Assessment of Turbulence Models for Prediction of Intermixed Amount with Free Surface Variation Using Coupled Level-Set Volume of Fluid Method

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In continuous casting tundish steelmaking, old ladle is replaced by new one to ensure continuous supply of steel from tundish to mold. Bath height changes in case of ladle change-over. To bring the bath height level to normal height, the flow rate of liquid steel from the new ladle is increased. This has a direct bearing on the fluid flow pattern and resultant intermixed amount formed. In the present work, assessment of Reynolds-averaged Navier-stokes (RANS) equations based standard k-ε, Renormalization group (RNG) k-ε, Realizable k-ε standard k-ω, and Shear-stress transport (SST) k-ω turbulence models have been carried out for prediction of free surface level of steel in tundish during ladle change-over and the intermixed amount formed. Coupled Level-Set Volume of Fluid (CLSVOF) method was used for free surface tracking in the three dimensional, multi-phase numerical model. Physical investigations were carried on water model setup of reduced scale tundish. Inflow rate of steel into the tundish from second ladle was varied due to which free-surface height of water varied and grade mixing in tundish was analyzed. Results obtained through physical investigations were compared with that of numerical investigations. The predictions revealed that RNG k-ε model have good approximation of F-curves as well as the interface between the two phases. Predictions made by all models except SST k-ω model have shown a satisfactory approximation with the experimental values. Free-surface interface profiles predicted by variants of k-ε models were seen to closely match with experimental data.

KEY WORDS: tundish; grade mixing; turbulence model; free surface.

1. Introduction

Generally, researches on tundish technology are carried using water models or numerical models. Numerical modeling of tundish operation based on Navier-Stokes equations have been done by various researchers. Most of the authors validated numerical codes with results obtained through in-house built reduced scale water models. It is well known that flow inside tundish is a turbulent flow and hence each numerical investigation reported in literature, has used one of the available turbulence models to precisely approximate the physical phenomenon. To approximate the physics inside tundish by numerical modeling is a complex work because several phenomena are simultaneously happening. This means that flow phenomenon in tundish is inherently multi-dimensional, multi-phase, reacting and turbulent. Specialy fluid flow modeling in a tundish necessitates significant efforts. There are various turbulence models available in pre-coded CFD softwares for characterization of fluid flow in different state of conditions. Few researchers working on tundish technology have made assessment of turbulence models and compared numerical investigations output with experimental results. In most of the previous numerical investigations, authors have assumed single-phase and constant bath height of molten metal in tundish during operation.1–7 In recent time, there has been few work by employing multi-phase model i.e., considering the phases of molten steel, slag and air. However, an assessment of turbulence models with multi-phase system is required. Since majority of numerical investigations have been carried out on Reynolds-averaged Navier-stokes (RANS) equation models, therefore present work includes five RANS equation based turbulence models for assessment. RANS based two equation models on eddy viscosity model has gain popularity because of better predictability in simulation of industrial problems. The computing resources required for reasonably accurate flow computations are modest, so this approach has been the mainstay of engineering flow calculations over the last three decades.8–10 These models includes two additional transport equations to characterize the turbulent properties of the fluid flow. These turbulent properties helps eddy viscosity models to account for past effects like convection and diffusion of turbulent energy. A review of the literature on numerical modeling in tundish reveals that the most of researchers9–13 used the standard k-ε turbulence model of Launder and Spalding.14 Q. Hou et al.15–17 have compared the standard and Renormalization group (RNG) k-ε turbulence models in a case of swirling flow tundish. H. Odenthal et al.17–19 have used Realizable k-ε model20 which is an advance model of standard k-ε model. They found good approximation by using RANS equation combined with the realizable k-ε model.

A detailed study have been carried by Jha et al.21 In that

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research work, nine different turbulence models have been studied. The observations carried out by them shows that the prediction from the standard k-ε model, the k-ε Chen-Kim (ck) and the standard k-ε with Yap correction (k-ε Yap), matches fairly with the mean residence time and the ratio of mixed to dead volume. Jha et al.\textsuperscript{22}\) have studied the three different turbulence models namely standard k-ε, RNG k-ε and Low Reynolds number Lam-Bremhorst model. It was found that the standard k-ε model performed better than the other two turbulence models for gross quantities. Robert et al.\textsuperscript{23}\) have also investigated turbulent flow in four different tundish designs and found that there was large difference between measurement and numerical results except for single strand tundish system.

In present study, detailed assessments of five different RANS two equation turbulence models have been done. The turbulence models have been compared for prediction of intermixed amount as well as correct free surface level of steel in the tundish during ladle change-over. A reduced scale boat shape billet caster has been used for water modeling experiments in laboratory for validation studies. Numerical investigation has been carried out on transient, 3-dimensional, multiphase, iso-thermal, turbulent flow. During the teeming operation of steel from ladle to tundish, the phase volume fractions continuously change. Coupled level-set volume of fluid (CLSVOF),\textsuperscript{24}\) a combination of volume of fluid (VOF)\textsuperscript{25}\) method with Level-set (LS) method\textsuperscript{26}\) has been employed to predict the free surface level during the ladle change over process.

2. Physical Description

A 1/4th reduced scale six strand boat shape (rectangular) billet caster tundish has been considered for physical modeling. Figure 1 shows the detailed dimensions of tundish. Geometric similarity has been maintained by keeping the dimensions of the model and prototype tundish in the same ratio and dynamic similarity is achieved by considering the inertial, viscous and gravitational forces.\textsuperscript{27,28}\) The dimensional analysis requires similarity of different dimensionless numbers such as Re, FR, Ri and We. The flow behavior inside tundish is largely affected by these dimensionless numbers. However, for the quasi steady-state, isothermal and single-phase flow of water in a reduced scale tundish model, only the Re and Fr number similarity suffices. It is impossible to respect both Reynolds and Froude similarity simultaneously in a reduced scale modeling studies using fluid of similar kinematic viscosity.\textsuperscript{29}\) In many earlier studies therefore, it has been assumed that flow phenomena in tundish are largely dominated by the inertial and gravitational forces (i.e., Froude number) rather than the viscous forces (Reynolds number).\textsuperscript{30}\) Water was used as fluid medium (at room temperature) as it has equivalent kinematic viscosity to that of molten steel. Concentrated water (homogenous salted water) was used as a tracer and its injections were done as continuous input to the tundish. Thermo Scientific\textsuperscript{TM} were used for monitoring the concentration of tracer due to change in the conductivity of the fluid because of mixing of salted water at outlets of the tundish. The conductivity is reported in the unit of micro-siemens per meter (µS/m). Figure 2(a) shows schematic diagram of experimental setup. Before initialization of experiment, tundish was half filled with pure, salt free water. At the beginning to experiment, inlet was opened and homogenous salted water was supplied into tundish at 1.5 times of steady state flow rate of tundish. The pre-calibrated steady state inflow & outflow rate of the tundish is 12 litres/min. The new incoming salted water was treated as a new grade of steel coming to the tundish. Due to increase in the inflow rate, bath height of tundish raised with respect to time. As the bath height level started to rise, outflows were maintained to nearly constant rate. After attaining the maximum free-surface height, the inflow rate was again adjusted to the steady state flow rate value and inlet & out flow rates were made equal. The temporal value of conductivity were recorded on the computer using data logger and further non-dimensionalized to get concentration value C as:

$$C = \frac{F(t) - F_{\text{old}}}{F_{\text{new}} - F_{\text{old}}} \quad \text{........................ (1)}$$

Where $F(t)$ is the fraction of a given element i.e. conductivity in the fluid, $F_{\text{old}}$ and $F_{\text{new}}$ are the fractions of that element measured in the previous and new grades respectively.\textsuperscript{31}\) Figure 2(b) illustrates the process involved in variation of flow rate throughout the ladle change-over process.

![Fig. 1. Dimensions of full scale tundish.](image)

![Fig. 2. (a) Schematic diagram of experimental setup, (b) Graphical representation of experimental procedure.](image)
3. Mathematical Model

3.1. Governing Equations

The volume-averaged continuity equation and momentum conservation equation describing the fluids flow are respectively:

\[ \nabla \cdot \mathbf{u} = 0 \] \hspace{1cm} (2)

\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla P + \nabla \cdot \mathbf{T} \] \hspace{1cm} (3)

The flow front is advanced by solving the following equation for species concentration given as:

\[ \frac{\partial C}{\partial t} + (\mathbf{u} \cdot \nabla) C = 0 \] \hspace{1cm} (11)

The governing equations of mass and momentum conservation are solved coupled with different turbulence models. The governing equation for the turbulent quantities for different turbulence methods has been presented in the next section. The velocity field obtained after solving the governing equations is used as an input to solve the governing equation for species concentration given as:

\[ \frac{\partial \rho}{\partial t} + \rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + \nabla \cdot \mathbf{T} \] \hspace{1cm} (4)

Equation for rate of dissipation of turbulent kinetic energy

\[ \frac{\partial}{\partial t} (\rho \epsilon) + \nabla \cdot (\rho \epsilon \mathbf{u}) = \frac{-\rho \epsilon}{\epsilon} \left( \nabla \cdot \mathbf{u} \right) + \frac{\rho}{k} \left( \nabla \cdot \mathbf{T} \right) \] \hspace{1cm} (8)

where \( \epsilon \) is the turbulent kinetic energy, \( k \) is the turbulent kinetic energy, and \( \mu_t \) is the turbulent viscosity.

3.2. Turbulence Models

3.2.1. Standard k-\( \epsilon \)

The standard k-\( \epsilon \) is simplest two-equation model of turbulence in which the solution of two separate transport equations allows the turbulent velocity and length scales to be independently determined. Equation for turbulent kinetic energy:

\[ \frac{\partial}{\partial t} \left( \rho k \right) + \nabla \cdot \left( \rho k \mathbf{u} \right) = \frac{\rho}{\epsilon} \left( \nabla \cdot \mathbf{u} \right) + \frac{\rho}{k} \left( \nabla \cdot \mathbf{T} \right) \] \hspace{1cm} (9)

where \( k \) is the turbulent kinetic energy, \( \epsilon \) is the turbulent dissipation rate, and \( \mu_t \) is the turbulent viscosity.

3.2.2. RNG k-\( \epsilon \)

The RNG model was developed using a mathematical technique called Re-Normalization Group (RNG) methods. The equations for k and \( \epsilon \) are given as follows:

\[ \frac{\partial \rho k}{\partial t} + \nabla \cdot \left( \rho k \mathbf{u} \right) = \nabla \cdot \mathbf{T} - \epsilon \] \hspace{1cm} (10)

Where \( \epsilon \) is the turbulent dissipation rate, \( k \) is the turbulent kinetic energy, and \( \mu_t \) is the turbulent viscosity.

3.2.3. Realizable k-\( \epsilon \)

This model satisfies certain mathematical constraints on the Reynolds stresses, consistent with the physics of turbulent flows. The transport equation for k is similar to Eq. (8)
and a transport equation for $\varepsilon$ in the realizable model is written as:

$$\frac{\partial}{\partial t}(\rho \varepsilon) + \text{div} (\rho \varepsilon U) = \text{div} \left[ \frac{\mu}{\sigma_c} \text{grad} \varepsilon \right] + \rho \varepsilon \left( C_{1e} \varepsilon - \rho C_{2e} \frac{\varepsilon^2}{k + \sqrt{\varepsilon \sigma_c}} \right) + C_{3e} \frac{\varepsilon \sigma_\omega}{k} C_{4b} \gamma \varepsilon$$

... (16)

Where,

$$C_1 = \max \left[ 0.43, \frac{\eta}{\eta + 5} \right], \eta = \frac{S_k}{\varepsilon}, S = \sqrt{2S_k S_\gamma},$$

Where values of constants have been taken as: $C_{1e} = 1.44, C_{2e} = 1.9, \sigma_c = 1.0, \sigma_\omega = 1.2.$

3.2.4. Standard k-ω Model

The standard k-ω model is based on the Wilcox k-ω model\(^{34}\), which incorporates modifications of low-Reynolds-number effects, compressibility, and shear flow spreading. The turbulence kinetic energy, $k$, and the specific dissipation rate ($\omega$), are obtained from the following transport equations:

$$\frac{\partial}{\partial t}(\rho K) + \text{div} (\rho KU) = \text{div} \left[ \frac{\mu}{\sigma_\omega} \text{grad} (K) \right] + \rho \omega - \beta' \rho K \omega$$

... (17)

And

$$\frac{\partial}{\partial t}(\rho \omega) + \text{div} (\rho \omega U) = \text{div} \left[ \frac{\mu}{\sigma_\omega} \text{grad} (\omega) \right] + \gamma_1 \left( 2 \rho S_\gamma - \frac{2}{3} \rho \omega \frac{\partial U}{\partial x_j} \delta_{ij} \right) - \beta_2 \rho \omega^2$$

... (18)

Where

$$P = \left( 2 \mu S_\gamma S_\gamma - \frac{2}{3} \mu K \frac{\partial U}{\partial x_j} \delta_{ij} \right)$$

The models constant are as follows:\(^{35}\)

$$\sigma_c = 2.0, \sigma_\omega = 2.0, \gamma_1 = 0.553, \beta_1 = 0.075, \beta^* = 0.09$$

3.2.5. Shear-Stress Transport (SST) k-ω Model

SST k-ω model\(^{36}\) combines the robustness of k-ω turbulence model on the near wall regions with the capabilities of k-ε model on regions away from the walls. The Reynolds stress computation and $k$ equation are the same as in Wilcox’s original k-ω model (Eq. (17)), but the $\varepsilon$-equation is transformed into an $\omega$-equation by substituting $\varepsilon = k \omega$. The $\omega$-equations are provided as:

$$\frac{\partial}{\partial t}(\rho \omega) + \text{div} (\rho \omega U) = \text{div} \left[ \frac{\mu}{\sigma_\omega} \text{grad} (\omega) \right] + \gamma_2 \left( 2 \rho S_\gamma - \frac{2}{3} \rho \omega \frac{\partial U}{\partial x_j} \delta_{ij} \right) - \beta_2 \rho \omega^2 + \frac{2 \rho \omega}{\sigma_\omega \omega} \frac{\partial \omega}{\partial x_k} \frac{\partial \omega}{\partial x_k}$$

... (19)

The models constant are as follows:\(^{35}\)

$$\sigma_c = 1.0, \sigma_\omega = 2.0, \alpha_{1e} = 1.168, \gamma_2 = 0.44, \beta_2 = 0.0828, \beta^* = 0.09$$

3.3. Computation Details

A three dimensional unstructured tetrahedral mesh was used for numerical simulations. Computation was carried out for half of the tundish because of prevalence of symmetry at the centre plane. A control volume based technique has been used to convert the governing equations to algebraic equations. Second-order upwind discretization scheme was used to discretize the transport equations. The SIMPLE algorithm was used for pressure-velocity coupling and body force (due to gravity) has been considered. The species equation was solved in the complete flow domain.

3.4. Boundary Conditions

Fluid was assumed to be incompressible. The symmetry boundary condition has been implied at the symmetry plane. The walls were set to a no slip condition with zero velocity. At the inlet, turbulence intensity value was taken as 2% and at the outlets, atmospheric pressure condition was assumed. The bottom of the tundish was treated like a wall where no slip conditions were used for the velocity. Tundish outlet nozzles were maintained to give constant throughput rate. Solutions through implicit discretization schemes are said to be stable and less time consuming than explicit scheme solutions. So therefore, implicit discretization scheme with modified-HRIC (High Resolution Interface Capturing) interface interpolation scheme has been used. Academic version of CFD software Ansys Fluent 13.0 was used for solving the set of equations.

3.5. Grid Independence Test

A transient numerical analysis for grid independence test has been carried out to control the solution convergence and quantification of discretization errors. This numerical test has been based on the Richardson extrapolation method and numerical procedure illustrated by Celik et al.\(^{36}\) It has been advocated by the said authors that iterative convergence must be achieved with at least three orders of magnitude decrease in the normalized residuals for each equation solved. This method for grid convergence test is widely acceptable and recommended for numerical work. For numerical solution, the cell size of the model $\lambda$ was calculated by Eq. (20).

$$\lambda = \left[ \frac{1}{N} \sum_{i=1}^{N} (\Delta V_i) \right]^{1/3}$$

... (20)

Where, $\Delta V_i$ is volume of ith cell and $N$ is number of cell. Three different set of grids were selected to determine the velocity magnitude ($V_{1s}, V_{2s}, V_{3s}$). For $\lambda_1 < \lambda_2 < \lambda_3$, the grid refinement factors were determined as $r_{21} = \lambda_2 / \lambda_1$, $r_{32} = \lambda_3 / \lambda_2$, and calculated the order of convergence using following expression:

$$P = \frac{1}{\ln(r_{21})} \ln \left| \frac{\varphi_{21}}{\varphi_{21}} + q(P) \right|$$

... (21)

in which $\varphi_{21} = V_2 - V_1$, $\varphi_{32} = V_3 - V_2$. For constant value of grid refinement ($r$), the term of $q(P)$ was selected as zero. The estimated relative errors were calculated as:

$$e_{21} = \left| \frac{V_2 - V_1}{V_1} \right|$$

... (22)
Thus, the grid convergence indexes (GCI) of fine and coarse grid were calculated as:
\[
GCI_{\text{fine}}^{21} = \frac{1.25e_{21}}{r_{e21}^p - 1} \tag{23}
\]
\[
GCI_{\text{coarse}}^{21} = \frac{1.25e_{21}r_{e21}^p}{r_{e21}^p - 1} \tag{24}
\]

Similarly, \( GCI_{\text{fine}}^{32} \) and \( GCI_{\text{coarse}}^{32} \) can be calculated.

Three sets of grid (with 160 185, 294 464 and 462 500 three dimensional elements) were used to calculate the GCI. In the test, the grid refinement factor \( r = 1.2 \) was considered as constant based on the grid sizes defined from Eq. (20). The key variables, \( V_1 \), \( V_2 \) and \( V_3 \) were considered as the velocity magnitudes at \( X/X_0 = 0.13 \) and \( Y/Y_0 = 0.09 \) for different grid sizes. The \( GCI_{\text{fine}} \) was reported as between 0.04% and 3.97% from calculations. The local order of accuracy \( P \) calculated from Eq. (21) ranges from 0.006 to 0.934, with global average, \( P_{\text{avg}} \) of 0.533, which is good indication of the hybrid method applied for this calculation. The maximum discretization error with the averaged apparent order was found as 2.27% for fine grid solutions, which corresponds \( \pm 0.04098 \) m/s. Figure 3(a) presents velocity magnitude profiles for three sets of different grids. In the Fig. 3(b), the numerical uncertainty was indicated by the discretization error bars using GCI. By analyzing the results obtained from different grids, fine grid solution was selected for numerical investigations.

4. Result and Discussion

4.1. Analysis of F-curves

During the process of ladle change over, there is mixing of the previous and the new grade steel in the tundish. The extent of intermixing is represented by a curve, known as F-curve. This curve is nothing but the temporal variation of tracer concentration at the tundish outlet when tracer (representing new grade steel from the new ladle) starts flowing into the tundish. The concentration of the tracer which is initially zero starts increasing with time and a tracer concentration value of 1 is representation of the tundish completely filled with new grade steel i.e. tracer. A value of F in between 0 to 1 represents the mixture of previous and new grade steels. As the value of the concentration is dependent upon the existing flow vector inside the tundish, which is further influenced by the selection of turbulence model, the assessment of different turbulence models namely Standard k-\( \epsilon \), Realizable k-\( \epsilon \), RNG k-\( \epsilon \), Standard k-\( \omega \) and SST k-\( \omega \) have been done in the present study to see its effect on various tundish parameters like F-curve, filling rate, interface prediction etc. by comparing the quantities numerically obtained versus the experimentally measured ones.

The F-curves obtained from experiment & by using five different turbulence models have been shown in the Figs. 4(a)–4(c) at near outlet, middle outlet and far outlet respectively. These characteristic graphs are used to evaluate the intermixed amount formed in a tundish during ladle change over process. The abscissa of the graph is the time during which the steel coming out through the outlet is of mixed

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![Fig. 3](image-url). (a) Velocity magnitude profile for tundish at \( X/X_0 = 0.13 \), \( Y/Y_0 = 0.09 \), (b) Calculated discretization error bars for fine-grid solution.

![Fig. 4](image-url). F-curve (a) near outlet (b) middle outlet and (c) far outlet.
grade quality not acceptable to the steelmaker. This time, multiplied with the volumetric flow rate through a particular outlet will give the volume of such intermixed steel formed. Hence the time is taken as the parameter representing the intermixed amount. The ordinate value gives the concentration of new grade steel coming out of the tundish. Depending upon the specified grade specification limits, the intermixed amount is computed for a particular curve.

In Fig. 4(a), F curves are drawn at the near outlet for experimental as well as different turbulence model cases. It is seen that all the curves show the concentration value of the tracer gradually increasing from 0 at initial time (representing only previous grade steel from tundish outlet) and approaching 1 (representing completely new grade steel) as the time progresses. The curve obtained by the experiment is well matched by most of the turbulence models during the initial time up to 100 seconds except the SST k-ω and Realizable k-ε. After 100 seconds, these two turbulence models predict closer to the experimental curve whereas the prediction by two other turbulence models namely standard k-ω and k-ε start to deviate from the experimentally obtained value. RNG k-ε is seen to match very much closer to the experimental curve.

In Figs. 4(b) and 4(c), F curves are drawn at the middle and far outlet respectively for experimental as well as different turbulence model cases. In these curves, a somewhat more visible difference in the concentration value predicted even during the initial time (of up to 100 sec) can be noticed. RNG model seems to be predicting the concentration values closer to the experimentally observed one in these cases too.

It is difficult to quantify the extent of match between the curves obtained by experimental and the turbulence model predicted ones in these curves. Hence intermixed amount has been calculated for three different grade specification values at these outlets and a comparison has been made among the turbulence models as to which of the turbulence models gives intermixed amount close to the predicted one by experiment. Grade specification is expressed as the percentage difference in composition between the previous and new grades of cast. For example, a 20:80 grade specification means contamination of either grade with only 20% of the other grade is enough to move the steel composition outside of the specification range and cause regarding. Tables 1 to 3 gives the intermixed amount obtained from the three outlets for three grade specification values and a variation of the predicted value for individual turbulence model as compared to the experimental curve is presented in the form of bar graph in Figs. 5(a)–5(c) for near, middle and far outlets respectively. The variation of intermixed amount predicted is both negative and positive. A negative or positive value on the ordinate axis of the bar graph means under-prediction or over-prediction of the intermixed amount as compared to the experimental value. It is seen from the bar graphs that the variations in intermixed amount prediction is largest for SST k-ω and minimum of the variation is for RNG k-ε model for all the outlet curves and for most of the grade specification limits. Standard k-ε model seems to match reasonably well for near and middle outlet cases and the variation is more for outlet cases. The reason for this may be attributed to the presence of more turbulent region in the vicinity of near and middle outlet and the fact that standard k-epsilon model gives more accurate results in the turbulent region. The region far away from the inlet i.e. close to far outlet, a nearly laminar condition prevails and so the prediction by standard k-ε model is not much closer to the experimental curve. It is also a notable point that in all the F curves obtained, the standard k-ε model matches well till the initial 100 sec of experiment and afterwards the divergence is seen. The turbulence is generated and maintained by shear in the mean flow. Turbulence quantities are generally anisotropic in nature & directly affected by shear. The model lacks in prediction of anisotropic diffusion of new grade steel in tundish due to Boussinesq’s analogy of isotropic assumption. It is seen that Realizable k-ε model predicts F-curve fairly, though not much precisely. It has fairly predicted the initial 100 seconds and last 150 seconds of experiments. It may be said that Realizable k-ε model predicts closer to the experimental value as compared to the standard k-ε model. It is further noted that RNG k-ε model has predicted the F-curve at all three outlets fairly accurately as compared to all other turbulence models. This model has advantage of special treatment of statistical mechanics. RNG model contains the analytically derived differential formulas for effective viscosity, which includes the low Reynolds number effect. The fluid flow in tundish has some swirling behavior and RNG model is said to predict the swirling flow more accurately.

A k-ω model turbulence model proposed by Wilcox has also been employed in which length scale is calculated along with another second variable called turbulence frequency $\omega$. This model has similar benefits like k-ε model. It can be noted from Figs. 4(a) & 4(b) that curves obtained by both models are nearly overlapping each other. However, in case of far outlet (Fig. 4(c)) k-ω model did not perform well. In case of SST k-ω model, it is noted from Figs. 4(a)–4(c) that F curves are under predicted. SST k-ω turbulence model

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**Table 1.** Calculated intermixed amount for near outlet.

<table>
<thead>
<tr>
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<th>Near Outlet</th>
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<tbody>
<tr>
<td></td>
<td>20:60</td>
<td>40:80</td>
<td>20:80</td>
<td></td>
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<tr>
<td>RNG k-ε</td>
<td>37.6</td>
<td>80.9</td>
<td>97.5</td>
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<tr>
<td>Realizable k-ε</td>
<td>64.25</td>
<td>114.3</td>
<td>140.05</td>
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<tr>
<td>SST k-ω</td>
<td>64.2</td>
<td>202.6</td>
<td>227.1</td>
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<tr>
<td>Standard k-ε</td>
<td>43.11</td>
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<tr>
<td>Standard k-ω</td>
<td>33.45</td>
<td>59.08</td>
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<tr>
<td>Experiment</td>
<td>37.86</td>
<td>94.3</td>
<td>107.5</td>
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**Table 2.** Calculated intermixed amount for middle outlet.

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<th>Middle Outlet</th>
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<tr>
<td>RNG k-ε</td>
<td>79.3</td>
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<tr>
<td>Realizable k-ε</td>
<td>94.2</td>
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<td>SST k-ω</td>
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<tr>
<td>Standard k-ε</td>
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<tr>
<td>Standard k-ω</td>
<td>54</td>
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<td>107.3</td>
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<tr>
<td>Experiment</td>
<td>59.5</td>
<td>154.3</td>
<td>168.5</td>
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</table>

**Table 3.** Calculated intermixed amount for far outlet.

<table>
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<th>Far Outlet</th>
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<td></td>
<td>10:40</td>
<td>20:40</td>
<td>10:50</td>
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<tr>
<td>RNG k-ε</td>
<td>66.5</td>
<td>43.3</td>
<td>93</td>
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<tr>
<td>Realizable k-ε</td>
<td>98.8</td>
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<tr>
<td>SST k-ω</td>
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<td>115</td>
<td>265</td>
<td></td>
</tr>
<tr>
<td>Standard k-ε</td>
<td>56.5</td>
<td>39.2</td>
<td>70.3</td>
<td></td>
</tr>
<tr>
<td>Standard k-ω</td>
<td>132</td>
<td>92</td>
<td>203</td>
<td></td>
</tr>
<tr>
<td>Experiment</td>
<td>82.7</td>
<td>60.5</td>
<td>119.2</td>
<td></td>
</tr>
</tbody>
</table>
The hybrid model is a combination of the standard $k$-$\varepsilon$ model and standard $k$-$\omega$ model. This combines the Wilcox $k$-$\omega$ model for the near-wall region and the standard $k$-$\varepsilon$ model in the fully turbulent region, away from the wall. However, due to the sensitive nature of this model, mixing curves are slightly offset from experimental curves.

### 4.2. Prediction of Free Surface Height

When the new ladle replaced the previous ladle, the bath height or free surface height of steel in the tundish is at a lower level than the normal height. Hence, the flow rate from the ladle is increased because of which the bath height starts increasing. The rise of tundish bath height with respect to time has been obtained numerically by using various turbulence models and is shown in Figs. 6(a), 6(b) at two time step values. These show the prediction of interface by turbulence models at two different time steps. The values of free surface (water/air) interface are straight and parallel to the Y plane. It can be noted from Fig. 6(a) that the different variants of $k$-$\varepsilon$ models have shown good approximation to the experimental values of free surface height. The $k$-$\omega$ turbulence models have not performed well in predicting the position of free surface and approximation of interface profile. From the initial time steps of calculation, standard $k$-$\omega$ models under predicted the interface profile. Figure 6(b) supports the fact that SST $k$-$\omega$ model over predicted the interface profile for each step of calculation time. The prediction of the interface has been carried out for all the time step values and they are represented in Figs. 7(a), 7(b). The ordinate refers to the free surface height of water in the tundish and abscissa shows the time spent during filling of the tundish. Further, among all the turbulence models used for prediction of intermixed amount and interface profiles, RNG $k$-$\varepsilon$ model is seen to perform most satisfactorily with experimental results.

### 4.3. Turbulent Structure

In two-phase flow, the relation between the phases and their turbulence structure plays an important role in the mass and momentum transfer between the phases. The turbulence characteristics are dependent upon various factors. The turbulent kinetic energy is one of the important parameters used to develop turbulence models. A proper understanding of the turbulence kinetic energy is important for analyzing flow characteristics and mixing of different grades in the tundish. In contrast to single phase modeling, turbulence characteristics are significantly different for multi-phase situations. Figure 8 shows the comparison of normalized values of turbulent kinetic energy employing each of the above mentioned turbulence models. The graph is plotted at three...
different locations along the Y axis. The abscissa represent Y plane of tundish i.e., transverse plane of tundish. The mid plane of tundish is located along X axis and positioned at $Y/Y_0=0.5$. The values of ordinate i.e., turbulent kinetic energy has been normalized with corresponding maximum local value. It is seen that the general characteristics of curves are similar for RNG, Realizable $k$-$\varepsilon$ models and SST $k$-$\omega$ model. For these three turbulence models, the maximum value of turbulence kinetic energy is found near at $Y/Y_0 =0.25$. While curves of standard $k$-$\varepsilon$ model and standard $k$-$\omega$ model have similar profile and maximum turbulence kinetic energy obtained at middle plane of tundish. The turbulent kinetic energy predicted by the RNG model seems to be more realistic as compared to other models because the maximum value of turbulent kinetic energy can’t be expected at the central plane of the tundish. This is so because the fluid plume after striking the tundish bottom moves towards the side walls and further towards the outlets by following the side wall. Hence at any point along the tundish length (in X direction) except the inlet plane, the turbulent kinetic energy is expected to be maximum somewhere in between the central plane ($Y/Y_0 = 0.5$) and the side wall ($Y/Y_0 = 0$) of the tundish. This has been rightly predicted by RNG model. Hence it can be said that RNG model is most accurate model among all the turbulence models used in present study.

Figure 9 shows the contours of turbulent kinetic energy on the inlet symmetry plane represented by different turbulence models. It can be observed that turbulent kinetic energy is maximum at the inlet stream zone, where velocity is high. In case of $k$-$\varepsilon$ models, representation of contours of turbulent kinetic energy is somewhat similar to each other. In contrast to this, standard $k$-$\omega$ model has over predicted, which is indicative of not being a realistic representation. It can be seen here (except standard $k$-$\omega$ model) that the slip velocity between the two-phases results in an additional turbulence energy production near the air-liquid phases interface. This phenomenon is more widely illustrated by standard $k$-$\varepsilon$ model. In addition to this, turbulent kinetic energy plays important role in mixing of fluid. If the turbulent kinetic energy is more, it means better mixing and larger dispersed volume. On the other hand side, if there is less turbulent kinetic energy, so therefore poor mixing and larger plug and dead volumes.

Figure 10 shows comparison of normalized values of eddy viscosity. It is seen that the profiles of curves made by RNG, Realizable $k$-$\varepsilon$ models and SST $k$-$\omega$ model have same characteristics. For these three turbulence models, the maximum value of eddy viscosity is found at $Y/Y_0 =0.25$. Curves of standard $k$-$\varepsilon$ model and standard $k$-$\omega$ model have similar profile and maximum eddy viscosity is found at middle plane of tundish. It can be noted that standard $k$-$\omega$ model estimate a larger eddy viscosity in the middle of tundish. In contrary, SST $k$-$\omega$ model calculate the least value. It is well known that velocity magnitude of fluid is not same everywhere in tundish and have fluid recirculation regions. Eddy viscosity is a function of flow and it is greater for flows with more turbulence. The turbulent transfer of momentum by turbulent eddies give rise to an internal fluid friction and can
be modeled with an eddy viscosity.

**Figure 11** illustrate the contours of turbulent eddy dissipation predicted by three different k-ε models on the inlet symmetry plane. Turbulent fluid flows inside steelmaking tundish contain eddy motions of wide range of time and scales. This phenomenon promotes the mixing of different grades of steels. The degree of dispersion of tracer can be determined by the dissipation rate of turbulent kinetic energy. A visualization of dissipation rate over the flow field is important to understand the mixing behavior. Here it can be seen that turbulence eddy dissipation rate represented by k-ε models have higher values in the direction of inlet stream. Eddy dissipation rate is low in transverse direction because of least flow movement. It is also observed that Realizable k-ε model predicts a lower magnitude of dissipation rate in wide area.

**4.4. Velocity Profile and Streamlines**

**Figure 12** shows normalized mean velocity profiles at three locations of tundish in longitudinal direction. The values of ordinate i.e., velocity has been normalized with respect to corresponding maximum local value. These velocity profiles are obtained through different turbulence models. It is seen here that the Realizable k-ε model predicts the maximum velocity magnitude as compared to others models. Realizable k-ε, RNG k-ε & SST k-ω models show higher velocity magnitude at middle (0.5 Y/Yo) and near region of wall. The fluid flowing at middle plane of tundish has larger velocity due to a larger mass of fluid plume are directed from inlet towards longitudinal direction. Fluid stream coming from inlet are equally directed and those fluid plumes which have been moved to transverse direction has been rebounded further to far zones of tundish. And this phenomenon causes an increment of velocity magnitudes close to side walls of tundish. However, standard k-ε model and standard k-ω models have shown similar velocity profile in the mid-plane of tundish. The objective of the discussion is to give an insight of mean velocity magnitude profile at transverse section (YZ plane) of tundish.

**Figure 13** shows velocity contours with streamlines shown at YZ (near inlet symmetry) plane (time step: 120 second).
fluid available near free surface. However this occurrence is suppressed by Realizable k-ε model and predicted very short. The streamlines represented by standard k-ω model has shown excessive recirculation of fluid at the corner of tundish. This has been over predicted due to flaw of high stream sensitivity of model. SST k-ω model have combined property of original k-ω model and standard k-ε model but illustration depicts poor representation of velocity streamlines. The effect of swirl on turbulence is included in the RNG k-ε model, thus it represents enhanced streamlines as compared to standard k-ε turbulence model. Flow inside tundishes are in transitional to turbulent zone and the suitability for transitional flow makes RNG k-ε model a good choice for tundish metallurgy. It can be said that realizable k-ε and RNG k-ε models have fairly predicted the velocity contours and streamlines.

5. Conclusions

Free surface level of steel in the tundish during ladle change-over plays an important role in ascertaining the intermixed amount formed. As the intermixed amount as well as the level of steel depend upon the flow conditions and flow of steel is normally turbulent in the tundish, an appropriate choice of turbulence model is a key factor in getting meaningful results. In the present work, assessment of RANS equations based k-ε and k-ω turbulence models have been carried out on multiphase flow in steelmaking tundish to predict free surface height variation of steel during ladle change-over process and the consequent intermixed amount formed. Navier stokes equations coupled with various turbulence models andCLSVOF method was used for solving three dimensional, multi-phase numerical models to obtain velocity and concentration field as well as prediction of free surface level of steel. Inflow rate of fluid available near free surface. However this occurrence is suppressed by Realizable k-ε model and predicted very short. The streamlines represented by standard k-ω model has shown excessive recirculation of fluid at the corner of tundish. This has been over predicted due to flaw of high stream sensitivity of model. SST k-ω model have combined property of original k-ω model and standard k-ε model but illustration depicts poor representation of velocity streamlines. The effect of swirl on turbulence is included in the RNG k-ε model, thus it represents enhanced streamlines as compared to standard k-ε turbulence model. Flow inside tundishes are in transitional to turbulent zone and the suitability for transitional flow makes RNG k-ε model a good choice for tundish metallurgy. It can be said that realizable k-ε and RNG k-ε models have fairly predicted the velocity contours and streamlines.

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Nomenclature

\( \rho \): Density of fluid (kg m\(^{-3}\))
\( \rho : \): Pressure (Pa)
\( F \): Body force (N)
\( \mu_{eff} \): Liquid effective viscosity (kg m\(^{-1}\) s\(^{-1}\))
\( \phi \): Level-set function
\( \alpha \): Volume of Fluid function
\( \mu \): Viscosity of fluid (kg m\(^{-1}\) s\(^{-1}\))
\( \mu_t \): Liquid turbulent viscosity (kg m\(^{-1}\) s\(^{-1}\))
\( \nu \): Kinematic viscosity of fluid (m\(^2\)/s)
\( K \): Turbulent kinetic energy (m\(^2\) s\(^{-2}\))

\( c_{in}, c_{fr}, c_{fr}, \gamma, \beta, \beta, \beta' \): Turbulent constant

\( \epsilon \): Turbulence dissipation rate (m\(^2\) s\(^{-3}\))
\( \omega \): Turbulence frequency (s\(^{-1}\))
\( F \): Volume fraction of the fluid
\( \tau_s \): Viscous stress tensor
\( \rho_0, \sigma_t, \sigma_t, \sigma_t, \sigma_t : \) Turbulent constant
\( \delta_s \): Kronecker delta
\( u \): Velocity vector
\( \sigma \): Rate of deformation
\( \eta \): Strain parameter
\( G_{\text{t}} \): Generation of turbulence kinetic energy due to the mean velocity gradients
\( G_{\text{b}} \): Generation of turbulence kinetic energy due to buoyancy,
\( Y_{M} \): Contribution of the fluctuating dilatation in compressible turbulence to the overall dissipation rate

\( C_{1t}, C_{2t}, C_{3t}, \gamma, \beta, \beta, \beta' \): Turbulent constant

REFERENCES