Effect of Random Pore Shape, Arrangement and Non-adhesion Grain Boundaries on Coke Strength

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In this study, the rigid bodies-spring model (RBSM) was used to numerically investigate how the fracture behavior of coke is affected by pore structure and non-adhesion grain boundaries. To study the effects of pore structure, randomly shaped pores were generated and randomly positioned in a coke matrix. The random shapes of pores were controlled by pore roundness and their random sizes were controlled by equivalent circle diameters. Non-adhesion grain boundaries were also randomly located in the coke matrix. First, results for a coke model with realistic pore structures showed that large distorted pores decrease coke strength. Second, fracture behavior was analyzed for a coke model composed of a coke matrix, pores, and non-adhesion grain boundaries. Coke strength decreased as the number of non-adhesion grain boundaries increased; these numerical results agreed with previous experimental data. Further, coke strength decreased even in the presence of only a relatively small number of non-adhesion grain boundaries. This is because, when non-adhesion grain boundaries occur in stress-concentrated regions, those boundaries become origins for fracture. This indicates that the presence of non-adhesion grain boundaries is one factor that decreases the strength of coke when it has been blended with low-quality coal.

KEY WORDS: ironmaking; coke; low-quality coal; coke strength; non-adhesion grain boundary; RBSM.

1. Introduction

To effectively use low-quality coals (e.g., non-caking coal, slightly caking coal, and steam coal) as metallurgical coal, high-strength coke is required as a spacer to sustain the flow of liquid metal and reducing gas in a blast furnace. Coke, which is generally produced from caking coal, is a porous material, and its strength is known to be determined by its pore structure rather than by the hardness of the coke matrix.1) For example, Patrick and Stacey2,3) and Hiraki et al.4) observed coke structures with an optical microscope and discussed the correlation between structure and strength using the average pore diameter and bulk density of the coke. For caking coal, the relationship could be determined using mean values because few large defects appeared inside the coke specimens. However, when low-quality coal is blended, it is known that defects, defined as non-adhesion grain boundaries, are generated around coal particles because the dilatability of low-quality coal is low compared to that of caking coal.5) Non-adhesion grain boundaries are thought to exist at interfaces between coal particles and arise by inhibition of adhesion of coal particles during carbonization. Kubota et al.6) studied pores with low roundness below 0.2 in coke from caking and low-quality coals; they considered such pores to be defects and suggested that the boundary length of a pore is related to coke strength. In this case, a connected pore that was joined by free expansion of coal particles could be extracted because of its low roundness. However, since a pore with low roundness below 0.2 would contain not only the connected pore but also the non-adhesion grain boundary, it is difficult to identify the factor that decreases coke strength in terms of only the low roundness pores. This is because a non-adhesion grain boundary is generated at the interface of coal particles, and when the polished surface of coke from low-quality coal is observed using a microscope, the boundary would be identified as a pore. Kanai et al.7) performed a diametral-compression test for coke from low-quality coal, observed fracture surfaces of the coke using a scanning electron microscope, and then quantified the non-adhesion grain boundaries. They found that coke strength, which was measured using their test and an I-shaped drum index one for the coke with several blending ratios of low-quality coal, was related to the existence ratio of non-adhesion grain boundaries. In particular, the presence of only a few non-adhesion grain boundaries was

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found to drastically decrease tensile strength, and the effects of the boundaries on coke strength were experimentally evaluated.

To investigate coke strength numerically, fracture analyses have been conducted using the rigid bodies-spring model (RBSM). Ogata et al.10 performed numerical fracture analyses using a four-point bending test for two objects: (a) a sample in which no pore appeared in the computational domain (i.e., only the apparent property of the coke was considered) and (b) a sample in which 100 circular pores were placed uniformly. The analytical results indicated that in the sample with pores, fracture occurred in the vertical direction of the pores and differed from fracture in the sample without pores; the numerical results with pores correlated with experimental data. Therefore, to reproduce the fracture behavior of coke, it appears that the pore structure of coke has to be considered. From the above results, Ueoka et al.9 analyzed coke models with simple pores to understand how fracture behavior is affected by the size and shape of pores, porosity, and pore wall thickness. They demonstrated that the onset of plastic deformation decreased with lower stress concentration coefficients of pore shape (i.e., square, circle, lozenge, and ellipse) and predicted that pore shape affects fracture phenomena in coke. Moreover, they pointed out that the effects of stress concentration increased with a decrease in pore wall thickness and the fracture load also decreased. They suggested that structures with small pores were harder to fracture than those with large pores. Hiraki et al.10 also carried out fracture analyses using four-point bending tests to study the effects of non-adhesion grain boundaries; in particular, they investigated the effects that location of a non-adhesion grain boundary have on fracture strength of coke. They placed a boundary at the upper center, center, or lower center of the computational domain. They supposed that when a non-adhesion grain boundary exists in a stress-concentrated part, a fracture would progress from that part and its behavior would depend on the arrangement of the boundaries. Although Ogata et al.10 and Ueoka et al.9 analyzed coke models with pores, they did not reproduce the complicated shapes of pores that are observed in actual coke, and their pores were placed uniformly. Non-uniform pore distributions were not considered. Furthermore, though Hiraki et al.10 numerically analyzed a coke model with non-adhesion grain boundaries, the positions of the boundaries were fixed, and the analysis corresponding to the existence ratio of the boundaries with an increase in low-quality coals, as shown by Kanai et al.,7 was not conducted. Moreover, since pores and the coke matrix were not distinguished in their analyses, a coke model containing non-adhesion grain boundaries, pores, and coke matrices has not been analyzed.

In the present study, randomly shaped pores were generated and randomly arranged to reproduce the complicated shapes and distributions of pores. The effects of the shapes and sizes of pores, as controlled by pore roundness and equivalent circle diameters, on fracture behavior were numerically investigated using fracture analyses. Furthermore, to evaluate the effects of existence ratio of non-adhesion grain boundaries on fracture strength, randomly shaped pores were randomly arranged, based on previous experimental data, and a coke model involving coke matrices, non-adhesion grain boundaries, and pores was developed and analyzed.

2. Numerical Methods

2.1. Randomly Shaped Pores

In previous fracture analyses of coke, pores with simple shapes (e.g., circles and squares) and certain sizes were placed in the computational domain. However, based on observations of structures of real coke, pores have complicated shapes and their various sizes. In the present study, we modeled randomly shaped pores, based on the method of Wang et al.,11 in which the shapes and sizes of pores were expressed by roundness and equivalent circle diameter, thereby mimicking realistic coke. To do so, the following operations were used: (1) a pore with a polygonal shape was generated, (2) then the polygon was distorted into arbitrary roundness, (3) and was calibrated to an arbitrary equivalent circle diameter. (4) Finally, the shape was rotated to avoid artifacts.

2.1.1. Generation of a Polygonal Pore

To generate a polygonal pore, the position for the center point of the pore was assigned. Next, a vertex was located at distance \( r_i \) from the center. The line segment from the vertex to the center was rotated through an angle \( \phi_i \), then the next vertex was located at distance \( r_{i+1} \) from the center. After \( n \) vertices were located, as shown in Fig. 1(a), the polygonal pore has been generated. The distance \( r_i \) from the center to vertex \( i \) and the angle \( \phi_i \) between rays from vertex \( i \) and vertex \( i + 1 \) are expressed by

\[ r_i = A_0 + (2\alpha - 1) \times A_1, \quad \phi_i = \frac{2\pi}{n} + (2\beta - 1) \times \delta \times \frac{2\pi}{n}, \]

where \( A_0 \) is the reference pore radius and \( A_1 \) is the maximum allowed variation in the length of the radius. The parameters \( \alpha \) and \( \beta \) are random variables between zero and one, and \( \delta \) is the maximum allowed variation in the angle. Figure 1(b) shows an example of a pore when the parameters were set to \( n = 7, A_0 = 1, A_1 = 0.5, \delta = 0.5 \).

2.1.2. Distortion of Pore Shape Based on Roundness

The pore generated by the procedure in Section 2.1.1 is randomly shaped. But since actual coke has complicated forms, the effects of pore shape on coke fracture strength cannot be evaluated using only complicated pore shapes. Therefore, roundness of the pore is used as a measure of pore shape. Roundness is defined by

\[ \text{Roundness} = \frac{4\pi S}{L^2}, \]

where \( S \) is the area of the pore and \( L \) is its perimeter. Using the coordinates of the vertices \((X_i, Y_i)\), the area of a polygonal pore can be calculated from

\[ S = \sum_{i=1}^{N} \frac{1}{2} (X_i - X_{i+1}) (Y_i + Y_{i+1}), \]

while the perimeter can be computed from the vertex coordinates. Then Eq. (3) is used to calculate the initial roundness, and the \( y \) coordinates of the vertices are moved until
the pore attains an arbitrary roundness. Figure 1(c) shows the polygonal pore in Fig. 1(b) after it was deformed to a roundness of 0.6.

2.1.3. Deformation of Pore Shape Based on Equivalent Circle Diameter

When the roundness of a pore is changed, the area and size of the pore also change. Since pores in actual coke have various sizes, the sizes of the pores should be adjusted. The equivalent circle diameter is used as a measure of pore size, so a pore with arbitrary roundness is resized by varying the equivalent circle diameter. In particular, the area of the polygonal pore is calculated and the pore is then expanded or contracted by moving its vertices so as to obtain the prescribed area. Figure 1(d) shows the pore in Fig. 1(c) after the vertices were moved to generate a pore with an equivalent circle diameter of 25 μm.

2.1.4. Rotation of Pore

After the steps in Sections 2.1.2–2.1.3, the shape of the pore is strained in the x direction because the pore has been scaled only in the y direction. Therefore, the shape would still retain some uniformity, and randomness would not be complete. To avoid this, the pore is rotated to represent a randomly shaped pore.

2.2. Random Arrangement of Pores

Although randomly shaped pores are acquired by the steps in Sections 2.1.1–2.1.4, if they are arranged uniformly in the computational domain, the pore structure of realistic coke will not be reproduced. Therefore, pores are randomly positioned in the computational domain. After a pore is generated, as described in Sections 2.1.1–2.1.4, the pore is placed in the computational domain. The location of the pore is determined by adding random variables \( (c_x, d_y) \) to the coordinates of all pore vertices. If the coordinates for a vertex do not fall in the domain, the operation is repeated, and we again attempt to place the new pore in the domain. In placing all pores after the first, it is necessary to avoid overlapping any pore that is already in the domain. In this study, overlap of pores is determined by (1) whether any vertex of a new pore is included in an existing pore and (2) whether any side of a new pore lies across a side of an existing pore. If an overlap is found, the new pore is randomly repositioned to prevent any overlap. Figure 2 shows randomly shaped pores that have been randomly positioned by the above method. In the figure, the number of vertices per pore was set between four and ten, and 90 pores were placed in the computational domain, each having equivalent circle diameters of 707 μm. The figure demonstrates that disoriented and random pore positions can be achieved.

2.3. Development of a Coke Model with Non-adhesion Grain Boundaries

To perform analyses that include non-adhesion grain boundaries, a model including coke matrices, pores, and non-adhesion grain boundaries must be developed. The pores should be randomly shaped and randomly arranged in the computational domain and non-adhesion grain boundaries should also be randomly placed in the coke matrix. When a fracture analysis of this coke model is conducted,
realistic results can be expected if the pore structure of actual coke has been reproduced. In this study, the random arrangement of pores with diameters and roundness based on the measurements reported by Kanai et al.,12) shown in Fig. 3, yields the pore structure shown in Fig. 4. However, in the figure, many acute pores exist, and they obviously differ from structures observed by Kanai et al.12) This is because pore roundness is adjusted only in the y-direction, and large low-roundness pores tend to be acute. In the present study, to avoid generating such peculiar pores, the roundness was set to between 0.5 and 1.0, and the equivalent circle diameter was made larger than 200 μm. Then, pores with the parameters given in Table 1 were placed in the domain. Non-adhesion grain boundaries were randomly placed at interfaces of the elements (i.e., coke matrices) in the domain where pores and coke matrices were arranged under the above conditions.

2.4. Rigid Bodies-spring Model

In the present study, fracture analyses were conducted using the rigid bodies-spring model (RBSM) developed by Kawai.13) The methodology was the same as that used in previous studies.9,10)

#### Table 1. Pore size distribution and pore roundness used in this analysis.

<table>
<thead>
<tr>
<th>Equivalent circle diameter [μm]</th>
<th>Pore area ratio in porosity [-]</th>
<th>Pore roundness [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>243</td>
<td>0.103</td>
<td>1.00</td>
</tr>
<tr>
<td>341</td>
<td>0.086</td>
<td>0.89</td>
</tr>
<tr>
<td>445</td>
<td>0.073</td>
<td>0.73</td>
</tr>
<tr>
<td>542</td>
<td>0.052</td>
<td>0.72</td>
</tr>
<tr>
<td>644</td>
<td>0.057</td>
<td>0.65</td>
</tr>
<tr>
<td>752</td>
<td>0.029</td>
<td>0.66</td>
</tr>
<tr>
<td>844</td>
<td>0.043</td>
<td>0.57</td>
</tr>
<tr>
<td>947</td>
<td>0.021</td>
<td>0.57</td>
</tr>
<tr>
<td>1500</td>
<td>0.307</td>
<td>0.50</td>
</tr>
</tbody>
</table>

[Fig. 2. Example of randomly shaped pores placed randomly in the computation domain.](image2)

[Fig. 3. Experimental results for relationship between pore shape (roundness) and pore size (equivalent circle diameter).](image3)

[Fig. 4. Example of randomly sized and shaped pores placed in a computational domain area 10×10 mm.](image4)

[Fig. 5. Illustration of the diametral-compression test used in numerical fracture analyses (White areas represent pores, black area represents the coke matrix).](image5)
2.5. Numerical Object and Conditions

2.5.1. Fracture Analysis for Randomly Arranged Pores with Random Shapes

Fracture analyses for several coke models using the diametral-compression test were performed as illustrated in Fig. 5. Values of relevant material properties are summarized in Table 2. The shear strength was estimated by the following approximate equation for brittle materials, such as glass and rock, according to Hiraki et al.:

\[ c = 3^{0.5} F_t \] ................................. (5)

where the compressive strength was assumed to be fifteen times larger than the tensile strength. As the fracture condition for coke, when the strain at the center of the coke sample was over 0.0017, it was judged that the coke was fractured. This fracture condition is the same one used in reproducing the results reported by Hiraki et al.\(^4\) Pore sizes were 300, 600, and 1000 \(\mu m\) in equivalent circle diameters and pore shapes were 0.6, 0.8, and 1.0 in roundness. Figure 6 shows the pores arranged in the computational domain for these values of equivalent circle diameter and roundness. The porosity was 38.5% and the number of vertices on each polygon was ten.

2.5.2. Fracture Analysis for a Coke Model with Non-adhesion Grain Boundaries

To investigate the effects of non-adhesion grain boundaries on coke strength, fracture analyses of diametral-compression tests on a coke model with coke matrices, pores, and boundaries, which were randomly arranged were performed. The analytical domain is shown in Fig. 7 and material properties are listed in Table 2. The porosity and the number of vertices of the polygon were 38.5% and ten, respectively. Non-adhesion grain boundaries were randomly arranged at interfaces of coke matrix elements at existence ratios of 0, 1, 5, 10, 30, 50, and 70%. Considering the combinations of boundaries and pores, fracture analyses for twelve kinds of analytical objects were conducted for each condition; these twelve differed only in the arrangement of pores and non-adhesion grain boundaries. Note that the coke model without non-adhesion grain boundaries is just coke from caking coal. The fracture criterion for non-adhesion grain boundaries was set to one tenth of that for the coke

| Table 2. Material properties used for coke models. |
|-----------------------------------|----------|
| Elastic modulus \([GPa]\)         | 60       |
| Poisson’s ratio \([-\]          | 0.2      |
| Critical breakage strain for tensile direction \([-]\) | 6.4 \times 10^{-4} |
| Tensile strength \([MPa]\)       | 80       |
| Critical breakage strain for compressive direction \([-]\) | 1.1 \times 10^{-4} |
| Compressive strength \([MPa]\)   | 1200     |
| Shear strength \([MPa]\)         | 138.6    |
| Internal friction angle \([\gamma]\) | 32      |

Fig. 6. Analytical objects on which numerical fracture analyses were performed using diametral-compression tests (Porosity is 38.5%).

Fig. 7. Analytical object containing pores, coke matrix, and non-adhesion grain boundaries.

Fig. 8. Arrangement of non-adhesion grain boundaries at 5% existence ratio (red lines indicate non-adhesion grain boundaries, gray area is coke matrix). (Online version in color.)
matrix based on the research of Hiraki et al.\textsuperscript{10} Figure 8 shows the analytical object with an existence ratio of 5\% for the boundaries. In the figure, the red lines locate non-adhesion grain boundaries.

3. Results and Discussion

3.1. Fracture Analysis for Randomly Arranged Pores with Random Shapes

First, the effects of equivalent circle diameter and roundness of pores on fracture phenomena are investigated. As an example of the results, Fig. 9 shows the state of the springs in the coke model after fracture for pores of 600 $\mu$m equivalent circle diameter and roundness of 0.8. The figure indicates a number of coke matrices in which tensile fracture progressed at the upper and lower sides of pores around the vertical centerline in the computational domain. This is because a vertical tensile stress developed around the centerline in the load direction, and the fracture progressed from the center of the sample. Similar behavior was found in the experimental results of Hiraki et al.\textsuperscript{4} Moreover, since coke matrices marked with the black circle between pores were fractured, cracks in the diametral-compression test were caused by fracture of coke matrices between pores around the centerline in the load direction. Fracture behavior in other coke models having different equivalent circle diameters and roundness was similar to that in Fig. 9. Consequently, equivalent circle diameter and roundness of pores have small effects on the fracture behavior of coke.

Next, the effects of equivalent circle diameter and roundness of the pores on the fracture strength are evaluated. The analytical object was a coke model in which pores were placed randomly in the computational domain. A unique fracture might be caused by the arrangement of pores because coke matrices between pores were fractured, as shown in Fig. 9. In the present analysis, the effects of the random number and meshes on the numerical result were eliminated by conducting fracture analyses for twelve kinds of coke models that differed only in the arrangement of the pores; the equivalent circle diameter and roundness were the same for all twelve models. Figure 10 shows the resulting fracture stresses for different pore diameters and various degrees of roundness. When the porosity of coke and the equivalent circle diameters were constant, the fracture stress was small when the roundness was low (i.e., the pore shape was distorted). When the equivalent circle diameter was large (i.e., the sizes of pores were large), the fracture stress was also small. These results show trends similar to the homogenized elastic coefficients calculated by Ueoka et al.\textsuperscript{14} for coke models with various pores.

Now, the effects of roundness of pores on the relationship between stress and strain in coke are investigated. Figure 11 shows the stress-strain curves with pores having roundness of 0.6, 0.8, and 1.0 and a constant equivalent circle diameter of 600 $\mu$m. In coke models with low roundness pores, plastic deformation started in the region where the load was

![Fig. 9. Locations of fractured springs (blue) in the coke model after fracture. The pores in this model had roundness = 0.8 and equivalent circle diameter = 600 $\mu$m. (Online version in color.)](image)

![Fig. 10. Effect of the shape and size of pores on fracture load.](image)

![Fig. 11. Effects of pore shape on horizontal stress-strain curves. In all these cases, pore sizes were the same (equivalent circle diameter = 600 $\mu$m).](image)
small and the apparent elastic coefficient was also small. This is because that distorted pores decrease resistance to transformation, i.e., stiffness. Moreover, pores with low roundness are sharp and form where the stress tends to be locally concentrated. Therefore, when pores with low roundness exist, a crack occurs in the region where the load is small because of the small apparent elastic coefficient and generation of stress concentration, and the fracture load is thought to be small.

The above numerical results indicate that large distorted pores remarkably decrease coke strength. Based on diametral-compression tests and observations of polished surfaces of coke, Kanai et al. found that the strength of coke with large low-roundness pores was low; the present results support those observations. These results also show that realistic coke can be successfully simulated by randomly shaped and randomly arranged pores with roundness and equivalent circle diameter as parameters.

3.2. Fracture Analysis on a Coke Model with Non-adhesion Grain Boundaries

In Section 3.1, coke models with randomly shaped and randomly arranged pores were fractured, and the results indicate that large distorted pores decrease coke strength. In this section, the effect of the existence ratio of non-adhesion grain boundaries on tensile stress is investigated. Figure 12 shows numerical and experimental results for the relationship between fracture stress and the existence ratio of non-adhesion grain boundaries. When the existence ratio for non-adhesion grain boundaries is zero, i.e., no boundaries are present, the analytical object was composed of only coke matrices and pores, so coke from caking coal was assumed. For this situation, Fig. 12 shows that the analytical result for fracture strength is in excellent agreement with the experimental one. Hence, the fracture of coke can be successfully studied by conducting fracture analyses of diametral-compression tests for a coke model with realistic pore structures.

Figure 12 also shows that the numerical and experimental data correspond, and the tensile strength decreases with an increase in the existence ratio of non-adhesion grain boundaries. These results demonstrate that non-adhesion grain boundaries cause a decrease in strength after coke is blended with low-quality coal. In particular, when the existence ratio of boundaries increased from 0 to 10%, in other words, when only a few non-adhesion grain boundaries were present, the coke strength drastically decreased. This is because non-adhesion grain boundaries become origins for fracture when those boundaries exist in stress-concentrated regions. It is also predicted that fracture strength would be high when there are few boundaries in stress-concentrated regions, but the strength would be low when boundaries exist in those regions. However, when the existence ratio is more than 50%, fracture strength only decreases slightly with an increase in the number of boundaries. This means that a number of non-adhesion grain boundaries exist in regions other than stress-concentrated regions; i.e., some boundaries would be origins for fracture and others would not. In other words, although some non-adhesion grain boundaries would be origins for fracture (when the boundaries exist in stress-concentrated regions), all boundaries would not be. The above results show that when there were a few non-adhesion grain boundaries, coke strength drastically decreased in both the experimental and numerical results. Now, the arrangement of non-adhesion grain boundaries and the state of springs after fracture when the existence ratio of boundaries was 5% are discussed. In Fig. 13(a), red lines mark non-adhesion grain boundaries. In the region enclosed by the blue circle, there are thin coke matrices between pores with boundaries between them. In the same region of Fig. 13(b), the spring in the region with the non-adhesion grain boundary is fractured. As mentioned above, when boundaries exist in regions where pore walls are thin and stress tends to be concentrated, the boundaries can be defects. However, in the region marked with the black circle in Fig. 13(a), stress was not concentrated because the pore wall was thick. Even though non-adhesion grain boundaries existed, fracture did not occur.
4. Conclusions

Based on the rigid bodies-spring model, a numerical coke model that contained randomly shaped and randomly arranged pores was developed. The model was numerically fractured and the effects of shape and size of pores on coke strength were investigated. Furthermore, a coke model with coke matrices, pores, and non-adhesion grain boundaries was analyzed, and the effects of the existence of boundaries on the fracture strength were evaluated. The results can be summarized as follows:

(1) In the coke model with randomly shaped and randomly arranged pores, the presence of large distorted pores caused coke strength to decrease.

(2) From the fracture analyses on the coke model with coke matrices, pores, and non-adhesion grain boundaries, which were randomly arranged on the matrices, coke strength decreased with an increase in the existence ratio of the boundaries. Our numerical results agree with previous experimental data and show that non-adhesion grain boundaries can decrease coke strength.

(3) When there were a few non-adhesion grain boundaries, coke strength drastically decreased. This is because non-adhesion grain boundaries become origins for fracture when the boundaries occur in the stress-concentrated regions.

These results indicate that non-adhesion grain boundaries, which arise from blending coke with low-quality coal, remarkably decrease coke strength. To produce strong coke, production of the boundaries should be minimized.

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Nomenclature

- \( A_0 \): standard pore radius [m]
- \( A_l \): variation in length of pore radius [m]
- \( c \): shear strength [MPa]
- \( c, d \): random coordinates [m]
- \( F_t \): tensile strength [MPa]
- \( L \): perimeter of pore [m]
- \( n \): number of vertices
- \( r \): length between pore vertex and center [m]
- \( S \): pore area [m²]
- \( X, x, Y, y \): coordinates [m]
- \( \alpha, \beta \): random variables
- \( \delta \): degree of variability of center angle [degree]
- \( \phi \): angle

Subscripts

- \( i \): vertex

REFERENCES