Note

A Simple Mathematical Model for Estimating Plume Hydrodynamics of Metallurgical Ladles

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In secondary steelmaking, the refining operation is typically carried out in a metallurgical ladle that is commonly furnished with gas injection facilities. Since gas injection plays an intrinsic role in determining the efficiency of secondary steelmaking and thus the quality of final products, a great volume of work has been conducted mostly utilizing air-water systems at room temperature. Despite a portion of the air-water correlations have been adopted in the literature, their applicability to the real argon-metal system is still questionable. The main motivation behind this paper is to present a simple mathematical model for plume hydrodynamics of metallurgical ladles based on the characteristic phenomena and underlying mechanisms of a buoyant plume, where bubble breakup and coalescence occur simultaneously. The main assumptions/simplifications and governing equations are firstly introduced. After that, the accuracy of the model is demonstrated by comparing predicted plume velocities with the ones measured in an industrial ladle.

KEY WORDS: metallurgical ladle; gas injection; buoyant plume; bubble breakup/coalescence.

1. Introduction

The widespread use of continuous casters and the ever-increasing demand for high-quality products in the steel-making industry require the molten product, *i.e.*, liquid steel to be precisely controlled in terms of temperature, composition and cleanliness. In order to fulfill these requirements, many refining processes, which are also named as secondary steelmaking or ladle metallurgy, have been developed and some have rapidly gained their popularity over the years. The refining operation is typically carried out in a metallurgical ladle that is commonly furnished with gas injection and/or vacuum facilities. During the operation, an inert (usually argon) gas is injected into the melt through single/multi porous plugs or nozzles located at the ladle bottom. The rising gas eventually shatters into numerous bubbles and thus induces a vigorous turbulent recirculating flow in the bath, providing a high level of bath mixing for accelerating a host of heat and mass transfer controlled processes, *e.g.*, adjustment of chemical composition, enhancement of gas-metal and slag-metal reactions, temperature homogenization and inclusion removal.¹⁾

Since gas injection plays an intrinsic role in determining

the efficiency of secondary steelmaking and thus the quality of final products, a great volume of work has been conducted on different aspects of the gas-liquid system in order to clarify the characteristic phenomena and underlying mechanisms. In general, the relatively large scale, high visual opacity and operating temperature preclude direct observations and examinations on an industrial ladle and therefore, most of the pertaining work lends itself to physical modeling involving air-water systems at room temperature and numerical modeling utilizing CFD techniques. It has been revealed that the gas-liquid mixture core caused by bottom gas injection can be subdivided into four physically distinct zones along the axial direction, *i.e.*, primary bubble, free bubble, plume and spout.^{1–3)} Of these, the spout regime appears within top 3 to 4% of the static bath height and the influence of kinetic energy embodied by the injected gas is of significance only in the primary and free bubble regimes. Anagbo and co-workers³⁾ were probably the first who quantitatively demarcated the depth of each regime by deriving a dimensionless dispersion group on the basis of extensive data from various sources. The authors also implied that under practically (moderate) gas flow rates adopted in ladle metallurgy, the plume regime that is solely governed by buoyancy due to the large density difference occupies most of the bath height. This essentially proves that the assumption of buoyant plume in metallurgical gas-stirred ladles is fundamentally reasonable and the plume hydrodynamics has then become the subject of considerable interest, where extensive attention has been paid to measure the local structures in terms of gas fraction, bubble shape and size, and liquid and gas velocities.

The thorough investigations of air-water system gave rise to a number of useful (empirical) correlations. In the literature, a portion of the correlations have been adopted, particularly in numerical studies focused on the real argon-metal system, for evaluating some integral condition and/or parameters, *e.g.*, gas fraction at the inlet and characteristic bubble diameter used for computing the drag force. The state-of-the-art schemes for numerical modeling of metallurgical ladle phenomena can be categorized into Eulerian-Lagrangian and Eulerian-Eulerian approaches.³⁾ It must be noted that the coupling terms between the phases in question are crucial for both approaches and a variety of constitutive equations and coefficients for them have been applied. This is somewhat ambiguous and implausible since there exist no criteria for choosing appropriate constitutive equations and coefficients of the terms. Also, the feasibility and rationality of a proposed CFD model is probably more attributed to the joint adjustment of the term coefficients that typically differ from one model to another and therefore, the correlations based on air-water system are of minor importance.

Figure 1 further illustrates the comparisons between measured data of average plume velocity in a 60-tonne ladle by Hsiao and co-workers⁴⁾ and the corresponding predicted ones using the correlations (for gas fraction, gas and liquid velocities at plume centerline) by Krishnapisharody and Irons,^{5,6)} who recently suggested a ‘plume’ Froude number for guaranteeing full similarity between a water model and its prototype. The average plume velocity in Fig. 1 is calculated as $\alpha U_g + (1 - \alpha)U_l$, where α , U_g and U_l are the gas fraction, gas (bubble) velocity and liquid velocity, respectively. In addition, the gas volumetric flow rates are calculated for conditions at bath mid-height.

It can be seen clearly in Fig. 1 that the measured velocities are generally underestimated despite their variations along the bath height are correctly predicted by the empirical correlations, which indicates that the applicability of the air-water correlations to the real system is still questionable. Therefore, the need for a more generic and accurate approach to characterizing plume hydrodynamics in metallurgical ladles becomes readily apparent. This is

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Fig. 1. Comparisons between measured data of average plume velocity and the predicted ones.

basically one of the motivations behind the present work. The emphasis of the current paper is on presenting a simple mathematical model that can be employed for estimating the plume hydrodynamics in terms of gas fraction, gas and liquid velocities, interfacial area concentration and plume width. The principle of the model is based on the characteristic phenomena and underlying mechanisms of a buoyant plume, where bubble breakup and coalescence occur simultaneously. The main assumptions/simplifications and governing equations are firstly introduced in the following text and after that, the model is demonstrated and verified using the measured data plotted in Fig. 1.

2. Model Description

2.1. Assumptions and Simplifications

Figure 2 shows a simplified outline of the hydrodynamics within a gas-stirred metallurgical ladle. As argon is injected into liquid steel, it expands quickly due to the sudden change of pressure and temperature. The resulting gas envelopes break into numerous bubbles over a short distance (*i.e.*, the primary and free bubble zones) and the bubbles proceed vertically upwards, forming the so-called buoyant plume. In parallel, liquid steel around the bubbles is accelerated by the bubbles to move upwards and this motion can be transferred to the steel bulk mainly by lateral turbulent fluctuations, leading to a recirculating flow in the bath. When the bubble-steel mixture reaches bath surface, all bubbles break into the atmosphere and the momentum of the liquid stream is converted into a radially spreading surface flow (*i.e.*, the spout zone). In order to necessitate a mathematical model for describing the aforementioned sophisticated process, the following assumptions and simplifications are made.

a) The bath flow is steady and the plume is axially symmetric and non-rotating. Cylindrical coordinates are therefore applied.

b) Temperature and pressure of bubbles are in equilibrium with the surrounding melt and there exist no mass transfer between the two and no temperature stratification along the bath height. In addition, the effect of the overlying slag and spout zone is neglected.

c) As for liquid or gas velocity in the plume, the radial component is much smaller compared to the axial one and is thus neglected. It is assumed that the axial gas velocity is the sum of the axial liquid velocity (u_l) and the gas-liquid slip velocity (u_s), which is nearly constant along the axial direction.

d) Radial profiles of axial liquid velocity and gas fraction are assumed to be Gaussian.^{1,2)} The width ratio in Fig. 2 (*i.e.*, σ), which is principally equivalent to the turbulent Schmidt number, is a constant.⁷⁾ The velocity profile width at the origin point, w_{v0} , is proportional to the initial height of

Fig. 2. Schematic of hydrodynamics within a gas-stirred metallurgical ladle.

primary and free bubble zones, *i.e.*, h_0 in Fig. 2. The initial height is defined as the distance along which gas fraction rapidly decreases from approximately unity to a critical value, which is usually set as 0.5 in the literature.

2.2. Conservation Equations

The governing laws involved in the current model are the continuity of gas, continuity of liquid and momentum conservation of gas-liquid mixture. The derivation is summarized as follows.

The continuity of gas at a specific level of z in the bath can be written as

$$\int_0^R (u_l + u_s) \alpha 2\pi r dr = Q_{\text{exit}} \frac{P_{\text{top}} + \rho_l g H}{P_{\text{top}} + \rho_l g (H - z)} \quad \dots\dots\dots (1)$$

where R , Q_{exit} , P_{top} , ρ_l , g and H are the bath radius, gas volumetric flow rate at the injector exit, pressure at the top, density of liquid steel, gravity and static bath height, respectively. The Gaussian forms of axial liquid velocity and gas fraction are

$$u_l = u_{l,c} \exp\left(-\frac{r^2}{w_v^2}\right); \alpha = \alpha_c \exp\left(-\frac{r^2}{(\sigma w_v)^2}\right) \quad \dots(2a, b)$$

where $u_{l,c}$ and α_c are the centerline velocity and gas fraction.

Substitution of Eqs. (2a) and (2b) into Eq. (1) followed by integration and rearrangement, it gives

$$\pi \sigma^2 w_v^2 \alpha_c \left(u_s + \frac{u_{l,c}}{1 + \sigma^2}\right) = Q_{\text{exit}} \frac{P_{\text{top}} + \rho_l g H}{P_{\text{top}} + \rho_l g (H - z)} \quad \dots\dots\dots (3)$$

For a cylindrical control volume with a depth of Δz (cf. Fig. 2), the increase of liquid steel along the depth is equal to the amount of liquid steel entrained by lateral turbulent fluctuations, as expressed by

$$d \left[\int_0^R \rho_l u_l (1 - \alpha) 2\pi r dr \right] = \rho_l U_t 2\pi w_v \Delta z \quad \dots\dots\dots (4)$$

With the limit of $\Delta z \rightarrow 0$, it reaches

$$\frac{d}{dz} \left[\int_0^R \rho_l u_l (1 - \alpha) 2\pi r dr \right] = \rho_l U_t 2\pi w_v \quad \dots\dots\dots (5)$$

where U_t is the mean lateral velocity stem from lateral turbulent fluctuations.

It is assumed here that the ratio of U_t to $u_{l,c}$ is equal to the one of bath recirculating velocity (U_r) to the average plume velocity (U_p), as expressed by

$$\frac{U_t}{u_{l,c}} = \frac{U_r}{U_p} \quad \dots\dots\dots (6)$$

Substitution of Eqs. (2a), (2b) and (6) into Eq. (5) fol-

lowed by integration and rearrangement, the following equation can be obtained

$$\frac{d}{dz} \left[w_v^2 u_{l,c} \left(1 - \frac{\sigma^2 \alpha_c}{1 + \sigma^2} \right) \right] = 2 \frac{U_r}{U_p} u_{l,c} w_v \dots (7)$$

For detailed description of U_r and U_p , the reader is referred to the comprehensive compilation by Mazumdar and co-workers.^{8,9)}

For the same control volume, the growth in momentum of gas and liquid is balanced by the net buoyancy induced by bubbles, as expressed by

$$\frac{d}{dz} \left[\int_0^R (\rho_l u_l^2 (1 - \alpha) + \rho_g (u_l + u_s)^2 \alpha) 2\pi r dr \right] = \int_0^R (\rho_l - \rho_g) \alpha g 2\pi r dr \dots (8)$$

where ρ_g is the gas density.

Considering $\rho_g \ll \rho_l$, the terms containing gas density can be neglected and Eq. (8) becomes

$$\frac{d}{dz} \left[\int_0^R \rho_l u_l^2 (1 - \alpha) 2\pi r dr \right] = \int_0^R \rho_l \alpha g 2\pi r dr \dots (9)$$

Substitution of Eqs. (2a) and (2b) into Eq. (9) followed by integration and rearrangement, the momentum conservation can be transformed into

$$\frac{d}{dz} \left[w_v^2 u_{l,c}^2 \left(0.5 - \frac{\sigma^2 \alpha_c}{1 + \sigma^2} \right) \right] = \sigma^2 w_v^2 \alpha_c g \dots (10)$$

In addition to the above equations, the interfacial area concentration model proposed by Hibiki and Ishii¹⁰⁾ is employed to include the effect of bubble breakup and coalescence on bubble behaviors in the plume. The model has been thoroughly described by the authors and only the breakup (S_B) and coalescence (S_C) kernels are outlined below.

$$S_B = 3.7 \times 10^{-4} \frac{(1 - \alpha) \eta^{1/3} a^{5/3}}{(0.75 - \alpha) \alpha^{2/3}} \exp \left[-0.07 \frac{\phi}{\rho_l \eta^{2/3}} \left(\frac{a}{\alpha} \right)^{5/3} \right] \dots (11)$$

$$S_C = 2.6 \times 10^{-4} \frac{\eta^{1/3} \alpha^{1/3} a^{5/3}}{(0.75 - \alpha)} \exp \left[-0.57 \frac{\rho_l^{1/2} \eta^{1/3}}{\phi^{1/2}} \left(\frac{a}{\alpha} \right)^{5/6} \right] \dots (12)$$

where ϕ and a are the liquid surface tension and interfacial area concentration. The stirring power per unit mass of liquid steel (η) is given as

$$\eta = 8.314 \frac{n}{m} T_{\text{bath}} \ln \left(\frac{P_{\text{top}} + \rho_l g H}{P_{\text{top}}} \right) \dots (13)$$

where n , m and T_{bath} are the molar flow rate of gas, mass of liquid steel and (average) bath temperature.

It is worth noting that the characteristic bubble size at plume centerline can be obtained by

$$d_b = \frac{6\alpha_c}{\alpha_c} \dots (14)$$

Therefore, the slip velocity can be calculated using the correlation suggested by Szekely¹¹⁾

$$u_s = 1.02 \left(\frac{d_b}{2} g \right)^{0.5} \dots (15)$$

2.3. Model Parameters and Results

The aforementioned equations are solved adopting an iterative algorithm based on the Runge-Kutta method. As for boundary conditions at the origin point, the starting bubble size is calculated using the correlation by Sano and co-workers¹²⁾ for the detachment size of bubbles in molten iron and mercury. Moreover, as a first approximation, the starting liquid velocity and its profile width are suggested as¹³⁾

$$u_{l,c0} = \left[\frac{25gQ_{\text{exit}}(1 + \sigma)^2}{24\pi h_0(U_r/U_p)^2} \right]^{1/3}; w_{v0} = \frac{6}{5} \alpha_0 h_0 \dots (16a, b)$$

Fig. 3. Comparisons between measured average plume velocities and the ones predicted by the present model.

Providing that the critical gas fraction α_0 is 0.5, the starting parameters, i.e., h_0 , $u_{l,c0}$ and a_0 , can therefore be determined iteratively.

The model is then applied to the industrial ladle mentioned above and for the calculation, the bath temperature, density and surface tension of liquid steel are set as 1 898 K, 7 000 kg/m³ and 1.7 N/m, respectively. The width ratio (σ) is set as 0.7. The comparisons between the measured velocities and the ones predicted by the present model are illustrated in Fig. 3. It is shown in the figure that all sets of the measured data can be approximated by the predicted curves to a satisfactory extent, thus confirming the validity of the model established.

3. Summary and Future Prospects

A simple mathematical model for plume hydrodynamics of metallurgical ladles has been built on the basis of the characteristic phenomena and underlying mechanisms of a buoyant plume. The advantage of the present model over the air-water correlations has been fairly demonstrated by comparing predicted plume velocities with the ones measured in an industrial ladle. As the model provides a fundamental framework for further development, more capabilities, e.g., gas purging for both hydrogen and nitrogen, slag-metal reaction for desulfurization and bubble dynamics under elevated and reduced pressures, will be considered. Also, the aforementioned versatilities of the model will be verified by both experimental work in laboratory and industrial trials.

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