Analysis of Asymmetrical Rolling of Unbonded Clad Sheet by Slab Method Considering Vertical Shear Stress

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A novel analytical approach for asymmetrical rolling of two unbonded clad sheet layers is creatively proposed to calculate the rolling force and torque. In this new approach, the vertical stresses which make calculation results be agree better with results measured in experiments are first applied in clad sheet asymmetrical rolling process. During varying of different roll speeds, roll radii and yield stress ratio of the two materials as well as different friction coefficients, two neutral points and one bonding point are obtained, then the plastic deformation region is divided into four zones. The neutral points, the bonding point, distribution of the rolling stress, the horizontal stress of the whole clad sheet and clad thickness ratio at the exit can be calculated by the present solution easily and rapidly. In order to verify the accuracy of the new model, the results received have been compared with those calculated by other traditional models as well as measured in the experiments conducted by previous scholars where the effects of vertical shear stress were ignored. As it can be seen, the maximum of errors is less than 9.8%.

KEY WORDS: vertical shear stress; asymmetrical rolling; analytical solution; bond clad rolling.

1. Introduction

Nowadays, clad sheet have been applied widely in various industries because of its many advantages such as strong anti-corrosion, anti-friction, and anti-electrical conductivity. Asymmetrical rolling which is a more economical method compared with other types of process used in manufacture of clad sheets and becomes a hot area of researches. The published researches about clad sheets rolling can be divided into two parts, one is most rely on the experiments,1,2) the other one is based on the analyses with upper bound theorem or numerical methods. Under the assumption of constant shear friction, upper bound method was also used by Kiuchi4) to research the process of the clad sheets rolling, which, unfortunately, needed be emphasized that the major disadvantage in using this method was that the rolling pressure distribution still could not be known. Suzuki used a numerical method to analyse the stress distribution in the roll gap, however, a large computer calculation time was required and only the same speeds or radii of the upper and lower rolls could be used in the numerical method as the application condition. Hamazu5) proposed a model based on finite element method to research the effects of the thickness ratio and shear yield stress ratio on the effective stress and strain, rolling force. But it should be pointed out that the shear stress on vertical sides of each slab is not considered by most of the above mentioned researchers that may make errors larger than acceptable limits for rolling pressure.6) As it is recorded in M. Salimi’s paper7) and S. H. Zhang’s,8) the vertical shear stress plays a vital part in asymmetrical rolling, thus obtaining an analytical solution which accounts for the shear stress on vertical sides of each slab and can accurately calculate all the parameters like rolling force, torque, neutral points and bonding point as well as the clad thickness ratio during the rolling process is necessary.

In the present work, a new stress field with vertical shear stress applied is first proposed, and an analytical solution based on slab method for asymmetrical bond rolling is obtained. By the solution rolling pressure distributions, rolling forces, torque, two neutral points, bonding points and clad thickness ratio are all obtained rapidly and easily. The results reveal good agreement with those received in experiments by other researchers and reported in the papers mentioned above.

2. Mathematical Model

In order to simplify the formulation involved in developing the analysis for asymmetrical bond rolling of clad sheet, the following assumptions are listed.

1. Work rolls are assumed to be rigid and the sheets are regarded as rigid-plastic materials with no strain-hardening effects.
2. The plastic deformation is seen as a plane strain
deformation and no spread in transverse direction during the rolling process.\(^9\)

3 The normal stresses on vertical sides of each slab are uniform distribution along the sides,\(^10\) while the friction between the rolls and sheets are constant along the contact arc. But the friction coefficient may be different because of the different materials and the shear frictions are assumed as \(\tau_1 = m_1k_1, \tau_2 = m_2k_2\). Where \(m_1\) is the friction coefficient for upper surface and \(m_2\) is for the lower one.

4 No curvature is considered\(^{11}\) that means the flow direction of the sheet is always horizontal both at the entrance and exit of the roll gap.

5 The total roll contact is much smaller than the circumference of the roll.

3. Modeling

The schematic diagram of asymmetric bond rolling of clad sheet is shown in Fig. 1. In order to research a more general situation, roll speeds, diameters and friction coefficients at upper and lower may be different.\(^{12}\) The plastic deformation region in the roll gap is divided into four zones which are different from each other. The origin of coordinates is set in the middle of the exit cross section. Since the clad sheet is unbonded before rolling, in zone one, the soft layer (layer 1) is the first one to be yielded, but the hard sheet (layer 2) is not yielded at this time. When the clad sheets are in the \(x = x_b\) (bonding point) the hard sheet is finally yielded and the clad sheets are bonded as a whole. That is to say when \(x_b \leq x \leq l\) there is zone I and the clad sheets are unbonded in this zone, but when \(x_{s1} \leq x \leq x_{s2}\) zone II, \(x_{s2} \leq x \leq x_{s3}\) zone III and \(0 \leq x \leq x_{s2}\) zone IV the clad sheets are bonded. Where \(x_{s1}\) and \(x_{s2}\) are the positions of the upper and lower neutral points respectively, \(l\) is projected length of roll/slab contact arc, \(v_1\) and \(v_2\) are speed of the upper and lower rolls, \(v_A\) represents the ratio of the two speeds \(v_A = v_2/v_1\). Also \(R_1\) and \(R_2\) are upper and lower work radii respectively, \(R_{eq}\) means the equivalent work roll radius. For simplicity we assume that \(R_1 < R_2\) and \(v_1 \leq 1\).\(^{13}\) Sheet thickness at inlet and outlet are \(h_1\) and \(h_{o2}\), as it is also shown in Fig. 1 \(h_{o1}\) and \(h_{o2}\) are the thickness of layer 1 and 2 at the inlet cross section, \(h_{o1}\) and \(h_{o2}\) are those at the outlet.

In Fig. 2(a) the slab stress state in zone I can be received, where the shear stress at the interface between layer 1 and 2 is \(\tau = m_{ik}k_{i2} = m_{i1}k_{12}\). The slab state in zone II is shown as Figs. 2(b) and 2(c) shows that for zone III as well as Fig. 2(d) for Zone IV. When it is in zones I and II, the speed of slab is less than both two rolls’ speed, but when it is in zone III the speed of slab is still less than that of lower roll but higher than the speed of upper roll. Zone III is a cross shear region where the frictional shear stresses are opposite, and the frictional shear stresses in zone IV is just contrary to those in zone II because in zone IV the speed of slab is higher than both of the rolls’ speeds.

3.1. Zone I (\(x_b \leq x \leq l\))

Since the rolling force is not reach the value that the hard layer can be yielded in zone I, the contact angle \(\theta_1\) between the two layers is equal to lower roll contact angle \(\theta_2\). The thickness of the zone I is \(h = h_{l1} + h_{l2}\) \((h_2 = h_{o2})\). From Fig.
2(a),
\[ \frac{d(\sigma_i h_i)}{dx} + p \tan \theta_i + p \tan \theta_i - \tau_i - \tau_m = 0 \] .......(1)

where \( p = p_1 + \tau_1 \tan \theta_1 + p_1 + \tau_m \tan \theta_1 + \tan d h_i \).

So Eq. (1) can be changed as follow:
\[ \frac{d(\sigma_i h_i)}{dx} + p(\tan \theta_i + \tan \theta_i) - \tau_i (\tan \theta_i + 1) \]
\[ - \tau_m (\tan \theta_i + 1) - \frac{d h_i}{dx} \tan \theta_i = 0 \]

\[ h \frac{d p}{dx} = (\sigma_i + p) \frac{d h_i}{dx} - \tau_i \left( \frac{x^2}{R_i^2} + 1 \right) \]
\[ - \tau_m \left( \frac{x^2}{R_i^2} + 1 \right) - \frac{d h_i}{dx} \frac{x}{R_i} = 0 \] .......(2)

where \( \tan \theta_i = \frac{x}{R_i}, \tan \theta_i = \frac{x}{R_i}, R_{eq} = \frac{2R_i R_j}{R_i + R_j}, h = h_1 + h_2 = h_o + \frac{x^2}{R_{eq}^2}, \frac{d h_o}{dx} = \frac{2x}{R_{eq}}, dp = -d\sigma_i \).

Also in zone I, for the soft layer (layer 1), \( p + \sigma_1 = 2k_i \).

Finally Eq. (1) becomes as
\[ R_{eq} \left( h_o - h_{eq} \right) + x^2 = 0 \]

After the calculation of the above \( p_1 \) is obtained as follows.
\[ p_1 = A_i x + 2k_i \ln \left( B_i + x^2 \right) \]
\[ + \left[ -A_i \sqrt{B_i + R_{eq} \left( -\tau_i - \tau_m \right)} \right] \arctan \left( \frac{B_i}{h_o} \right) + A_{le} \] .......(4)

where \( A_i = R_{eq} \left( -\tau_i - \tau_m - \frac{2\tau_i}{R_i R_j} \right); B_i = R_{eq}(h_o - h_2). \)

When we take hard layer as the analysis object, the followings are shown as:

\[ d(\sigma_i h_i) + p_1 \tan \theta_1 - p_1 \tan \theta_1 - \tau_1 + \tau_m = 0 \] .......(5)

where \( p = p_2 + \tau_2 \tan \theta_2 = p_3 + \tau_m \tan \theta_2 \).

Equation (5) can be expressed as follow:
\[ h \frac{d \sigma_2}{dx} = \left( \tau_2 - \tau_m \right) \left( \frac{x^2}{R_j^2} + 1 \right) \] .......(6)

After integrals Eq. (7) can be received:
\[ \sigma_2 = \left( \frac{x^2 - \tau_m}{h_2} \right) \left( \frac{x^3}{3R_j^2} + x \right) + C_{le} \] .......(7)

The constants \( C_{le} \) and \( C_{le} \) are obtained by following conditions:

At \( x = l \), \( \sigma_1 = \sigma_1 \), and \( \sigma_2 = \sigma_2 \), that means when it is in entrance of bite zone, the tensions of the both two layers are equal to their initial tensions. Also with Mises yield criterion \( p = 2k_i - \sigma_1, C_{le} \) is expressed:

\[ C_{le} = 2k_i - \sigma_1 - \frac{A_i l}{2} - 2k_i \ln \left( B_i + l^2 \right) \]
\[ - \left[ A_i \sqrt{B_i + R_{eq}} \left( \tau_i - \tau_m \right) \right] \arctan \left( \frac{l}{\sqrt{B_i}} \right) \] .......(9)

With the \( C_{le} \) and \( C_{le} \), of course the bonding point can be obtained.

At \( x = x_0 \) the hard layer metal is yielded, so \( p + \sigma_2 = 2k_i \).

\[ 2k_i = A_i x_0 + 2k_i \ln \left( B_i + x_0^2 \right) \]
\[ + \left[ -A_i \sqrt{B_i + R_{eq} \left( -\tau_i - \tau_m \right)} \right] \arctan \left( \frac{x_0}{\sqrt{B_i}} \right) \]
\[ + C_{le} + \left( \frac{\tau_2 - \tau_m}{h_2} \left( \frac{x_0^3}{3R_j^2} + x_0 \right) + C_{le} \right) \] .......(10)

\( x_0 \) is solved through Eq. (10).

3.2. Zone II (\( x_{o1} \leq x \leq x_o \))

In Zone II, two kinds of metal have been bonded completely, however the ratio of the clad sheet is not totally the same at the entrance and exit zone, that means when \( 0 \leq x \leq x_o \), \( \beta_1 \) is set as a constant value but not equal to that in entrance \( \beta_1 \) can be shown as \( \beta_1 = h_i/h_1, \beta_2 = h_2/h_0 \).

From the Fig. 2(b)
\[ \frac{d(\sigma_i h_i)}{dx} + p_1 \tan \theta_1 - p_1 \tan \theta_1 - \tau_1 + \tau_m = 0 \] .......(11)

where \( p = p_1 + \tau_1 \tan \theta_1 = p_3 + \tau_m \tan \theta_1 + \frac{d h_b}{dx} \)
\[ \frac{d(\sigma_i h_i)}{dx} + p_2 \tan \theta_2 + p_2 \tan \theta_2 - \tau_2 - \tau_m = 0 \] .......(12)

where \( p = p_3 + \tau_m \tan \theta_2 = p_2 + \tau_2 \tan \theta_2 + \frac{d h_b}{dx} \).

From Eqs. (11) and (12) the follow equation is received:

\[ h \frac{d a}{dx} + (\sigma_i + p) \frac{d h_i}{dx} - \tau_1 \left( \frac{x^2}{R_i^2} + 1 \right) - \tau_2 \left( \frac{x^2}{R_j^2} + 1 \right) \]
\[ - \frac{d h_b}{dx} \tan \theta_2 = 0 \] .......(13)

Then Eq. (13) is changed as
\[ h \frac{d p}{dx} = 2k_{oe} \frac{2x}{R_{eq}} \tau_1 \left( \frac{x^2}{R_i^2} + 1 \right) - \tau_2 \left( \frac{x^2}{R_j^2} + 1 \right) \]
\[ - \frac{d h_b}{dx} \tan \theta_2 = 0 \] .......(14)

where \( \frac{d h_i}{dx} = \tan \theta_i + \tan \theta_2 = \frac{2x}{R_{eq}}, \frac{d h_b}{dx} = (1 - \beta_2) \frac{d h_i}{dx} = (1 - \beta_2) \frac{2x}{R_{eq}} \)
\[ \frac{2x}{R_{eq}}, h = h_o + \frac{x^2}{R_{eq}}, k_o = k_i \beta_2 + k_2 = 1 - \beta_2. \]

So \( p_i \) can be obtained as
\[ p_i = A_i x + 2k_{oe} \ln \left( B_i + x_0^2 \right) \]
\[ - \left[ A_i \sqrt{B_i + R_{eq}} \left( \tau_1 + \tau_2 + \tau_m \right) \right] \arctan \left( \frac{x}{\sqrt{B_i}} \right) + C_{le} \] .......(15)
where, \( A_\Pi = -R_{eq} \left[ \frac{\tau_1}{R_1^2} + \frac{\tau_2}{R_2^2} + \frac{2\tau_{eq}}{R_{eq}R_1R_2} \right] \), \( B_\Pi = h_s R_{eq} \).

Also, at \( x = z_b \) as it is known to us, the value of \( p_i \) is equal to that of \( p_{II} \), then \( C_{II} \) is given,

\[
A_{II} z_b + 2k_{so} \ln \left( B_I + x_b^2 \right) + \left[ -A_1 \sqrt{B_I} + \frac{R_{so} (-\tau_1 - \tau_m)}{\sqrt{B_I}} \right] \arctan \frac{x_b}{\sqrt{B_I}} + C_{II}^* \]

\[
= A_{II} z_b + 2k_{so} \ln \left( B_I + x_b^2 \right)
- \left[ A_II \sqrt{B_{II}} + \frac{R_{so} (\tau_1 + \tau_m)}{\sqrt{B_{II}}} \right] \arctan \frac{x_b}{\sqrt{B_{II}}} + C_{II}^* \quad \ldots (16)
\]

It derived as

\[
C_{II}^* = (A_{II} - A_1) z_b + 2k_{so} \ln \left( B_I + x_b^2 \right) - k_{so} \ln \left( B_I + x_b^2 \right)
+ \left[ -A_1 \sqrt{B_I} + \frac{R_{so} (-\tau_1 + \tau_m)}{\sqrt{B_I}} \right] \arctan \frac{x_b}{\sqrt{B_I}}
+ \left[ A_II \sqrt{B_{II}} + \frac{R_{so} (\tau_1 + \tau_m)}{\sqrt{B_{II}}} \right] \arctan \frac{x_b}{\sqrt{B_{II}}} + C_{II}^* \quad \ldots (17)
\]

### 3.3. Zone III (\( x_{a2} \leq x \leq x_{a1} \))

Between zone III and other zones, there is a difference which is the friction between two rolls and clad sheet are totally opposite that means it is under a cross shear condition.

\[
\frac{d(\sigma_I h_I)}{dx} = p_1 \tan \theta_1 - p_1 \tan \theta_1 + \tau_1 - \tau_m = 0 \quad \ldots (18)
\]

where \( p = p_1 - \tau_1 \tan \theta_1 + \tau_1 h_{so} \frac{dh_{so}}{dx} = p_1 - \tau_m \tan \theta_1 \).

\[
\frac{d(\sigma_{II h}_I)}{dx} = p_2 \tan \theta_2 + p_1 \tan \theta_1 - \tau_2 + \tau_m = 0 \quad \ldots (19)
\]

where \( p = p_2 + \tau_2 \tan \theta_2 + p_1 \tan \theta_1 + \tau_{II} \frac{dh_{II}}{dx} = p_2 + \tau_2 \tan \theta_2 + p_1 \tan \theta_1 + \tau_m \tan \theta_2 \).

From Eqs. (18) and (19), the follows are received.

\[
\frac{h}{dx} \left[ \frac{\sigma + p}{\sqrt{R_{eq}}} + \tau_1 \frac{dh_{so}}{dx} - \tau_2 \left( \tan^2 \theta_2 + \tau_m \right) + \tau_{II} \frac{dh_{II}}{dx} - \tau_m \tan \theta_1 = 0 \right] \quad \ldots (20)
\]

\[
\frac{h}{dx} = 2k_{so} R_{eq} \frac{2x}{x_{a1}^2 + x_{a2}^2 + x^2}
+ \tau_1 \left( \frac{x}{R_1^2 + 1} - \tau_2 \left( \frac{x}{R_2^2 + 1} \right) - \tau_{II} \frac{2\beta_m R_{eq} x}{R_{eq} R_1} \right) \quad \ldots (21)
\]

Then \( p_{II} \) is obtained as:

\[
p_{II} = A_{II} z_{a1} + 2k_{so} \ln \left( B_{II} + x_{a1}^2 \right)
+ \left[ -A_II \sqrt{B_{II}} + \frac{R_{so} (\tau_1 - \tau_2)}{\sqrt{B_{II}}} \right] \arctan \frac{x_{a1}}{\sqrt{B_{II}}} + C_{II}^* \quad \ldots (22)
\]

where \( A_{II} = R_{eq} \left( \frac{\tau_1}{R_1^2} - \frac{\tau_2}{R_2^2} - \frac{2\tau_{eq} \beta_m}{R_{so} R_1} \right) B_{II} = h_s R_{eq} \).

In the same way, at \( x = x_{a1} \) with the value \( p_{II} \) is equal to that of \( p_{III} \) in neutral point on upper surface of the clad sheet. It can be expressed as:

\[
C_{III}^* = A_{III} x_{a1} + 2k_{so} \ln \left( B_{III} + x_{a1}^2 \right)
- \left[ -A_III \sqrt{B_{III}} + \frac{R_{so} (\tau_1 + \tau_2)}{\sqrt{B_{III}}} \right] \arctan \frac{x_{a1}}{\sqrt{B_{III}}} + C_{III}^* \quad \ldots (23)
\]

### 3.4. Zone IV (\( 0 \leq x \leq x_{a2} \))

From the Fig. 2(d) the follow equations are derived.

\[
\frac{d(\sigma_{IV h})}{dx} = \frac{p_1}{p_1} \tan \theta_1 - p_1 \tan \theta_1 + \tau_1 - \tau_m = 0 \quad \ldots (24)
\]

where \( p = p_1 - \tau_1 \tan \theta_1 + \tau_1 \frac{dh_{II}}{dx} = p_1 - \tau_m \tan \theta_1 \).

\[
\frac{d(\sigma_{IV h})}{dx} = p_2 \tan \theta_2 + p_1 \tan \theta_1 + \tau_2 + \tau_m = 0 \quad \ldots (25)
\]

From Eqs. (24) and (25), then the follows are obtained:

\[
\frac{hR_{eq} + (\tau_1 + \tau_2) R_{eq} + \frac{2x}{R_{eq}}}{\sqrt{R_{eq}}} + \frac{2\beta_m x^2}{R_{eq} R_1} R_{eq} x_{a1}^2 + x_{a2}^2 + x^2
\]

So, the rolling pressure distribution in zone IV is obtained as:

\[
p_{IV} = A_{IV} x + 2k_{so} \ln \left( h_{IV} R_{eq} + x^2 \right)
+ \left[ -A_{IV} \sqrt{B_{IV}} + \frac{(\tau_1 + \tau_2) R_{eq} + \frac{2x}{R_{eq}}}{\sqrt{B_{IV}}} \right] \arctan \frac{x_{a1}}{\sqrt{B_{IV}}} + C_{IV} \quad \ldots (27)
\]

where \( A_{IV} = R_{eq} \left( \frac{\tau_1 + \tau_2}{R_1^2} - \frac{2\tau_{eq} \beta_m}{R_{eq} R_1} \right), B_{IV} = R_{eq} h_{so} \).

When \( x = x_{a2} \) since \( p_{IV} \) is equal to \( p_{II} \) at this point. Then it can be expressed as

\[
C_{IV} = A_{IV} x_{a2} + 2k_{so} \ln \left( B_{IV} + x_{a2}^2 \right)
+ \left[ -A_{IV} \sqrt{B_{IV}} + \frac{R_{so} (\tau_1 - \tau_2)}{\sqrt{B_{IV}}} \right] \arctan \frac{x_{a2}}{\sqrt{B_{IV}}} + C_{IV} \quad \ldots (28)
\]

At exit of the bite zone \( x = 0 \), and \( \sigma_{IV} = \sigma_0 \) it can be
expressed as follow.

\[ C_{IV} = 2k_e \left[ 1 - \ln \left( \frac{h_R}{h} \right) \right] - \sigma_a \]  \hspace{1cm} (29)

With the principle of volume constancy the relationship between \( x_{n1} \) and \( x_{n2} \) can be obtained as follow:

\[ x_{n1} = \sqrt{\frac{v_{A} x_{n2}^2 + \left( v_A - 1 \right) h_o}{R_x}} \]  \hspace{1cm} (30)

where \( v_A = \frac{v_2}{v_1} \), \( R_x = \frac{1}{R_o} - \frac{h_o}{2R_o^2} \).

Then from the Eqs. (23), (28) and (29) as well as (30), the parameters \( x_{n1} \), \( x_{n2} \), \( C_{m1} \) and \( C_{m2} \) are received.

### 3.5 Vertical Shear Stress

The distributions of the shear stress from Prantl’s solution\(^{[6,17]} \) in any vertical plane is given by

\[ \tau_y = \frac{mk}{h} v \]  \hspace{1cm} (31)

\( V \) means the resultant shear force in horizontal direction at any cross section in the bite zone.

\[ \int_{h/2}^{h} \tau_y \, dy = V \]  \hspace{1cm} (32)

where friction shear stress is shown as \( \tau_y = mk \) on \( y = h \) is considered.

The shear stress in vertical plane can be assumed as linearly distributed and on the surfaces the maximum values \( m k_1 \) and \( m k_2 \) are reached, so Fig. 3(a) can yield the following equation.\(^{[18]} \)

\[ m_k d_1 - m_k d_2 = V \]  \hspace{1cm} (33)

\( d_1 \) and \( d_2 \) are the thicknesses affected by the corresponding friction stress on the surface and also can be seen from the figure \( d_1 + d_2 = h \).

Also \( V \) is expressed as

\[ V = a k_e h. \]  \hspace{1cm} (34)

Where \( k_1 = k_\beta, k_2 = k_\alpha(1 - \beta) \). For convenience, here \( k_\alpha \) represents \( k_\alpha \) in zone I and \( k_\alpha \) in zone II III and IV, and in the same way \( \beta \) is used to stand for \( \beta_1 \) and \( \beta_2 \) in different zones respectively.

So it can be expressed as:

\[ d_1 = \frac{2a + m_1 \left( 1 - \beta \right)}{m_1 \beta + m_2 \left( 1 - \beta \right)} h, \quad d_2 = \frac{m_1 \beta - 2a}{m_1 \beta + m_2 \left( 1 - \beta \right)} h \]  \hspace{1cm} (35)

Also from Fig. 3, in zones I II and IV, the direction of friction stress from upper is opposite to that from the lower roll, so the effects of the friction stresses in these zones make the mean vertical shear stresses \(( \tau_{upper}, \tau_{lower} \) \) have a different sign. However, when it comes to the zone III, the effects of the friction stresses from both upper and lower have the same direction with each other. So the mean vertical shear stresses \(( \tau_{upper}, \tau_{lower} \) \) have a same sign.\(^{[19]} \)

Defined that \( b = m_1/m_2 \), and \( c_l \) is coefficient for the mean vertical shear stress.

So \( \tau_{upper} = c_l \tau_1 = c_l m_k k_1 \) can be obtained.

\[ c_l = \frac{\tau_{upper}}{\tau_1} = \frac{m_k k_1}{m_k k_1} \]

\( (2/h) \int_{h/2}^{h} \tau_y \, dy = V \)  \hspace{1cm} (56)

Noticing that \( d_2 = \frac{h}{2} - d_1 \).

So it yields that

\[ c_l = \left( \frac{2a + m_1 \left( 1 - \beta \right)}{m_1 \beta + m_2 \left( 1 - \beta \right)} \right) \beta - \frac{2a}{m_2} \]

\[ = \frac{4b}{m_2} \left( \frac{a}{m_2} + \left( 1 - \beta \right) \right) - \frac{1 - \beta}{\beta^2} \left( \frac{b \beta - 2a}{m_2} \right) \]

\[ = \frac{4b}{m_2} \left( \frac{a}{m_2} + \left( 1 - \beta \right) \right) - \frac{1 - \beta}{\beta^2} \left( \frac{b \beta - 2a}{m_2} \right) \]

\[ = \frac{4b}{m_2} \left( \frac{a}{m_2} + \left( 1 - \beta \right) \right) - \frac{1 - \beta}{\beta^2} \left( \frac{b \beta - 2a}{m_2} \right) \]

\[ = \frac{4b}{m_2} \left( \frac{a}{m_2} + \left( 1 - \beta \right) \right) - \frac{1 - \beta}{\beta^2} \left( \frac{b \beta - 2a}{m_2} \right) \]

The similar method is used to receive \( c_2 \).

\[ c_2 = \frac{\tau_{lower}}{\tau_2} = \frac{2a + m_2 \left( 1 - \beta \right)}{m_1 \beta + m_2 \left( 1 - \beta \right)} \]

\[ c_2 = \frac{\tau_{lower}}{\tau_2} = \frac{2a + m_2 \left( 1 - \beta \right)}{m_1 \beta + m_2 \left( 1 - \beta \right)} \]

So \( c_2 \) is got as

\[ c_2 = 1 - \frac{b \beta + \left( 1 - \beta \right)}{4 \beta^2 - \frac{2a}{m_2}} \]  \hspace{1cm} (36)

With the condition of zero resultant force \( V = 0 \), so \( a = 0 \) and these are simplified as:

\[ c_1 = \left( \frac{b \beta - 1}{4} \right) (3b \beta + \beta - 1) \left( 1 - \beta \right), \quad c_2 \approx \frac{3}{4} \frac{1 - \beta}{4b \beta} \]  \hspace{1cm} (40)

If \( b \) is regarded as 1 (\( b = 1 \)),

\[ c_1 = \frac{1 - \beta}{4 \beta^2}, \quad c_2 = \frac{3}{4} \frac{1 - \beta}{4 \beta^2} \]  \hspace{1cm} (41)

Specially, when \( \beta = 0.5 \), \( c_1 = 0.5, c_2 = 0.5 \).

In zone III the shear stresses in vertical elements are in the same direction as shown in Fig. 3(b) that makes Eq. (33) change into

\[ m_k d_1 + m_k d_2 = V \]  \hspace{1cm} (42)
And also Eqs. (36) and (38) can be changed as
\[
\sigma_{k1} = \frac{\int_0^{h/2} \tau_{xy} \, dy}{m_{k1}} + \frac{1}{2} \left( \frac{m_{k1} - m_{k2}}{h} y + \frac{m_{k1} + m_{k2}}{2} \right) \int_0^{h/2} \tau_{xy} \, dy
\]

Thus, the rolling force can be found by integrating the normal rolling pressure over the arc length of the contact.21) Thus, the rolling force can be obtained from Eqs. (27) and (28) as
\[
P = \int_0^{r_2} p_v \, dx + \int_0^{r_1} p_m \, dx + \int_{r_1}^{r_2} p_d \, dx
\]

Rolling torques, \(T_1\) and \(T_2\) which effect respectively on the upper and lower rolls can be also calculated by the following equations
\[
T_1 = R_1 \left( \int_0^{r_1} m_{k1} \, dx - \int_0^{r_1} m_{k2} \, dx + \int_0^{r_1} m_{k1} \, dx + \int_0^{r_1} m_{k2} \, dx \right) = R_m m_{k1} (l - 2 x_{c1})
\]
\[
T_2 = R_2 \left( \int_0^{r_1} m_{k2} \, dx - \int_0^{r_1} m_{k1} \, dx + \int_0^{r_1} m_{k2} \, dx + \int_0^{r_1} m_{k1} \, dx \right) = R_m m_{k2} (l - 2 x_{c2})
\]

And the total rolling torque becomes
\[
T = T_1 + T_2
\]

So the results are calculated by the approach proposed in present paper. In order to verify the new approach, the followings are shown and discussed.

4. Results and Discussions

From the Fig. 4(a) the distribution of \(p/2k_1\) and \(\sigma/2k_1\) along \(l\) can be seen, and (b) is shown that of \(\sigma_j/2k_1\) and \(\sigma_2/2k_1\) along the projected contact arc. All the stresses increase with increasing \(k_j/k_1\), on both of Figs. 4(a) and 4(b). Comparing the two models shown on the figure, \(p\) is a little higher calculated by Pan model than that in present model, for example, in zone III with \(k_j/k_1 = 2\) and zone I with \(k_j/k_1 = 1.2\). That is because the vertical shear stress can make the deformation easier to appear, and less stress is needed when vertical shear stress is considered. Here it needs be emphasized that lower rolling force will agree more with the value measured in experiment, which is shown in the last in more detail.

From the Figs. 5(a) and 5(b) different \(v_p/v_1\) are applied in the calculation which yields different curves above. Also the
distribution of stresses can be seen from both two figures. The stresses decrease with increasing $v_2/v_1$, but we also can find that the main reason for this situation is the velocity differences in zone III. In zones I, II, and IV, the velocity differences are in the same direction from both upper and lower, so the effect of $v_2/v_1$ is not obvious. But when it is in zone III which is a cross friction shear zone, $v_2/v_1$ profoundly affects the stresses here. From Fig. 5(b) as it is shown, more tension will be yielded when velocity difference is higher which may give a new control method in clad sheet rolling. When more tension is needed, $v_2/v_1$ needs be increased.

From Fig. 6, effects of reduction on clad thickness ratio, bonding point and two neutral points respectively on upper and lower surfaces with different $k_2/k_1$ are shown. From the Fig. 6(a), $\beta_o$ increases with increasing reduction but decreas-

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**Fig. 5.** Distribution of $\rho/2k_1$, $\sigma^2/2k_1$, $\sigma^1/2k_1$ and $\sigma^2/2k_1$ along the projected length of roll/slab contact arc with different $v_2/v_1$. ($k_2/k_1 = 1.5$, $R_1 = 100$, $R_2 = 110$, $h = 2$ (mm); $\beta_i = 0.6$, $r = 30\%$, $m_1 = 0.5$, $m_2/m_1 = 1$, $m_3 = 0.9$, $k_1 = 98.1$ MPa, $\sigma_o = \sigma_i = \sigma_2 = 0$).

**Fig. 6.** Effects of reduction on $\beta_o$, $x_{b/l}$, $x_{n2/l}$, and $x_{n1/l}$ with different $k_2/k_1$. ($v_2/v_1 = 1.1$, $R_1 = 100$, $R_2 = 110$, $h = 2$ (mm); $\beta_i = 0.6$, $r = 30\%$, $m_1 = 0.5$, $m_2/m_1 = 1$, $m_3 = 0.9$, $k_1 = 98.1$ MPa, $\sigma_o = \sigma_i = \sigma_2 = 0$).
es with increasing $k_2/k_1$. As it is shown in Fig. 6(b), $x_{b/l}$ increases with increasing reduction and decreases with increasing $k_2/k_1$, that means yield stress ratio between the two layers are larger, then it will need more distance to be bonded. Fig. 6(c) shows that $x_{n1/l}$ decreases with increasing reduction and $k_2/k_1$. It is a little complicated in Fig. 6(d), when $k_2/k_1 = 1$, $x_{n2/l}$ decreases with increasing reduction, but when $k_2/k_1 = 1.5$ and 2, $x_{n2/l}$ increases with increasing reduction, then after reduction is higher than 40%, the $x_{n2/l}$ is almost a constant value. Also it can be seen in Fig. 6, the values calculated by present model are almost the same with those calculated by Pan model, the errors between them are less than 8.4%.

From Fig. 7(a), effects of reduction on $\beta_o$, $x_{b/l}$, $x_{n2/l}$, and $x_{n1/l}$ with different $\nu_2/\nu_1$ are shown. As it is seen in Fig. 7(a) the clad thickness ratio decreases with increasing reduction, but the ratio will remain as a fixed value with increasing $\nu_2/\nu_1$. In Fig. 7(b) $x_{b/l}$ increases with increasing reduction, and

![Diagram](image)

**Fig. 7.** Effects of reduction on $\beta_o$, $x_{b/l}$, $x_{n2/l}$, and $x_{n1/l}$ with different $\nu_2/\nu_1$. ($k_2/k_1 = 1.5$; $R_1 = 100$, $R_2 = 110$, $h_i = 2$ (mm); $\beta_i = 0.6$, $r = 30\%$, $m_1 = 0.5$, $m_2/m_1 = 1$, $m_3 = 0.9$, $k_1 = 98.1$ MPa, $\sigma_o = \sigma_i = \sigma_o = 0$).

![Diagram](image)

**Fig. 8.** Comparisons of analysis and experiment for rolling force and clad thickness ratio. $R_1 = 50$, $R_2 = 100$ (mm), $v_1 = v_2 = 94.4$ mm/s, $h_i = 10$ mm, $\beta_i = 0.2$, $m_1 = m_2 = 0.5$, $m_3 = 1.0$, layer 1 is aluminum with $K = 18.5$ kg/mm² and layer 2 is copper with $K = 58.8$ kg/mm².
it is the same with the clad thickness ratio remain as a constant value with different \( v_2/v_1 \). As it is in Fig. 7(c), \( x_0/l \) decreases with increasing reduction and \( v_2/v_1 \). But in Fig. 7(c) \( x_0/l \) increases with increasing reduction and decreases with increasing \( v_2/v_1 \). All the conclusions received above will verify that the results calculated by both Pan model and Present model are the same with each other. And the errors are less than 9.8%.

From Fig. 8, a clad sheet rolling experiment was conducted by Hwang\(^{(16)}\) as recorded in reference with copper and aluminum. Rolling force increases with increasing reduction, however, clad thickness ratio decreases with increasing reduction. All the results calculated by present model, Pan model and experiments verify the conclusions mentioned above. Also it can be seen that after reduction is higher than 10% the curves represent present model and experiment is lower than Pan model in Fig. 8(a), that means in higher reduction rolling force calculated by a model with vertical shear stress will be lower than that calculated by Pan model which is a model that the vertical stress is ignored. But from the Fig. 8(a) value calculated by this present model is closer to that measured in experiment. Also it can be seen that from Fig. 8(b) the \( \beta_0 \), calculated by this new approach is a little higher than that measured by the experiments and Pan model. The reason for that may be the vertical stresses play an important role in bonding process which means harder layer metal may more easier to reduce their thickness than the model ignoring the effects of vertical shear stress. So layer 2 copper can be thinner at the exit of the bite zone, \( \beta_0 \) can be higher.

5. Conclusions

(1) With shear stresses on the vertical sides of each slab taken into account, a new analytical solution of rolling force and torque is firstly proposed for asymmetrical clad bond rolling. The results calculated by the new solution are verified through comparison with those calculated by other traditional models and measured datum in the experiments written in previous reference where the effects of vertical shear stress were ignored.

(2) With vertical shear stresses considered, the rolling forces are far lower than the values calculated by Pan model, and more agree with measured ones. The bonding point \( x_{0u} \), the upper and lower neutral points \( x_{0u}, x_{0d} \) and the clad thickness ratio at exit \( \beta_0 \) are all obtained by the new method. With varying reduction, velocity ratio \( v_2/v_1 \) and yield stress ratio \( k_2/k_1 \), the change rules of all the parameters are obtained.

(3) The present analytical model can be used to calculate the parameters in asymmetrical clad bond rolling easily and quickly, and it is more suitable for online control because of its precision.

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