions of the Zener-Hollomon parameter \( Z = \dot{\varepsilon} \exp(Q_{def}/RT) \), where \( \dot{\varepsilon} \) is the strain rate, \( Q_{def} \) the activation energy of the deformation, and \( R \) the gas constant. This enables \( X \), \( \sigma_{wh} \) and \( \sigma_{rex} \) to be evaluated at any deformation temperature and strain rate. For this purpose, the dependences on \( Z \) of \( \sigma_0 \), \( \sigma_c \), \( \sigma_{sat} \), \( r \), \( r' \), \( \sigma_{wh} \), and \( \sigma_{rex} \) are described in equations A1 to A9 in Ref. 8). Finally the work hardening parameter \( r' \), is introduced to take into account the average recovery rate of the crystals that are currently at different stages of DRX in the recrystallized portion of the material.
n and k must first be determined experimentally.

3. Materials and Experiments

3.1. Materials and Deformation Conditions

Three different steels were investigated in the present study. The first was a Nb-modified plain carbon steel (steel A), the second a Nb-modified TRIP steel (steel B) and the third a Ti-stabilized low carbon steel (steel C). Their respective compositions are listed in Table 1.

Cylindrical compression samples were machined from rolled plates with their axes parallel to the rolling direction. The dimensions of the samples were 7.6 mm in diameter and 11.4 mm in height. Grooves were machined into the end faces of the samples to ensure efficient lubrication between the anvils and the samples by acting as lubricant reservoirs during deformation. The lubricant used was boron nitride powder. Constant strain rate compression testing was carried out on an MTS servo-hydraulic machine with a maximum load capacity of 100 kN. Samples were heated up to 1 200°C and held for 15 minutes and then cooled to the deformation temperature. The specimens were held for 5 minutes at temperature to permit homogenization prior to testing. The testing conditions in compression are listed in Table 1. The temperature and strain rate ranges chosen for each steel were selected so that there was clear evidence for the occurrence of DRX during testing.

3.2. Experimental Flow Curves

Employment of the deformation conditions listed in Table 1 led to a set of 57 stress-strain curves. A selection of these is presented in Fig. 2. It should be noted that all the steel C (plain C) flow curves exhibit well defined DRX peaks (Fig. 2(c)), whereas DRX peaks are no longer observed in the case of steels A (a microalloyed steel) and B (a TRIP steel) at the lowest temperature, i.e. at T=950°C and T=1 000°C, respectively (Figs. 2(a) and 2(b)). Multiple peaks were also observed at the higher temperatures and lower strain rates in steels B and C.

To begin the analysis, the work hardening parameters pertaining to the flow curve prior to the critical strain associated with the initiation of DRX must be derived. Here, a 2% offset was used to define the yield stress and therefore the beginning of work hardening. The part of each flow curve between ‘yielding’ and εc, was fitted with a 7th order polynomial using the MATLAB® software. These polynomials were employed to produce the 2θσ vs. σ2 plots and to determine the values of the r and h parameters. The critical stresses σc and strains εc, associated with the initiation of DRX were calculated by means of the double differentiation method.17)

The deformation activation energies Q_{def} were calculated from the peak or maximum stresses using the hyperbolic law in Arrhenius form to describe the relationship between Z and σ:

\[ Z = \dot{\varepsilon} \exp \left( \frac{Q_{\text{def}}}{RT} \right) = A \left[ \sinh \left( \alpha \sigma \right) \right]^{n} \] ............(11)

Here A, α and n are material constants.

<table>
<thead>
<tr>
<th>Steel</th>
<th>Composition (wt%)</th>
<th>T (°C)</th>
<th>ε (s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: Nb-modified steel</td>
<td>0.011C; 0.26Si; 1.1Mn; 0.038Nb; 0.003S; 0.004P; 0.004O; 0.03Al; 0.003N</td>
<td>950</td>
<td>0.5, 0.25, 0.1, 0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0.1, 0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1050, 1075, 1100, 1150</td>
<td>0.5, 0.25, 0.1, 0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1200</td>
<td>0.05</td>
</tr>
<tr>
<td>B: Nb-modified TRIP steel</td>
<td>0.22C; 1.5Si; 1.56Mn; 0.045N; 0.017Ti; 0.004S; 0.006P; 0.002O; 0.031Al; 0.004N</td>
<td>1000</td>
<td>0.5, 0.25, 0.1, 0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1050</td>
<td>0.5, 0.25, 0.1, 0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1100</td>
<td>0.5, 0.25, 0.1, 0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1150</td>
<td>0.5, 0.25, 0.1, 0.05</td>
</tr>
<tr>
<td>C: Ti-stabilized low carbon</td>
<td>0.019C; 0.2Si; 1.5Mn; 0.017Ti; 0.008S; 0.005P; 0.04Al; 0.0089N; 0.0003B</td>
<td>880</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>900</td>
<td>1, 0.5, 0.25, 0.1, 0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>950, 1000, 1050</td>
<td>1, 0.5, 0.25, 0.1</td>
</tr>
</tbody>
</table>

Fig. 2. Experimental flow curves determined for steels A (Fig. 2(a)), B (Fig. 2(b)) and C (Fig. 2(c)). Not all the curves are displayed here for clarity reasons. The critical strains are represented by black circles.
After determining the values of \( n \) and \( \alpha \) using the method described by Lin et al.,\(^{18} \) values of 351, 404 and 323 kJ mol\(^{-1} \) were obtained, respectively, for steels A, B and C. These were calculated from the slopes of plots of \( \ln \sinh(\alpha \sigma_p) \) vs. reciprocal absolute temperature.\(^8 \) The values for steels B and C are in good agreement with those reported by previous workers.\(^{5,6} \)

4. Work Hardening Parameters

4.1. \( 2\theta \sigma \) vs. \( \sigma^2 \) Plots Curves

As mentioned above, the work hardening parameters \( r \) and \( h \) were obtained by converting the stress-strain curves into \( 2\theta \sigma \) vs. \( \sigma^2 \) plots. The temperature dependences of \( b \)\(^{19} \) and \( \mu \)\(^{20} \) were taken into account and \( M \) and \( \alpha \) were set equal to 3 and 0.5,\(^{21} \) respectively. Some of the plots calculated from the steel A, B and C compression curves are illustrated in Fig. 3.

It can be seen that the data for the three steels are well represented by linear fits between yielding and the critical stress for the initiation of DRX. When the stress exceeds the critical stress, the \( 2\theta \sigma \) vs. \( \sigma^2 \) plot no longer follows the linear dependence as a result of the initiation of DRX. The softening attributable to dynamic recrystallization leads to a further decrease in the strain hardening rate beyond that associated with dynamic recovery. After the peak stress is reached (zero strain hardening rate), the stress decreases until a second condition of zero strain hardening rate is reached. This is the steady state stress \( \sigma_{ss} \). It is important to note that a critical stress for the initiation of DRX can be defined even when the flow curve does not exhibit a DRX peak. The four highest plots corresponding to \( T=950^\circ \text{C} \) in Fig. 2(a) provide examples of this phenomenon. This means that even when a DRX peak is not apparent, the occurrence of DRX can still be detected on the stress-strain curve.

4.2. Values of the Work Hardening Parameters

The \( r \) and \( h \) values obtained from the \( 2\theta \sigma \) vs. \( \sigma^2 \) plots can now be presented as functions of \( Z \), as shown in Fig. 4. Here it can be seen that \( h \) increases with \( Z \) for the three steels; this represents the dependence of the saturation stress on \( Z \). By contrast, the value of \( r \) decreases with \( Z \), indicating that dynamic recovery becomes less effective as the temperature is decreased or the strain rate is increased. Nevertheless, \( r \) appears to saturate at high \( Z \) values and can therefore be considered as a constant under such conditions. As observed in our earlier study,\(^{15} \) the \( r' \) values are lower than the \( r \) values, although they exhibit the same trends (e.g. saturation at high \( Z \) values).

\[
\begin{align*}
\sigma_{wh} & = 4 + 9.6 \times 10^5 Z^{-0.5} \quad (12) \\
\sigma_{ex} & = 6 + 2.5 \times 10^6 Z^{-0.5} \quad (13) \\
\sigma_{wh} & = 6 + 1.2 \times 10^7 Z^{-0.4} \quad (14) \\
\sigma_{ex} & = 7.5 + 3 \times 10^4 Z^{-0.3} \quad (15) \\
\sigma_{wh} & = 4 + 1.3 \times 10^7 Z^{-0.4} \quad (16) \\
\sigma_{ex} & = 6 + 1.5 \times 10^4 Z^{-0.3} \quad (17)
\end{align*}
\]

5. Evaluation of the Avrami Parameters \( n \) and \( k \)

With the aid of these quantities, the \( \sigma_{wh} \) and \( \sigma_{ex} \) flow curves were constructed for each set of experimental conditions. An example of such a construction is displayed in Fig. 5(a) for steel C at \( T=1000^\circ \text{C} \) and \( \varepsilon=0.5 \text{ s}^{-1} \). Values of \( X \) were then determined from each experimental curve using Eq. (1), after which the Avrami coefficients were evaluated for each experimental condition using Eq. (10), as shown in Fig. 5(b). The Avrami parameters, \( n \) and \( k \), were derived from these plots using Eq. (9).

6. Modeling High Strain Rate Flow Curves

In order to model flow curves for \( Z \) values that are higher than those employed in the laboratory, the dependences of
σ_{wh}, r, \ r', n and k. The dependences of the work hardening parameters \ r and \ r' on Z have been specified above for steels A, B and C (Eqs. (12) to (17)). It now remains to provide expressions for the characteristic stresses, \ σ_0, \ σ_c and \ σ_{sat}, as well as the Avrami parameters, n and k.

6.1. The Characteristic Stresses

The dependences of \ σ_0, \ σ_c and \ σ_{sat} on Z are presented in Fig. 6 for steels A (Fig. 6(a)), B (Fig. 6(b)) and C (Fig. 6(c)). As outlined previously, the experimental stress-stress curves covered the range from the multiple peak regime to the range where DRX is initiated but has not spread sufficiently to produce a peak. The trends illustrated in Fig. 6 can therefore be considered to cover the full range of Z values. These relationships are expressed by Eqs. (18) to (20) for steel A, Eqs. (21) to (23) for steel B and Eqs. (24) to (26) for steel C. As these are linear dependences, they can be readily extrapolated to higher Z values.
6.2. Avrami Kinetics

The dependence of the Avrami exponent \( n \) on \( Z \) is presented in Fig. 7 for steels A, B and C. The coefficient \( n \) decreases as \( Z \) increases, a dependence that corresponds to a power law. This relationship is expressed for steels A, B and C by Eqs. (27), (28) and (29), respectively.

\[
\begin{align*}
\sigma_0 &= 15.7 \times \log(Z) - 157 \quad \text{(18)} \\
\sigma_c &= 29.7 \times \log(Z) - 309 \quad \text{(19)} \\
\sigma_{sat} &= 32.7 \times \log(Z) - 327 \quad \text{(20)} \\
\sigma_0 &= 16.6 \times \log(Z) - 193 \quad \text{(21)} \\
\sigma_c &= 29.5 \times \log(Z) - 359 \quad \text{(22)} \\
\sigma_{sat} &= 30.2 \times \log(Z) - 284 \quad \text{(23)} \\
\sigma_0 &= 15.9 \times \log(Z) - 136 \quad \text{(24)} \\
\sigma_c &= 30.2 \times \log(Z) - 340 \quad \text{(26)} \\
\end{align*}
\]

The dependence of the Avrami constant \( k \) on \( Z \) is not as straightforward as it is for \( n \). In Fig. 8(a), the crosses represent experimental values of \( k \) for steel C obtained by employing the Avrami formalism and the data of Fig. 5(b). It can be seen that the dependence of \( k \) on \( Z \) is not a straightforward one. For this reason, \( k \) was calculated from the \( t_{50} \) values, the time of half recrystallization, where the influences of \( T \) and \( \dot{\varepsilon} \) were derived separately (not in combination as a function of \( Z \)). The dependences of the experimental \( t_{50} \) of steel C on temperature and strain rate are illustrated in Fig. 8(b), where they have been fitted using power laws. These dependences are expressed by means of Eq. (30), where the strain rate exponent is \( q \). The dependences of the two parts of Eq. (30) on \( T \) and \( \dot{\varepsilon} \) were then assessed independently, leading to Eqs. (31), (32) and (33) for steels A, B and C, respectively. These expressions are illustrated in Figs. 8(c) and 8(d). In this way, the Avrami constant \( k \) can be calculated for any \( T \) and \( \dot{\varepsilon} \) by transforming \( t_{50} \) into \( k \) using Eq. (34).

\[
t_{50} = A \dot{\varepsilon}^{-q} \exp \left( \frac{-Q_{DRX}}{RT} \right) \quad \text{(30)}
\]
Here $d_0$ is the initial grain size, $v$ the grain size exponent, $q$ the strain rate exponent and $Q_{\text{DRX}}$ the activation energy associated with DRX.

\[ t_{50} = \left(9.2 \times 10^{21} \times T^{-7.3}\right) \times \dot{\varepsilon}^{(-0.00047 \times T + 1.2)} \]  
\[ t_{50} = \left(5.3 \times 10^{12} \times T^{-4.3}\right) \times \dot{\varepsilon}^{(-0.0007 \times T + 1.5)} \]  
\[ t_{50} = \left(1.7 \times 10^{12} \times T^{-4.2}\right) \times \dot{\varepsilon}^{(-0.0009 \times T + 1.3)} \]

\[ k = -\ln 0.5 \]  
\[ t_{50} \]  
\[ (34) \]

The values of $k$ obtained by this means for steel C using Eq. (34) are represented by the solid lines in Fig. 8(a). It is worth noting that, at a constant temperature, the dependence of $k$ on $Z$ is not linear, which accounts for the inaccuracy of earlier extrapolations of $k$ to high $Z$ values.

### 6.3. Flow Curve Modeling

Using Eq. (1) and the dependences of $\sigma_0$, $\sigma_c$, $\sigma_{\text{sat}}$, $r$, $r'$, $n$ and $k$ on $Z$, reported above, flow curves can be predicted for arbitrary combinations of $T$ and $\dot{\varepsilon}$. Some simulated flow curves for steel A are illustrated in Fig. 9(a) and compared with the experimental flow curves corresponding to the same deformation conditions. The good agreement shown there demonstrates the ability of the modeling method to produce accurate results. It is of interest that the transition from a single peak flow curve ($T=1100^\circ C$ and $\dot{\varepsilon}=0.05$ s$^{-1}$) to one without any peak at all ($T=950^\circ C$ and $\dot{\varepsilon}=0.5$ s$^{-1}$) is well reproduced by the simulated curves. Thus the model used in this study describes the softening effect of DRX over a broad range of deformation conditions ($T$ and $\dot{\varepsilon}$) with considerable accuracy.

Simulation of the occurrence of DRX at rolling mill strain rates is particularly important as these deformation rates cannot readily be attained in laboratory tests. It was seen in Fig. 2 that steel C exhibits a clear DRX peak, at a strain rate of 1 s$^{-1}$, even at temperatures close to the $Ae_3$ (calculated to be between 867 and 875$^\circ C$ using Andrews’ and Grange’s formulas, respectively). Simulations were carried out at $T=900^\circ C$ and strain rates up to 100 s$^{-1}$ to see whether DRX was likely to be initiated under industrial rolling conditions. The results are displayed in Fig. 9(b) with some experimental flow curves determined at 900$^\circ C$ and strain rates below 1 s$^{-1}$.

At a strain rate of 10 s$^{-1}$, a clear peak is evident. When the strain rate is increased to 100 s$^{-1}$, although a peak is no longer evident, the model shows that DRX has been initiated at $\varepsilon=0.24$. Thus, beyond a strain of 0.24, the flow stress is controlled by the occurrence of DRX even though no stress maximum can be seen. Similar remarks apply to steels A and B, which contain higher concentrations of alloying elements. In Fig. 9(c), simulated flow curves are displayed for steel B and a temperature of 900$^\circ C$. At this temperature, the DRX peak is only apparent at a strain rate of 0.05 s$^{-1}$. Above 1 s$^{-1}$, even though no DRX peak can be seen, it is clear that DRX has been initiated and therefore controls the flow stress. Thus the effects of DRX softening must be taken into account when modeling separation force and torque in rolling mills for materials of the present type, even when stress peaks are not displayed on the flow curves.

### 7. Conclusions

Hot compression tests were carried out on three commercial steels in order to determine the effects of temperature, strain rate and strain on the flow curves. Those obtained on the Ti-modified low carbon steel displayed clear DRX peaks right down to the $Ae_3$ temperature. Conversely, no peaks were observed on the flow curves of the two Nb-modified steels in the temperature range 900–950$^\circ C$ at strain rates up to 1 s$^{-1}$. Nevertheless, it was evident that DRX had been initiated under the testing conditions employed.

The kinetics of DRX were established from the flow
curves and the relations obtained in this way were used to simulate the flow curves at industrial strain rates up to 100 s⁻¹. In the plain carbon steel, DRX is predicted to begin, for T=900°C, at strains of 0.2 to 0.3 in the strain rate range 10 to 100 s⁻¹, so that DRX clearly plays a role in determining the flow stress and therefore the rolling load. In the two Nb-modified steels, the initiation of DRX is delayed to strains of 0.3 to 0.4 in this strain rate range and for the same temperature. Despite the delay, the kinetics of DRX continue to play a role in establishing the flow stress in the microalloyed steels, particularly when strain accumulation is taking place.

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