Influence of Tensile Strain on Young’s Modulus in High-strength Cold-rolled Steel Sheets

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The change of Young’s modulus accompanying tensile deformation was investigated for an anisotropic high-strength cold-rolled steel sheet with a high Young’s modulus value in the transverse direction and an isotropic steel sheet with no prominent development of texture. The characteristic phenomenon was the decrease in Young’s modulus in the transverse direction by tensile deformation in the transverse direction in the anisotropic steel sheet. The drop of Young’s modulus in the transverse direction was attributable to a decrease in the ODF intensity around \{112\}<110>. Young’s modulus in the diagonal direction also decreased somewhat as a result of tensile deformation in the rolling and diagonal directions in the anisotropic steel sheet. This change was caused mainly by an increase in ODF intensity around \{001\}<110>. Young’s modulus in the rolling direction displayed little change as a result of tensile deformation because the texture on the \(\alpha\)-fiber and \(\gamma\)-fiber, which were mainly developed in cold-rolled steel sheets, showed an almost constant Young’s modulus in the rolling direction. In the isotropic steel sheet, Young’s modulus showed little change in any direction as a result of tensile deformation. Young’s modulus calculated by the Voigt model had relatively good correspondence with the measured results. Concerning simulation of crystal rotation, the Taylor model indicated that the crystals around the \(\alpha\)-fiber from \{112\}<110> to \{111\}<110> rotated to the vicinity by tensile deformation in the transverse direction. The remarkable change of Young’s modulus was explained by the crystal rotation simulated by the Taylor model.

KEY WORDS: Young’s modulus; texture; ODF intensity; Voigt model; crystal rotation; Taylor model.

1. Introduction

The elastic moduli of materials are one of dominant factors for stiffness. Therefore, increasing the Young’s modulus of the material accompanying increased strength is an effective way to reduce the thickness of structural parts. The representative methods for increasing Young’s modulus in steel are as follows: (1) Addition of alloy elements like Co, Cr, and Re,5) (2) formation of a complex with high Young’s modulus ceramics like TiB2,2) and (3) texture control such as the Thermo-Mechanical Control Process.3) In particular, texture control is the most rational alternative for improving Young’s modulus, as addition of alloy element or ceramics greatly increases costs.

Because steel sheets are usually press-formed, crystal rotation occurs in the steel. For this reason, there is possibility that the steel sheets in which Young’s modulus was improved by texture control may no longer maintain the original values after press forming. Therefore, clarifying the change of Young’s modulus corresponding to practical deformation mode is important. Plastic deformation of polycrystals is induced by active slip systems, which implies shearing on preferred planes in preferred directions. Classically, Taylor treated the problem of plastic deformation based on the minimum virtual shear.4) Bishop and Hill treated this problem based on the principle of maximum virtual work.5,6) In recent years, new models such as the advanced Lamel model, which considered interactions between neighboring grains, were proposed, and their accuracy for prediction of deformation texture has improved.7) However, because of the complex interaction between polycrystal grains, the hurdles for perfect prediction are still high.

The Young’s modulus of a single crystal is determined by its orientation. In polycrystals, Young’s modulus is commonly evaluated by the following three models: (1) the Voigt model based on the assumption of uniform local strains,8) (2) the Reuss model based on the assumption of uniform local stresses,9) and (3) the Hill model, which is the average of the Voigt and Reuss models.10) Various reports have indicated that the Hill model shows a relatively good approximation11,12) and an appropriate model depends on the texture of the polycrystal.13)

This study investigates the influence of tensile strain, which is one of the basic deformation modes affecting Young’s modulus. Young’s moduli were measured by the resonant method and compared with the results calculated
by the Voigt model. Furthermore, with a view to grasping the crystal rotation, the changes of texture by tensile deformation were simulated by using the Taylor model. The changes of Young’s modulus due to deformation were discussed from the viewpoints of the fluctuation of the integration degree of advantageous and disadvantageous crystal orientations to increase Young’s modulus.

2. Experimental Procedure

The steels used in this investigation were industrially manufactured cold-rolled high-strength steel sheets of 1.2 mm thickness. Their tensile properties are shown in Table 1. A tensile test was carried out in the transverse direction using JIS No. 5 type specimens with a 25 mm gauge width and 50 mm gauge length. The cross head speed in the tensile test was kept at 1.67×10⁻⁴ mm/s. The tensile strength, TS, of steel A is 550 MPa, while that of steel B is 620 MPa. Yield strength, YS, and elongation, EL, of the two steels are substantially the same. The two steels differ mainly in anisotropy. Table 2 shows Young’s modulus, E, and the r-value of the rolling direction, RD, the diagonal direction, DD, and the transverse direction, TD, respectively. The elastic modulus was measured by the resonant method using a non-contact free vibration bending mechanism. The specimens were ground to rectangular pieces of 5×40 mm² to avoid shear strain. Young’s modulus, E (GPa), is given by ASTM C 1259 - 98 as follows:

\[
E = 9.465 \times 10^{10} \left( \frac{f_i^2}{w} \right) \left( \frac{l}{t^3} \right) \quad \text{(1)}
\]

where \(f_i\) (Hz) is the fundamental resonant frequency of the specimen in flexure and \(m\) (g), \(w\) (mm), \(l\) (mm), \(t\) (mm) are the mass, width, length, and thickness of the specimen, respectively. The measurement of the r-value was carried out using JIS No.5 type specimens at pre-strain of 0.1. Steel A has large anisotropy, whereas steel B is almost isotropic. In particular, in steel A, Young’s modulus of the TD is very high, at 1.74. However, the r-value of the DD is also quite high, at 1.74.

The specimens were ground to rectangular sheets with dimensions of 50×300 mm² whose long sides were parallel to the RD, DD, and TD, respectively, to give tensile strains. Pre-strains of 0.02, 0.05 and 0.1 were introduced by a tensile machine. For measurement of Young’s modulus, the samples strained parallel to the RD, DD, and TD were ground to rectangular pieces of 5×40 mm² with the long sides parallel to the RD, DD, and TD, respectively, as shown in Fig. 1. Texture measurements were performed by using samples of 25×25 mm² which had been polished chemically to the quarter-thickness sections by XRD. The target, current, and voltage used were Cu, 60 kV, and 260 mA, respectively, and pole figures corresponding to (110), (200), and (211) were obtained. The orientation distribution function (ODF) was calculated from the pole figure data by using the discrete method and plotted in counter lines in Euler space (Bunge’s notation).

Table 1. Tensile properties of steels used.

<table>
<thead>
<tr>
<th>Steel</th>
<th>t (mm)</th>
<th>YP (MPa)</th>
<th>TS (MPa)</th>
<th>El (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.2</td>
<td>400</td>
<td>550</td>
<td>28</td>
</tr>
<tr>
<td>B</td>
<td>1.2</td>
<td>380</td>
<td>620</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 2. Anisotropy of Young’s modulus, E, and r-value in steels used.

<table>
<thead>
<tr>
<th>Steel</th>
<th>RD</th>
<th>DD</th>
<th>TD</th>
<th>RD</th>
<th>DD</th>
<th>TD</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>216</td>
<td>212</td>
<td>233</td>
<td>0.95</td>
<td>1.74</td>
<td>1.47</td>
</tr>
<tr>
<td>B</td>
<td>204</td>
<td>214</td>
<td>210</td>
<td>0.98</td>
<td>0.94</td>
<td>1.27</td>
</tr>
</tbody>
</table>

3. Calculation of Deformation Texture and Young’s Modulus

3.1. Crystal Rotation by Tensile Strain

The deformed texture was predicted by the Taylor full-constraints (FC) model and relaxed-constraints (RC) model. In bcc crystals, it has been reported that the slip direction is <111> and the slip planes are {110}, {211}, and {321}. However, to avoid the complexities associated with a large number of slip planes, the pencil glide model was used in this calculation. In this model, all planes containing the <111> slip directions are admissible. Dillamore and Kato showed the analytic solutions of this model. In the case of multiple slips, the shear strains are divided into strains on the {110} planes, \(\varepsilon_{11}\), and on the {112} planes, \(\varepsilon_{12}\). Since the count of the equivalent and independent directions of <111> is four, the slip systems and the shear strains on these are as shown in Table 3. The strain tensor of the crystal axis including rotation, \(F_{cr}\), is expressed as follows:

\[
F_{cr} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} - \varepsilon_{13} & \varepsilon_{13} + \varepsilon_{22} \\ \varepsilon_{21} + \varepsilon_{31} & \varepsilon_{22} - \theta_3 & \varepsilon_{23} - \varepsilon_{13} \\ \varepsilon_{31} - \theta_2 & \varepsilon_{32} + \theta_1 & \varepsilon_{33} \end{pmatrix} \quad \text{(2)}
\]

where \(\theta_1\), \(\theta_2\), and \(\theta_3\) are rotation angles. A strain state expressed along one set of orthogonal axes is transformed to another strain state expressed along a different set of orthogonal axes as follows:

\[
\varepsilon_{ij} = \sum l_{mn} l_{pj} \varepsilon_{mn} \quad \text{(3)}
\]

where \(l_{mn}\) and \(l_{pj}\) are the direction cosines between the axes of the two coordinate systems. When the shear strains along the slip coordinate system are transformed to the crystal coordinate system, \(x_1\) and \(y_1\) are given as follows:

\[
x_1 = A \left( -2\varepsilon_{11} - \varepsilon_{12} - 2\varepsilon_{13} + \varepsilon_{21} + 2\varepsilon_{23} - \varepsilon_{31} + \varepsilon_{32} - \varepsilon_{33} - \theta_1 - \theta_2 + 2\theta_3 \right) \quad \text{(4a)}
\]
Table 3. Slip systems and shear strains of steel used.

<table>
<thead>
<tr>
<th>Slip direction</th>
<th>Slip plane</th>
<th>Shear strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>[111]</td>
<td>(T10)</td>
<td>$x_1$</td>
</tr>
<tr>
<td></td>
<td>(TT2)</td>
<td>$y_1$</td>
</tr>
<tr>
<td></td>
<td>(110)</td>
<td>$x_2$</td>
</tr>
<tr>
<td></td>
<td>(T12)</td>
<td>$y_2$</td>
</tr>
<tr>
<td>[TT1]</td>
<td>(T0)</td>
<td>$x_3$</td>
</tr>
<tr>
<td></td>
<td>(112)</td>
<td>$y_3$</td>
</tr>
<tr>
<td></td>
<td>(TT0)</td>
<td>$x_4$</td>
</tr>
<tr>
<td>[TT1]</td>
<td>(T12)</td>
<td>$y_4$</td>
</tr>
</tbody>
</table>

\[ x_2 = A \begin{bmatrix} 2\varepsilon_{11} - \varepsilon_{12} + 2\varepsilon_{21} + 2\varepsilon_{23} \\ + \varepsilon_{31} + \varepsilon_{32} + \varepsilon_{33} - \theta_1 + \theta_2 + 2\theta_3 \end{bmatrix} \] ...

\[ x_3 = A \begin{bmatrix} -2\varepsilon_{11} - \varepsilon_{12} + 2\varepsilon_{21} - 2\varepsilon_{23} \\ + \varepsilon_{31} - \varepsilon_{32} - \varepsilon_{33} + \theta_1 - \theta_2 + 2\theta_3 \end{bmatrix} \] ...

\[ x_4 = A \begin{bmatrix} -2\varepsilon_{11} - \varepsilon_{12} - 2\varepsilon_{21} - 2\varepsilon_{23} \\ - \varepsilon_{31} - \varepsilon_{32} - 3\varepsilon_{33} + \theta_1 - \theta_2 + 2\theta_3 \end{bmatrix} \] ...

\[ y_1 = B(2\varepsilon_{11} - \varepsilon_{12} + \varepsilon_{31} + \varepsilon_{32} + \varepsilon_{33} - \theta_1 - \theta_2) \] ...

\[ y_2 = B(\varepsilon_{11} + \varepsilon_{21} - \varepsilon_{31} - \varepsilon_{32} - \theta_1 - \theta_2) \] ...

\[ y_3 = B(-\varepsilon_{11} - \varepsilon_{12} - \varepsilon_{31} + \varepsilon_{32} + \theta_1 + \theta_2) \] ...

\[ y_4 = B(\varepsilon_{11} + \varepsilon_{21} - \varepsilon_{31} + \varepsilon_{32} + \theta_1 + \theta_2) \] ...

where $A = \frac{\sqrt{6}}{8}$ and $B = \frac{3\sqrt{2}}{8}$.

Assuming that the elongation in the tensile direction is $\varepsilon_t$, the deformation tensor of a sample axis, $F_{ij}$, can be defined as follows: \(^{(21)}\)

\[
F_i = \begin{bmatrix} d\varepsilon & \gamma_{12} & \gamma_{13} \\ -d\varepsilon & \gamma_{23} & \gamma_{24} \\ 0 & 0 & -d\varepsilon \end{bmatrix} \] ...

where $\gamma_{12}$, $\gamma_{13}$, $\gamma_{23}$ are components of shear strain and 1, 2, and 3 are defined as the tensile, width, and thickness directions, respectively. In the FCM model, $\gamma_{12}$, $\gamma_{13}$, and $\gamma_{23}$ are zero. In the RC model, the constraints of the thickness direction were left free for bcc metals in perivous studies, so $\gamma_{23}$ was set to zero. \(^{(21-23)}\) The rotation matrix, $g$, is defined by Euler angles, $\phi_1$, $\Phi$, $\phi_2$ as follows: \(^{(25)}\)

\[
g = \begin{bmatrix} \cos \phi_1 \cos \Phi & \sin \phi_1 \cos \Phi & \sin \Phi \\ -\sin \phi_1 \sin \phi_2 \cos \Phi + \cos \phi_1 \sin \Phi & \cos \phi_1 \sin \phi_2 \cos \Phi & \cos \phi_1 \cos \Phi \\ -\sin \phi_2 \sin \phi_1 \cos \Phi + \cos \phi_2 \sin \Phi & \cos \phi_2 \sin \phi_1 \cos \Phi & \cos \phi_2 \cos \Phi \end{bmatrix} \] ...

The shear strains along the sample coordinate system are transformed to the crystal coordinate system by using the variety of Eq. (3) as follows:

\[
F_i = g \cdot F_i \cdot R^{-1} \] ...

where $F_i$ is the strain tensor of the crystal axis excluding rotation.

The summation of the shear strains along the slip coordinate system, $\Gamma$, is calculated as follows:\(^{(8)}\)

\[ \Gamma = \sum_{i=1}^{6} (\varepsilon_i^2 + \gamma_i^2)^{1/2} \]

From Eqs. (4) and (7), $x_i$ and $y_i$ are expressed as the function of $\theta_1$, $\theta_2$, $\theta_3$. The combination of $\theta_1$, $\theta_2$, $\theta_3$ which minimizes the value of $\Gamma$ is the rotation angles which should be found.

The Euler angles $\phi_1$, $\Phi$, $\phi_2$ are converted to the notation of $(hkl)[uvw]$ as follows:

\[ h = \sin(\phi_2) \sin(\Phi) \] ...

\[ k = \cos(\phi_2) \sin(\Phi) \] ...

\[ l = \cos(\Phi) \] ...

\[ u = \cos(\phi_2) \sin(\phi_1) \sin(\Phi) \] ...

\[ v = -\cos(\phi_1) \sin(\phi_2) \cos(\Phi) \] ...

\[ w = \sin(\phi_1) \sin(\Phi) \] ...

When $(hkl)[uvw]$ rotates to $(h'k'l')[u'v'w']$ by $\theta_1$, $\theta_2$, $\theta_3$, $(h'k'l')[u'v'w']$ is expressed as follows:

\[ h' = (h + k \cdot \theta_3 - l \cdot \theta_2) / m \] ...

\[ k' = (-h \cdot \theta_1 + k \cdot \theta_3) / l \] ...

\[ l' = (h \cdot \theta_2 + k \cdot \theta_1) / n \] ...

\[ u' = (u + v \cdot \theta_2 - w \cdot \theta_1) / n \] ...

\[ v' = (-u \cdot \theta_1 + v \cdot \theta_3 - w \cdot \theta_2) / n \] ...

\[ w' = (u \cdot \theta_2 - v \cdot \theta_3 + w \cdot \theta_1) / n \] ...

\[ m = \sqrt{(h + k \cdot \theta_3 - l \cdot \theta_2)^2 + (-h \cdot \theta_1 + k \cdot \theta_3)^2 + (h \cdot \theta_2 + k \cdot \theta_1)^2} \] ...

\[ n = \sqrt{(u + v \cdot \theta_2 - w \cdot \theta_1)^2 + (-u \cdot \theta_1 + v \cdot \theta_3 - w \cdot \theta_2)^2 + (u \cdot \theta_2 - v \cdot \theta_3 + w \cdot \theta_1)^2} \]

Moreover, $(h'k'l')[u'v'w']$ is returned to the Euler angles $\phi_1'$, $\Phi'$, $\phi_2'$ as follows:

\[ \phi_1' = \arcsin \frac{w'}{\sqrt{k'^2 + h'^2}} \] ...

\[ \Phi' = \arccos(l') \] ...

\[ \phi_2' = \arcsin \frac{k}{\sqrt{k'^2 + h'^2}} \]

3.2. Orientation Dependence of Young’s Modulus

Young’s modulus was predicted from ODF by using the ideal Young’s modulus. Young’s modulus along an arbitrary direction defined by Euler angles, $E_{\phi_1\phi_2\phi_3}$, is calculated as follows:\(^{(25)}\)

\[ \frac{1}{E_{\phi_1\phi_2\phi_3}} = \frac{1}{E_{(100)}} + \frac{3}{E_{(111)}} \beta^2 \gamma^2 + \gamma^2 \alpha^2 + \alpha^2 \beta^2 \left( \frac{1}{E_{(111)}} - \frac{1}{E_{(100)}} \right) \]

where $\alpha$, $\beta$, and $\gamma$ are the direction cosines of the direction defined by $\phi_1$, $\Phi$, $\phi_2$ with 1, 2, 3 orthogonal crystal axes, and $E_{(100)}$, $E_{(111)}$ are the Young’s moduli along [100], [111], respectively. For iron cubic, it is known that $E_{(100)}$ is 135
GPa and $E_{(111)}$ is 290 GPa. As it is likely that the influence of the grain boundary constraint is ignored except in the case of extremely fine grains, Young's modulus of a polycrystal is calculated by using the Voigt model as follows:

$$E = \frac{\sum \sum \sum E_{\phi\psi} f(\phi, \psi) \sin(\Theta)}{\sum \sum \sum f(\phi, \psi) \sin(\Theta)}$$

where $f(\phi, \psi)$ is the ODF intensity.

4. Results

4.1. Changes in Young’s Modulus by Tensile Deformation

The measured Young’s moduli of the specimens before and after tensile deformation in the TD are shown in Fig. 2. Young’s modulus in the TD of steel A decreases greatly with increasing tensile strain up to 0.05 and then decreases gradually afterward. The drop of Young’s modulus reaches 5 GPa at 0.05 tensile strain, whereas the drop is only 3 GPa from 0.05 to 0.1 tensile strain. On the other hand, Young’s moduli in the RD, DD of steel A and the RD, DD, and TD of steel B are almost constant, independently of tensile strain. Although Young’s modulus in the TD of steel A decreases, the value remains the highest.

Figure 3 shows the change of Young’s modulus, $\Delta E$, in the RD, DD, and TD in the case of tensile strain of 0.05 in the respective directions. $\Delta E$ is obtained by subtracting the original value from that after deformation. Here $E$ in the DD under DD deformation takes the average value of the DD1 and the DD2 as shown in Fig. 1. As shown in Fig. 2, the $\Delta E$ of steel A has a large minus value in the TD under TD deformation, and the change is noteworthy. Furthermore, the $\Delta E$ of steel A also has relatively large minus values in the DD under RD and DD deformation, these values being approximately −4 and −3 GPa, respectively. The other $|\Delta E|$ of steel A and all $|\Delta E|$ of steel B are low, with values of approximately 2 GPa and smaller.

Fig. 2. Influence of tensile strain in TD on Young’s modulus in various directions of steel A (a) and steel B (b).

Fig. 3. Change of Young’s modulus before and after tensile deformation with strain of 0.05 in various directions of steel A (a) and steel B (b).

Fig. 4. ODF sections for $\phi_2 = 45^\circ$ of original sheets of steel A (a) and steel B (b).
4.2. Changes in Texture by Tensile Deformation

The ODF sections for $\phi_2=45^\circ$ of the original sheets of steels A and B are presented in Fig. 4. Concerning steel A, the $<110>\parallel RD$ fiber is developed prominently, and the maximum intensity exceeds 8 at $\{223\}<110>$. The intensity of $\{112\}<110>$ is also high, at 8, and that of $\{001\}<110>$ is approximately 4. Furthermore, the normal direction, $<111>\parallel ND$ fiber, is relatively developed, and its intensities are between 4 and 6. In steel B, the development of texture is suppressed; although its intensity profile is similar to that of steel A, the maximum intensity is below 3.5.

Figure 5 shows the ODF sections for $\phi_2=45^\circ$ of the deformed sheets with strain of 0.05 in the TD for steels A and B, and the changes of intensity, which was obtained as the difference between the original intensity and that after deformation. The change of intensity is larger in steel A than in steel B. Concerning steel A, under deformation in the TD, the increases in the intensities around $\{111\}<112>$ and $\{001\}<110>$ are remarkable, with increments that reach the range from 2 to 6. On the other hand, the intensities around $\{223\}<110>$ and $\{112\}<110>$ decrease extremely under deformation in the TD, and the decrements reach around 3. For steel B, although the intensity profile of the change resembles that of steel A, the absolute value of the intensity of the change is within 0.8.

The changes of ODF intensity before and after deformation with strain of 0.05 in the RD and DD are shown in Fig. 6. Concerning the changes by deformation in the RD, the increments around $\{001\}<110>$ are large, at around 1.5, for steel A. The changes around $\{223\}<110>$ and $\{112\}<110>$ are also high, at about 1.0. For steel B, the intensity profile of the change is similar to that of steel A, but the absolute value of the intensity of the change is within 0.8. Under deformation in the DD, for steel A, the decrements around $\{112\}<110>$ and $\{223\}<110>$ and the increments around $\{111\}<112>$ are large, at around 1.5, and the change around $\{001\}<110>$ is about 0.5. For steel B, the increment around $\{111\}<112>$ is about 0.2, whereas the changes around $\{112\}<110>$ and $\{223\}<110>$ are about 0.

5. Comparison with Simulation

5.1. Calculation of Young’s Modulus

Young’s modulus is evaluated by Eqs. (12) and (13) by using the ODF intensity. Figure 7 presents the calculated results of the original sheets of steels A and B in comparison with the experimentally measured Young’s moduli. The calculated Young’s modulus of steel A decreases gradually as the angle from the rolling direction increases up to 30°, and then increases precipitously as the angle increases to 90°. The calculated values deviated slightly from the measured ones, whereas the anisotropic tendency of the calculated Young’s modulus is coincident with the measured values. Compared with the deviation in the RD, the deviations in the DD and TD are somewhat small. The calculated Young’s of steel B increases greatly as the angle from the rolling direction increases up to 50°, but then decreases gradually at angles up to 90°. The calculated values in the DD and TD are almost the same as the measured ones, while the calculated value in the RD deviates slightly from the measured result.

Figure 8 shows the relationship between all measured Young’s moduli, including those of the original and deformed sheets, together with the calculated values. The plots are distinguished by the directions of the evaluated Young’s moduli and the type of steel. Although the calculated Young’s moduli are evaluated to be slightly lower than the measured ones, the calculated and measured results show
good correspondence. Here, the deviations of the Young’s moduli in the RD of steels A and B are slightly larger than those in the DD and TD. Although it is thought that the measured Young’s moduli are slightly over-evaluated due to the residual stress caused by skinpass rolling in the RD excluding the texture, the particular reasons are unknown.

The comparison between the measured and calculated $E$ before and after tensile deformation with strain of 0.05 for steels A and B is shown in Fig. 9. Concerning steel A, the calculated $E$ under TD deformation is almost constant at zero when the angle from the rolling direction is from 0–45° and decreases extraordinarily when the angle increases beyond 45°. As the measured $E$ in the TD shows a very small minus value, the tendency of the calculation corresponds to the measured tendency. On the other hand, the calculated $E$ in the RD and DD are almost zero at all angles, whereas the measured $E$ is a somewhat small minus value at the angle of 45°. Concerning steel B, both the calculated and measured $E$ are within ±2 GPa at all angles. That is, although there is a slight deviation between the calculated and measured $E$, the calculation results reflect the measured ones.

5.2. Prediction of Crystal Rotation

The crystal rotation angles are evaluated by calculating the combination of $\phi_1$, $\phi_2$, $\phi_3$ for minimizing Eq. (8). For calculation of the crystal rotation angles due to tensile deformation, $d\varepsilon$ is increased in increments of 0.01. Figure 10 shows
the movement by using the FC model of crystal rotation due to tensile deformation in the TD with strain of 0.05 on the ODF sections for $\phi_2=45^\circ$ of steels A and B. Here, all the original directions on the ODF section for $\phi_2=45^\circ$ has ceased to exist after deformation, so the plots in Fig. 10 are distinguished by the value of $\phi_2$. The crystal rotations for steels A and B are almost the same because the difference of the $r$-value in the TD is small. In both steels, the crystals around the $\alpha$-fiber from $\{112\}<110>$ to $\{111\}<110>$ rotate largely to other directions. In contrast, the crystals around $\{001\}<110>$ and $\{111\}<112>$ converge from other directions.

Figure 11 shows the movement by using the FC model of crystal rotation by tensile deformation in RD with strain of 0.05 on ODF sections for $\phi_2=45^\circ$ of steel A (a) and steel B (c), and in DD with strain of 0.05 for steel A (b) and steel B (d).

5.3. Comparison of Young’s Modulus Calculated by Measured and Simulated Textures

Young’s modulus can be evaluated properly by calculation from the measured ODF intensity. Moreover, the change of Young’s modulus by simulated crystal rotation is verified by comparison with the results of calculations from the measured ODF intensity. Figure 12 shows the calculated $\Delta E$, which is obtained by subtracting the $E$ calculated by the measured original ODF intensity from the $E$ obtained by the calculated or measured ODF intensity after tensile deformation in the TD with strain of 0.05 for steel A. The results of simulations of crystal rotation by the FC and RC models are compared with the results calculated from the measured ODF intensity. With the RC model, $\gamma_3$ and $\gamma_{23}$ in Eq. (5) changed within $\pm 0.05$. In the calculations from the measured ODF intensity, $\Delta E$ is almost constant at zero at angles of $0–45^\circ$ from the rolling direction, and then decreases greatly to about $–3$ as the angle increases to $90^\circ$. In contrast, with the FC model, $\Delta E$ takes a value of around $–1$ at the angle of $0^\circ$, increases to about $2$ as the angle increases to $45^\circ$, then decreases to about $–3$ with further increases in the angle up to $90^\circ$. In the RC models, when $\gamma_3$ is $–0.05$, $\Delta E$ around the angle of $45^\circ$ becomes smaller than the value by the FC model and approaches the result by the...
measured ODF intensity independently of \( \gamma_{13} \). Moreover, \( \Delta E \) around the angle of 0° is almost the same as the result by the FC model. On the other hand, in the case of \( \gamma_{13} \) of +0.05 or -0.05 at \( \gamma_{23} \) of -0.05, \( \Delta E \) around the angle of 90° becomes larger than the value by the FC model and diverges from the result by the measured ODF intensity. When \( \gamma_{13} \) is 0 at \( \gamma_{23} \) of -0.05, \( \Delta E \) around the angle of 90° is similar to the result by the FC model. Furthermore, when \( \gamma_{13} \) is -0.05 and \( \gamma_{13} \) is 0, \( \Delta E \) around the angle of 45° becomes smaller, whereas \( \Delta E \) around the angle of 90° becomes larger than the value by the FC model. \( \Delta E \) around the angle of 0° is almost the same as the result by the FC model. That is, the RC model with \( \gamma_{13} \) of 0 and \( \gamma_{23} \) of -0.05 shows relatively good agreement with the results calculated from the measured ODF intensity, but the difference between the FC and RC models is not particularly large. On the basis of these results, the FC model is also able to simulate the crystal rotation to the same degree as the RC model.

**Figure 13** shows a comparison of \( \Delta E \) calculated by the FC model and the measured ODF intensity for steels A and B. Here, the pre-strain directions are the RD, DD, and TD, and the tensile strain is 0.05. Concerning steel A, \( \Delta E \) in the DD by the FC model shows slightly large plus values around 0 and 90° from the rolling direction and a slightly minus value around 45°, whereas \( \Delta E \) by the measured ODF intensity shows a slightly small minus value around 90° and is almost zero around 0 and 45°. \( \Delta E \) in the RD and TD display relatively good correspondences in the results by the FC model.

![Fig. 12. Comparison of change of Young's modulus by tensile deformation in TD with strain of 0.05 calculated from simulated ODF using FC and RC models and measured results for steel A.](image)

![Fig. 13. Comparison of change of Young's modulus by tensile deformation in RD, DD, and TD with strain of 0.05 calculated from simulated ODF using FC model and measured results for steel A (a) and steel B (b).](image)
FC model and measured ODF intensity. In the case of steel B, $\Delta E$ in the RD by FC model shows a slightly small minus value around 0° and is almost zero around 45 and 90°, whereas $\Delta E$ by the measured ODF intensity is almost zero around 0°, is slightly minus around 45°, and then displays a slightly large plus value around 90°. Furthermore, $\Delta E$ in the TD by the FC model is almost zero around 0°, shows a slightly large in plus value around 45°, and then a small minus value around 90°. $\Delta E$ in the DD shows relatively good correspondence between the results by the FC model and the measured ODF intensity.

Figure 14 shows the comparison between $\Delta E$ in the RD, DD, and TD, as calculated from the measured ODF, and the absolute value of the change of Young’s modulus calculated from the FC model, $\Delta E_m + |\Delta E_F - \Delta E_m|$, by tensile deformation in the RD, DD, and TD with strains of 0.02, 0.05, and 0.1 for steels A and B. Here, $\Delta E_m$ is the change of Young’s modulus calculated from the measured ODF, and $\Delta E_F$ is the change of Young’s modulus calculated from the FC model. The deviation between $\Delta E_m$ and $\Delta E_F$ is around 3 GPa for steel A but only around 1 GPa for steel B. This is because $\Delta E$ of steel B is originally small. That is, although there are slight deviations between $\Delta E_F$ and $\Delta E_m$, the characteristic changes can be simulated by the FC model and the calculation results reflect the measured ones.

6. Discussion

The ideal Young’s modulus of an arbitrary orientation is evaluated from Eq. (12). Figure 15 shows the calculation results of the ideal Young’s moduli in the RD, DD, and TD on the ODF sections for $\phi_2=45^\circ$, respectively. In the RD, Young’s moduli on the $\alpha$-fiber of \<110\> // RD and $\gamma$-fiber of \<111\> // ND are almost constant between 220 and 230 GPa. As shown in Fig. 4, the textures of cold-rolled steel sheets like steels A and B develop on the $\alpha$-fiber and $\gamma$-fiber. The reason why Young’s moduli in the RD hardly changes when tensile deformation is applied to steels A and B, regardless of the direction of tensile deformation, is presumably due to the restricted crystal rotation up to tensile strain of 0.1 and the almost constant Young’s moduli on the $\alpha$-fiber and $\gamma$-fiber.

In the DD, the Young’s moduli on the $\alpha$-fiber from \{001\}<110> to \{223\}<110> are low and are below 200 GPa; in particular, the Young’s moduli around \{001\}<110> are remarkably low at less than 140 GPa. Therefore, the decrease in Young’s modulus in the DD due to tensile deformation in the RD of steel A seems to be attributable to the increases in ODF intensity around \{001\}<110> and \{223\}<110>. Furthermore, the decrease in Young’s modulus due to tensile deformation in the DD is also understood from the decrease in the ODF intensity around \{001\}<110>, in spite of the slight decrease in texture around \{112\}<110>. On the other hand, the reason why the decrease in Young’s modulus due to tensile deformation in the TD is not particularly large, in spite of the increase in ODF intensity around \{001\}<110>, may be due to the remarkable decreases in the ODF intensity around \{112\}<110> and \{223\}<110>.

In the TD, the Young’s moduli around \{112\}<110> are particularly high, exceeding 280 GPa. In this study, the most conspicuous change of Young’s modulus is the decrease in
\( \Delta E \) in the TD in the case of tensile deformation in the TD in steel A. It appears that the decrease in the ODF intensity around \([112]<110>\) by tensile deformation in the TD in steel A is the reason for the large decrease in \( \Delta E \) in the TD.

Based on these results, the change of Young’s modulus due to tensile deformation is recognized to be due to the change of texture by crystal rotation.

7. Conclusions

The quantitative influences which affect changes in Young’s modulus and texture due to tensile deformation in various directions were investigated by an experimental method by using anisotropic and isotropic cold-rolled high-strength steel sheets, and the results were compared with the results of the calculated crystal rotation by the Taylor model and the calculated ideal Young’s modulus. The following conclusions were obtained:

(1) In the anisotropic steel sheet, which had a high Young’s modulus in the transverse direction, the change of Young’s modulus in the transverse direction due to tensile strain in the transverse direction showed a conspicuous minus value. Young’s modulus in the diagonal direction decreased as a result of tensile strain in the rolling and diagonal directions. The changes of Young’s modulus were not particularly remarkable in other directions. In contrast, the isotropic steel sheet displayed hardly any change in Young’s moduli in any direction as a result of tensile strain, regardless of the direction of strain.

(2) Crystal rotation due to tensile strain was distinguished at the texture on the \( \alpha \)-fiber. In the anisotropic steel sheet, the ODF intensities increased around \([001]<110>\), \([112]<110>\), and \([223]<110>\) when tensile strain was applied in the rolling direction. On the other hand, when tensile strain was applied in the diagonal and transverse directions, the ODF intensities increased around \([001]<110>\) but decreased around \([112]<110>\) and \([223]<110>\). The changes of the ODF intensities were especially marked in the case of tensile strain in the transverse direction. The tendency of the isotropic steel sheet was almost the same as that in the anisotropic steel sheet; however, the amount of change was smaller in the isotropic sheet.

(3) Crystal rotation due to tensile deformation could be simulated approximately by using the Taylor FC model. In particular, the large decrease of the ODF intensities around \([112]<110>\) and \([223]<110>\) associated with tensile deformation in the transverse direction, which is a characteristic change, was simulated as huge crystal rotation around the \( \alpha \)-fiber from \([112]<110>\) to \([111]<110>\) by the Taylor model.

(4) Young’s modulus was calculated by using the Voigt model accompanied by the ideal Young’s moduli along \([100]\), \([111]\) and the ODF intensity. Although the calculated Young’s moduli were evaluated to be slightly lower than the measured results, the calculated and measured results showed good correspondence.

(5) The fact that almost no changes were observed in Young’s modulus in the rolling direction as a result of tensile deformation, regardless of the direction of deformation, seemed to be attributable to the constancy of the ideal calculated Young’s moduli on the \( \alpha \)-fiber and \( \gamma \)-fiber, which are mainly developed in cold-rolled steel sheets. The decreases in Young’s modulus in the diagonal direction due to tensile deformation in the rolling and diagonal directions in the anisotropic steel sheet were probably caused by the increase in the ODF intensity around \([001]<110>\) and \([223]<110>\), which is attributed to the decrease in Young’s modulus in the diagonal direction. Moreover, it was likely that the decreases in Young’s modulus in the transverse direction by tensile deformation in the transverse direction in the anisotropic steel sheet originated in the decrease in the ODF intensity around \([112]<110>\), which is attributed to the increase in Young’s modulus in the transverse direction.

(6) The changes of Young’s moduli in high-strength cold-rolled steel sheets depending on the direction of tensile deformation were understood from the crystal rotation behavior simulated by the Taylor FC model and the Voigt model.

REFERENCES