1. Introduction

The main raw material in the Electric Arc Furnace (EAF) is scrap. This material has the advantage of lower energy consumption in comparison with Direct Reduced Iron (DRI), but the production of high quality steels with scrap is difficult due to the presence of residual elements and also due to severe fluctuations in both availability and price. On the other hand, with DRI as feedstock, the production of steels of higher quality is possible. However, DRI is a porous material containing gangue that increases the consumption of electric energy. In order to decrease energy consumption it is critical to understand the melting mechanism and increase the melting rate of DRI.

The melting mechanism of particulate additions in a liquid metal at high temperatures involves the formation of a frozen shell. Its liquidus temperature with respect to the liquidus temperature of the addition defines whether heat or mass transfer is the dominant mechanism during its melting process.\(^1\)-\(^3\) It has also been reported\(^4\)-\(^6\) that exothermic reactions and CO generated during heating, melting and dissolution of particulate additions, accelerate the melting rate of the frozen shell.

The melting rate of DRI depends on many variables: properties of the frozen shell, metallization, gangue content, temperature and size of the addition, temperature of liquid steel, superheat, stirring conditions and slag physicochemical properties. It has been reported that the melting rate increases with DRI of higher metallization,\(^7,8\) lower gangue content,\(^9\) DRI particles of higher density,\(^9\) smaller particle size,\(^2,10,14\) higher temperature of the solid particles,\(^10-14\) increasing the temperature of liquid steel (higher superheat),\(^10,15\) higher stirring conditions\(^7,12,14,16\) and a slag composition with a binary basicity ratio between 0.8 and 1.\(^17,18\)

A decrease in melting rate of 7% as the gangue content increases by 5% has also been reported.\(^19\) The large number of variables involved in the melting rate of DRI particles clearly explains the large differences in melting rate of DRI obtained in different processes.\(^20\) In addition to the previous research, some of the current authors have also reported an increase in melting rate as arc length increases\(^21\) and also a more realistic computation of the convective transfer coefficient in an industrial EAF.\(^22\)

Several researchers have developed mathematical models involving the melting of particles in their own bath or particles in a molten bath with different chemical composition. However, the modeling approach is the same in all of them. Their difference lies in the method used to solve the equation of the second law of Fourier, some researchers have used numerical solutions,\(^9,10,13,16,18,20,23,24\) and others have developed analytical solutions.\(^10,11,25\) Table 1 summarizes the mathematical models on the melting of particles in a metal bath. In all works a continuous profile for the thermal conductivity is used at the pellet-slag interface and in most of the research works an arithmetic mean of the interfacial thermal conductivity was calculated.

In this work a fundamental mathematical model was
Table 1. Summary of mathematical modelling proposes for the melting of particles in a metallic bath.

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Materials (solid in liquid)</th>
<th>Special Characteristics</th>
<th>Solution</th>
<th>Governing equation, boundary and initial conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ehrich10</td>
<td>DRI in iron DRI in DRI Iron in iron</td>
<td>Transforms differential equations in dimensionless form</td>
<td>Solve integral equations using Green functions</td>
<td>Governing equation, boundary conditions, Initial conditions</td>
</tr>
<tr>
<td>Seaton20</td>
<td>DRI in Steel</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jiao13</td>
<td>Slag in slag Al in Al DRI in slag Cu–Ni in CuS</td>
<td>Solve equations in dimensionless form</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taniguchi16</td>
<td>Al in Al Al in Al Alloy</td>
<td>Not transform equations in dimensionless form</td>
<td></td>
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<tr>
<td>Nauman9</td>
<td>DRI in slag</td>
<td></td>
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</tr>
<tr>
<td>O’Malley25</td>
<td>DRI in slag</td>
<td>Diffs from the boundary condition 2 in node convective when $t &gt; t_c$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>He18</td>
<td>DRI in slag</td>
<td>Implements a boundary condition with a heat transfer coefficient at the interface pellet-slag, $-\lambda \frac{\partial T}{\partial r}</td>
<td>r = h_{in} (T_{pellet} - T_b)$</td>
<td></td>
</tr>
<tr>
<td>Rohmen24</td>
<td>Al in Al</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zhang13</td>
<td>DRI in iron</td>
<td>Boundary condition 2 in $R = R_0$, $\lambda R_0^2 (T_b - T_0) = 4 \pi R_0^2 \frac{dT}{dt} \rho_s (\Delta H)$</td>
<td>Analytic</td>
<td></td>
</tr>
<tr>
<td>Engh25</td>
<td>Metallic additions with melting point less than liquid bath</td>
<td>• Compute the temperature in any time when the temperature profile is a straight line • Establishes relationships between the Biot number and the dimensionless time, $F_0$</td>
<td>Solve for: • Maximum radius ($R_{max}$) • Shell stage ($t_c$) • Radius evolve</td>
<td>Governing equation, boundary conditions, Initial conditions</td>
</tr>
</tbody>
</table>

developed and numerically solved with a finite differences explicit approach, based on heat transfer that simulates the melting kinetics of a DRI pellet in a slag bath. The model presents several numerical improvements such as the use of a harmonic mean pellet-slag interface and the representation of the thermal conductivity of the pellet as a function of the porosity, features that make this model unique. An extensive process analysis was performed to estimate the effects of the process variables (slag temperature, pellet initial size and temperature, its porosity and the stirring conditions of the slag) as well as the physicochemical properties of DRI and the slag on the melting kinetics. Besides the melting time estimations, this work provides estimations of the energy consumption and the process analysis gives a clue for optimum operational condition to improve the melting rate in the EAF.

2. Mathematical Model

The problem considers a pellet of sponge iron of radius $R_0$ initially at temperature $T_{0e}$ being fed to the EAF at a temperature $T_b$ at an initial time $t = 0$ where $T_b > T_{0e}$. A shell of solidified slag around the particle is formed; the shell grows to a maximum size and then melts back. If the slag bath has a high temperature, the sponge iron pellet will eventually melt. The temperature profile in both the slag shell and sponge iron can be seen in Fig. 1(a).

The phenomenon that describes the melting kinetics process is very complex and then in order to simplify the problem, the following assumptions are adopted:

(i) Thermo physical properties of all the materials involved in the calculation are considered to be constant.
(ii) Temperature profile inside the pellet is symmetrical respect to its angular coordinates.
(iii) Melting temperature of both metallic bath and pellet is constant.
(iv) Changes of chemical composition in the shell, metallic bath and pellet, are neglected.
(v) Heat transfer from the metallic bath to the solid-
liquid interface is governed by the convection mechanism. 

(vi) Shell begins to solidify instantly after the pellet is immersed in the metallic bath.

(vii) DRI particles are treated as low melting point additions.

(viii) The chemical reactions of phase transformations or gas emissions inside of the pellet, that accelerate the heating and melting of the pellet occurring before the shell disappears are not considered.

The computational domain shown in Fig. 1(b) represents the solid particle-liquid system; the discretization is divided in spherical elements with a finite thickness Δr. During the shell formation period (t < t_i), the governing equation is represented by a 1D energy conservation equation in transient state neglecting the latent heats due to phase transformations. The governing equations in spherical coordinates, boundary conditions and initial conditions for each one of the regions are described as follows below,

**DRI region:**

Governing equation:

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha_{\text{DRI}}} \frac{\partial T}{\partial t} \quad \text{for} \quad 0 < r < R_0 \]

**Initial conditions:**

\[ T = T_{\text{DRI}} \quad \text{for} \quad r \leq R_0, \text{ uniform temperature} \]  

**Boundary conditions:**

1) at \( r = 0, \frac{\partial T}{\partial r} = 0 \) symmetry \( (3) \)

2) at \( r = R_0, T = T_{\text{DRI}}, t > 0 \) \( (4) \)

**Slag Shell region**

Governing Equation:

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha_{\text{SLAG}}} \frac{\partial T}{\partial t} \quad \text{for} \quad R_0 < r < R \]

**Initial conditions:**

\[ T = T_{\text{DRI}} \quad \text{at} \quad r = R_0, \quad t = 0 \]  

\[ R = R_0, \quad t = 0 \] \( (7) \)

**Boundary conditions:**

1) at \( r = R_0, T = T_{\text{SLAG}}, t > 0 \) \( (8) \)

2) at \( r = R, \lambda_{\text{SLAG}} \frac{\partial T}{\partial r} - h(T_b - T_{\text{SLAG}}) = \frac{dR}{dt} \rho_{\text{SLAG}} \Delta H_{\text{SLAG}} \] \( (9) \)

Where \( r \) and \( T \) represent the radial coordinate position and temperature, respectively. \( \lambda_{\text{DRI}} \) represents the thermal diffusivity of DRI, \( R_0 \) is the sphere radius at \( t = 0 \), while \( T_{\text{DRI}} \) and \( T_{\text{SLAG}} \) are the initial and instantaneous DRI temperature, respectively. \( T_{\text{SLAG}} \) is the slag temperature while \( \lambda_{\text{SLAG}} \) represent the thermal diffusivity of slag. \( R \) is the sphere radius at time \( t \) and \( h \) is the heat transfer coefficient. \( \lambda_{\text{SLAG}}, T_{\text{SLAG}}, \rho_{\text{SLAG}} \) and \( \Delta H_{\text{SLAG}} \) represents the thermal conductivity, melting temperature, density and latent heat of melting of the slag, respectively.

In this stage the boundary condition at the pellet – bath interface given by Eq. (9) represents the energy balance at the liquid-solid interface (a dynamic interface) and takes into account the latent heat of solidification or melting of the slag shell formed. The heat balance at this boundary assumes that the surface temperature of the shell is equal to the melting temperature as expressed in Eq. (10).

\[ T = T_{\text{SLAG}} \quad \text{at} \quad r = R \] \( (10) \)

The temperature profile and the heat flux at the DRI particle – solidified slag interface at \( r = R_0 \) must be continuous. This continuity condition requires setting equal temperatures and heat flows at the pellet–shell interface, according to Eqs. (11) and (12).

\[ \lambda_{\text{DRI}} \left( \frac{\partial T}{\partial r} \right)_{\text{DRI-int}} = \lambda_{\text{SLAG}} \left( \frac{\partial T}{\partial r} \right)_{\text{SLAG-int}} \]  

(11)

\[ \left( \frac{\partial T}{\partial r} \right)_{\text{DRI-int}} = \left( \frac{\partial T}{\partial r} \right)_{\text{SLAG-int}} \]  

(12)

Finally, once that the slag shell has been melted (at time \( t = t_i \)) the model of pellet melting is given by Eq. (13).

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) = \frac{1}{\alpha_{\text{DRI}}} \frac{\partial T}{\partial t} \quad \text{at} \quad 0 < r < R \]  

(13)

The initial conditions are given by Eqs. (14) and (15).

\[ T = T(r, t_0), \quad r \leq R_0, \quad t = t_0 \]  

(14)

\[ R = R_0, \quad t = t_i \]  

(15)

The boundary conditions indicate symmetry in the center of the pellet by Eq. (16).

\[ \frac{\partial T}{\partial r} = 0 \quad \text{at} \quad r = 0, \]  

(16)

When the temperature on the surface of the pellet is less than the melting temperature, the equation for the boundary condition is given by Eq. (17).

\[ \lambda_{\text{DRI}} \frac{\partial T}{\partial r} = h(T_b - T_{\text{surface}}) \quad \text{at} \quad r = R_0 \]  

(17)

However, when the temperature on the surface of the pellet reaches the melting temperature, the boundary condition
is given by Eq. (18).
\[
\lambda_{\text{DRI}} \frac{\partial T}{\partial r} - h(T_b - T_m) = \frac{dR}{dt}\rho_{\text{DRI}} \Delta H_{\text{DRI}} \quad \text{at } r = R \ldots (18)
\]

2.1. Solution Method by Finite Differences

The model was solved numerically by the explicit finite differences technique. Improvements made for the solution are given at the slag-pellet interface where the material properties present a discontinuity. The temperature profile at this interface has to be continuous by means of Eq. (19).
\[
\lambda_{\text{DRI}} \frac{T_{\text{int}} - T_i}{\Delta r} = \lambda_{\text{SLAG}} \frac{T_{i+1} - T_{\text{int}}}{\Delta r} \ldots \ldots \ldots \ldots \ldots (19)
\]

where \( T_{\text{int}} \) and \( T_i \) represents the interface temperature and the temperature in the spherical element \( i \), respectively.

Patankar suggests a simple procedure for computing the thermal conductivity at the interface, \( \lambda_{\text{int}} \), consisting in averaging the conductivity with the values of the two neighbor nodes, \( i \) and \( i+1 \) (Eq. (20)).
\[
\lambda_{\text{int}} = f_{\text{int}} \lambda_i + (1 - f_{\text{int}}) \lambda_{i+1} \ldots \ldots \ldots \ldots \ldots (20)
\]

where \( \lambda_i \) and \( \lambda_{i+1} \) are the thermal conductivity in the \( i \) and \( i+1 \) nodes, while \( f_{\text{int}} \) represent the interpolation factor that defines the ratio between the distances of the nodes to the interface. Equation (20) is known as the arithmetic mean and has been used by most previous researches to represent the thermal conductivity at the interface between two dissimilar materials. In composite materials, like DRI-slag there is an abrupt change in \( \lambda \) driving to errors in the computation. In this work a harmonic mean was used to calculate the thermal conductivity between the \( i \) and \( i+1 \) nodes (at the interface between two different materials). The advantage of expressing the conductivity in this way is evident in cases where \( \lambda_{i+1} < \lambda_i \), such as in the case of Fe particle -Slag interphase. In such a case, the material of the \( i+1 \) node is a thermal insulator and then, Eq. (21) sets the thermal conductivity at the interphase equal to \( \lambda_i \), promoting insulation as it physically happens when an insulator is in contact with a conductor. Therefore, the thermal conductivity in the interface is given by Eq. (21).
\[
\lambda_{\text{int}} = \frac{2 \lambda_i \lambda_{i+1}}{\lambda_i + \lambda_{i+1}} \ldots \ldots \ldots \ldots \ldots (21)
\]

The thermal diffusivity, \( \alpha \), and the delta Fourier, \( \Delta F \), at the interfaces of both regions DRI and DRI shell can be computed by Eqs. (22) to (25) respectively.
\[
\alpha_{\text{DRI-int}} = \frac{\lambda_{\text{int}}}{\rho_{\text{DRI}} C_{\text{DRI}}} \ldots \ldots \ldots \ldots \ldots (22)
\]
\[
\alpha_{\text{SLAG-int}} = \frac{\lambda_{\text{int}}}{\rho_{\text{SLAG}} C_{\text{SLAG}}} \ldots \ldots \ldots \ldots \ldots (23)
\]
\[
\Delta F_{\text{DRI-int}} = \frac{\alpha_{\text{DRI-int}} \Delta M}{\Delta r^2} \ldots \ldots \ldots \ldots \ldots (24)
\]
\[
\Delta F_{\text{SLAG-int}} = \frac{\alpha_{\text{SLAG-int}} \Delta M}{\Delta r^2} \ldots \ldots \ldots \ldots \ldots (25)
\]

The problem has a moving boundary (increases the size of the pellet when the shell forms and decreases the size of the pellet when the shell and pellet are being melted). The change in the domain was adjusted by keeping constant the size of the nodes (\( \Delta r \)) and simultaneously adding nodes (during solidification) or deleting nodes (during melting). The melting process is completed when the number of elements is reduced to 2.

To solve the equations by finite differences with an explicit scheme, a program was written in Fortran.

3. Results and Analysis

3.1. Model Validation

The results obtained by this model were validated with the results of three previous investigations. Those investigations include the melting of a material in its own metallic bath such as (a) sponge iron in liquid iron, (b) aluminum in aluminum, and the melting of particles in a metallic bath with a different chemical composition such as (c) sponge iron in liquid iron and (d) sponge iron in liquid slag.

In Table 2 the thermo physical properties employed for model validation of different metallic baths are presented. The heat transfer model to the melting of a material in the same medium (such as with iron in iron or aluminum in aluminum) is the simplest model to solve, because there is only one melting interface, and therefore there are not abrupt changes in the thermo physical properties of the materials.

Table 2. Thermo-physical properties of iron, sponge iron, aluminum and slag.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density ( \rho ) (kg/m(^3))</th>
<th>Heat Capacity ( C_p ) (J/kg K)</th>
<th>Thermal Conductivity ( \lambda ) (W/mK)</th>
<th>Melting Point ( T_m ) (K)</th>
<th>Latent heat of fusion ( \Delta H_m ) (J/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron(^{10})</td>
<td>7 650</td>
<td>820</td>
<td>37.65</td>
<td>1 809</td>
<td>277 400</td>
</tr>
<tr>
<td>Sponge Iron(^{123})</td>
<td>2 600</td>
<td>820</td>
<td>2.13</td>
<td>1 809</td>
<td>277 400</td>
</tr>
<tr>
<td>Sponge Iron LR-2J-1(^{123})</td>
<td>3 370</td>
<td>820</td>
<td>3.7656</td>
<td>1 809</td>
<td>251 040</td>
</tr>
<tr>
<td>Sponge Iron LR-1B-1(^{123})</td>
<td>2 020</td>
<td>620</td>
<td>1.8828</td>
<td>1 812</td>
<td>251 040</td>
</tr>
<tr>
<td>Aluminum(^{123})</td>
<td>2 550</td>
<td>900</td>
<td>933</td>
<td>1 589</td>
<td>400 000</td>
</tr>
<tr>
<td>Slag(^{123})</td>
<td>2 900</td>
<td>1 004</td>
<td>1 171</td>
<td>1 3589</td>
<td>920 480</td>
</tr>
<tr>
<td>Slag 2(^{123})</td>
<td>2 900</td>
<td>1 171</td>
<td>1 3589</td>
<td>1 3589</td>
<td>426 768</td>
</tr>
</tbody>
</table>

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Figure 2 shows a comparison between the numerical results obtained by the model and the numerical and experimental results obtained by Ehrich. Melting of a single pure iron pellet with initial radius of 15 mm for two initial temperatures of the pellet at 303 K and at 1473 K in their own bath at temperatures of 1873 and 1848 K, respectively are showed. It is seen that the curve describing the melting kinetics of an iron pellet obtained by the model agrees satisfactorily with the experimental measurements from the time of immersion in the liquid bath until complete pellet melting. The temperature profiles measured and calculated at the center of the pellet under the conditions described earlier are shown in Fig. 2. A reasonable good agreement is found between the temperature profile predicted by the model (solid line) and the experimental measurements (symbols).

(b) Melting of aluminum in aluminum bath

To validate the mathematical model of pure aluminum pellets in a bath of melting pure aluminum, experimental data obtained by Taniguchi were used. In Fig. 3 it can be seen how the results obtained by the model are in good agreement with the experimental data measured by Taniguchi. A particle diameter of 11.9 mm for three different temperatures in the aluminum metallic bath was used for these calculations. It can also be seen a better prediction of experimental data by the numerical model developed in comparison with the numerical model proposed by Taniguchi.

(c) Fusion of sponge iron in pure iron

The robustness of the model could be proven by comparing the numerical solution of the melting of a pellet of sponge iron in a pure iron bath against analytical solutions proposed by Ehrich. The analytical solutions employ simple expressions to estimate the time at which the shell is melted, the maximum radius of the sphere, and the total melting time of the particle. In Fig. 4 it is seen that the prediction of the model agrees well with the analytical solutions. These analytical solutions are valid only under certain conditions. For example, in the case of the expression proposed to compute the maximum radius of the sphere, the thermal conductivities of both the shell and the pellet are assumed to be infinity. Therefore the maximum radius increases instantaneously from \( R_0 \) to \( R_{\text{max}} \). Then, the maximum radius computed with a finite thermal conductivity is lower than the analytical maximum radius, as shown in Fig. 4.

(d) Melting of sponge iron in slag

The model was validated with experimental results reported by O’Malley for sponge iron pellets in non-reactive slag, as can be seen in Fig. 5. The thermal histories obtained numerically show good agreement with the experimentally measured thermal history of the sponge iron in the center of the DRI pellets. The small discrepancies are due to disturbances that occur due to local melting in the slag shell when the temperature gradient between the slag shell and the pellet is small.

3.2. Model Predictions for Melting Sponge Iron in Non-reactive Slag

The predictions for the melting kinetics of sponge iron in non-reactive slag are presented in terms of the main characteristics and properties of the pellet, the physical properties of the slag and fluid-dynamic conditions:

- DRI particles: particle size and porosity–related variables such as density and thermal conductivity.
- Slag: thermal conductivity, density and specific heat.
Fluid-dynamic conditions: bath temperature, the initial pellet temperature and the heat transfer coefficient. The heat transfer coefficient employed in the simulations was 630 W/m²K, corresponding to the mean value reported by O’Malley.23 This author reviewed 6 different models predicting the heat transfer coefficient as a function of gas flow rate for a pellet of 2.5 cm immersed in a bath of slag. The values reported ranged from 300 to 900 W/m²K for a gas flow rate evolved from the pellet from 5 to 20 Ncm³/min.

(i) Effect of radius of sponge iron pellet
The melting time of the pellets is significantly affected when the initial radius of the particle changes. Figure 6 shows the size evolution of the pellet during melting by plotting the dimensionless pellet radius versus time. This trend is expected, because as the radius increases also increases the mass, and consequently more energy is needed to melt the pellet under the same conditions of agitation and bath temperature, resulting in longer melting times for bigger particles. The dimensionless radius is obtained by dividing the instantaneous radius over the initial pellet radius, and this quantity is a better and easier parameter for comparison purposes. Increasing the initial size of the pellet increases both the melting time and the maximum size of the shell, although the percentage of increment of the shell relative to the initial size is bigger for small pellets than for larger ones, also the size of the shell increases as the initial radius of the particle increases as shown in Fig. 7.

(ii) Pellet specific heat effect
This property depends on the chemical composition of the pellet as well as on the gangue composition. Impurities of high melting point increase the heat capacity. Increasing the specific heat of the pellet retards melting seen in Fig. 8. The duration and size of the shell increases significantly by increasing the specific heat of the pellet, which results in larger amounts of solidified slag per ton of sponge iron in contact with the slag. Furthermore, the melting time of the particle is directly proportional to the specific heat of the DRI as shown in Fig. 8.

(iii) Pellets porosity
One of the most important physical properties of the sponge iron is its porosity, because this property affects the thermal conductivity, specific heat and density of the pellet.

Effect of pellet porosity on its density
To compute the porosity of the pellet is necessary to know the volume fraction of each component, Yᵢ, present in the pellet:

\[ Yᵢ = Xᵢ \left( \rho_{\text{pellet}} / \rhoᵢ \right) \]  

(26)
where $\rho_{\text{pellet}}$ is the pellet density, $\rho_i$ and $X_i$ represents the density and weight fraction of the component $i$, respectively. Then, the porosity of the pellet, $P_0$, is calculated with Eq. (27).

$$ P_0 = 1 - \sum_i Y_i \quad \text{(27)} $$

The density of the pellet decreases when the pellet porosity increases. The density of the pores may be neglected and therefore the pellet density can be computed as a linear function of the porosity with Eq. (28), where $\rho_s$ is the density of the main substance which in this case has a value of 7837.2 kg/m$^3$.

$$ \rho_{\text{pellet}} = \rho_s (1 - P_0) \quad \text{(28)} $$

**Effect of pellet porosity on its thermal conductivity**

The thermal conductivity of iron sponge obtained experimentally oscillates in the range of values from 1.25 to 5.02 W/m K below 700°C, and above this temperature gradually increases up to 16.7 W/m K. An empirical correlation is widely accepted to predict the effective thermal conductivity, $\lambda_{\text{eff}}$, in terms of the porosity of porous metallic solids, which was proposed by Koh and Fortini and given by Eq. (29).

$$ \lambda_{\text{eff}} = \lambda_s \left(1 - nP_0\right) \quad \text{(29)} $$

where $\lambda_s$ is the thermal conductivity of the substrate and $n$ is a constant with a value of 11 for sintered metal powder. **Figure 9(a)** shows the thermal conductivity calculated by Eq. (29) as a function of the porosity of the pellet. When the particle porosity decreases, the thermal conductivity increases since the mass is more compact and heat can easily and uniformly flow through the pellet.

**Effect of pellet porosity on the melting time**

Porosity in the range between 25 to 75% in volume in the pellet was considered for practical purposes. As the porosity decreases, the thermal conductivity and density of the pellet increases, decreasing the melting rate of the solid particles, as shown in Figs. 9(a) and 9(b), this behavior is attributed to a larger mass to melt by decreasing the porosity.

The previous results assume that the only phases are solid particles and liquid slag. In a previous research conducted by Sadrnezhaad in a solid particle-liquid slag-liquid steel system, it was proved that DRI particles of higher density melt faster due to a change in chemical composition in the frozen shell from slag to steel when particles of higher density than the slag penetrate into the pool of liquid steel. A shell of steel has a much higher thermal conductivity and therefore its melting rate increases.

**Figure 10** shows the amount of energy required to melt a ton of DRI as a function of three parameters: the initial radius, its specific heat and porosity. In a previous section of this paper it was shown that a small particle size pellet yields shorter melting times, however based on the results shown in Fig. 10 it can be seen that the energy requirement increases significantly with decreasing the pellet initial radius and therefore small pellets are not the best choice economically, because as the particle size decreases, the liquid-solid interfacial area increases and consequently promotes more solidified mass of slag increasing the energy requirements to melt than the energy to melt a ton of steel with smaller particles. In real conditions some pellets in the slag may agglomerate and form colonies and not remain as individual particles. This condition presents complications to compute the total amount of solidified slag formed. Another limitation of small particle size DRI is a decrease in metallic yield because many of those small particles never reach the liquid slag due to entrapment by the off
gas stream.

Figure 10 also shows that the total amount of energy demanded significantly increases when the specific heat increases. Actually the increment in energy required to melt a ton of steel is higher with an increment in the specific heat than with an increment in porosity. These results suggest that an efficient melting process of a ton of sponge iron needs to avoid high levels of impurities (gangue) in the pellet, which are the reason of the increment in the specific heat. Feeding sponge iron pellets with high porosity represents more energy expenses to melt the particles because this type of pellets hardly touch the steel bath affecting the process efficiency, since a higher power in the electrodes is needed to remelt the slag shells and prevent a decrement in the bath temperature.

Previously it was also found that the thickness of the frozen shell in porous (low density) DRI particles decreases due to a lower thermal conductivity, enhancing the melting rate of DRI. However with low density DRI more pellets are involved in the melting process in contrast to high-density DRI particles and then the amount of solidified slag per ton is higher and the energy required to melt a ton of porous DRI is higher, as shown in Fig. 10.

(iv) Effect of the physical properties of the slag: thermal conductivity, density and specific heat

Effect of the slag thermal conductivity

Variations in the slag thermal conductivity do not have significant changes in the melting time of DRI pellet, as shown in Fig. 11(a). It is also shown that by increasing the thermal conductivity, the shell grows quickly to reach a maximum and then rapidly melts back. In other words, by decreasing the thermal conductivity in the slag, the solidification and melting of the shell is slower and the size of the shell is smaller, although in both cases the total melting time of the pellet is very similar. A higher conductivity, involves a higher thermal shock and consequently the shell grows faster and larger, but simultaneously it melts faster than those pellets with lower conductivity.

Effect of slag density

The melting time of DRI particles increases when both the slag density and slag heat capacity increase, as shown in Fig. 11(b). If the density of the slag is larger, bigger shells are formed, affecting the kinetics of fusion of the particle. Similarly, if the specific heat of the solid slag changes, the melting time of the pellet is affected, since an increment in the specific heat implies more energy to remelt the slag.

(v) Fluid-dynamic conditions: bath temperature, initial pellet temperature and heat convective transfer coefficient

Other important factors influencing the melting kinetics are the fluid-dynamic conditions of the slag bath. If the bath is well agitated, particle melting can be improved dramatically compared to the melting rate in a quiescent bath. This section provides an analysis on how these parameters affect the melting kinetics of sponge iron.

Initial temperature of sponge iron and slag bath

The effect DRI pellet initial temperature can be seen in Fig. 12. This figure shows the estimated melting time for a particle of 5 mm diameter fed in the slag bath at initial temperatures of 298, 598 and 1 758 K, which are 245, 220 and 100 seconds, respectively (discontinuous lines in Fig. 12). When the initial temperature of the pellet is low, the formation stage of the shell has a dominant effect on the melting process of the particle and the heat transferred from the bath is used for melting this shell. Figure 13 shows that the melting time decreases linearly by increasing the initial temperature of the pellet. According to the results, an increment from 298 K to 998 K in the initial pellet temperature promotes energy savings of around 20%. The technology to charge hot DRI is already available and in those cases where cold DRI is produced it is worth to explore the re-use of the
sensible heat of the off-gases to preheat the DRI.

In Fig. 12 it can be seen the evolution of the pellet radius with time for different temperatures of the metallic bath (continuous lines). Heat transfer to the center of the pellet is enhanced when the temperature of the slag bath is high; therefore, it is desirable that the metal bath is maintained at temperatures above of the pellet melting temperature. The pellet melting time decreases by increasing the temperature of the slag bath. The melting time has a behavior similar to an exponential decay when the slag bath temperature increases as it can be seen in Fig. 13, which indicates the bath temperature is an important parameter and suggests that pellets must be injected near the delta of the electrodes where the molten bath is hotter. It is important to take into account that in industrial furnaces there are large thermal gradients between the hot areas directly affected by the electrodes (center) and by the surrounding areas near the cold walls.

Convection heat transfer coefficient

In real operating conditions in the EAF there is some agitation due to various means such as injection of oxy-fuel jets impinging on the bath, by CO gas evolving when the bath is decarburized or even by electromagnetic stirring in DC-EAF, however, the general stirring conditions in the molten bath are poor. With poor stirring conditions, heat transfer from the molten bath to the solid particles is decreased. Previous research has reported a range of values for the convective heat transfer coefficient (h) in the case of melting of sponge iron immersed in slag under natural convection between 125.5 and 210 W/m² K.20) above this range of values forced convection is considered.

When the slag bath agitation is intense, the forced convection between the bath and the boundaries of the pellet causes a more a faster heat being transferred from the bath to the pellet, reducing the melting process, as shown in Fig. 14(a). Although the turbulent fluid-dynamic bath conditions reduce the time of existence of the shell, this stage still exists. A moderate increase in the agitation of the slag increases the heat transfer coefficient from 300 W/m² K to 900 W/m² K and reduces by half the melting time. When the heat transfer coefficient increases, it causes a drastic change in the melting time of the pellet, this change is even more noticeable when the initial radius of the pellet is small. In Fig. 14(b) it can be seen how the melting time decreases by increasing h. Melting time for pellets with an initial radius of 5, 10 and 20 mm are displayed in Fig. 14(b). It is appreciated that values of the coefficient of heat transfer greater than 1 000 W/m² K does not further reduce the melting time. In order to reach high stirring conditions and large values of the heat transfer coefficient, in addition to the small evolution of CO due to a chemical reaction between FeO and C contained in DRI particles and the electromagnetic forces around the delta region, stirring can be improved with higher rates of oxygen injection for decarburization and specially with bottom gas injection.

4. Conclusions

An improved mathematical model that can reproduce the kinetics of melting of a pellet of sponge iron immersed in a slag bath was successfully developed. The model is robust as it was validated with experimental and analytical results existing in the literature. According to the model,

- Is desirable to feed relatively larger pellets to reduce power consumption of the furnace per ton of steel and make the process more efficient
- Pellets with high porosity require a higher number of particles per ton and consequently there are larger amounts of solidified slag per ton of steel, resulting in a slow and more energy consuming process.
- When the bath temperature of slag and the initial temperature of the sponge iron are increased, the melting times

![Fig. 13. Melting time versus the initial temperature of the pellet and versus the temperature of the slag bath.](image1)

![Fig. 14. (a) Evolution of the radius of the pellet under different values of heat transfer coefficient and (b) Melting time variation changing the heat transfer coefficient for three initial radius of pellet.](image2)
of the particles are reduced.

- If the thermal conductivity, density and specific heat of the slag are reduced, significant energy savings can be achieved.
- If the forced convection between the bath and the particle is employed (incrementing the heat transfer coefficient) to obtain an average heat transfer coefficient of around 1 000 W/m²K, the melting time and the size of the shell decreases drastically and then the savings in time (productivity) and energy in the process (operational cost) will be significant.

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Nomenclature

- \( A \): Input heat flow area (m²)
- \( Cp \): Specific heat (J/Kg K)
- \( f_{int} \): Interpolation factor
- \( Fo \): Fourier number
- \( h \): Heat transfer coefficient (W/m² K)
- \( h_s \): Heat transfer coefficient in interface pellet-slag shell
- \( n \): Constant to compute the effective thermal conductivity of the pellet
- \( P_f \): Fraction of porosity
- \( Q_{tot} \): Heat total contributed to the particle
- \( r \): Radial coordinate
- \( R \): Radius of the sphere at time \( t \) (mm)
- \( R_0 \): Radius of the sphere at time \( t = 0 \) (mm)
- \( R_{max} \): Maximum radius of the sphere at time \( t \) (mm)
- \( t \): Time (s)
- \( T \): Temperature (K)
- \( T_{bs} \): Sphere temperature at time \( t = 0 \) (K)
- \( T_b \): Metallic bath temperature (K)
- \( t_c \): Time to form and remove the shell formed (s)
- \( T_r \): Temperature in radial position at a time \( t_c \) (K)
- \( T_m \): Melting temperature of the metal (K)
- \( V_i \): Volume of element \( i \) (m³) in Fig. 1(a)
- \( X_i \): Weigth fraction of component \( i \) in the pellet
- \( Y_i \): Volume fraction of component \( i \) in the pellet

Greek

- \( \alpha \): Thermal diffusivity (m²/s)
- \( \Delta \): Interface position (m)
- \( \Delta F_0 \): Delta Fourier
- \( \Delta H \): Latent heat of melting (J/kg)
- \( \Delta r \): Discretization size in the r coordinate
- \( \Delta t \): Time step (s)
- \( \lambda \): Thermal conductivity (W/m K)
- \( \rho \): Density (kg/m³)

Subindex

- \( 0 \): Initial condition
- \( 0 \) DRI: Initial DRI condition
- \( i \): Component properties
- \( b \): Bath
- \( s \): Substrate of main substance
- \( SLAG \): Slag
- \( SLAG\text{-int} \): Slag-DRI interface on the DRI side
- \( DRI\text{-int} \): At the slag-DRI interface on the DRI side

REFERENCES