Effects of Carbide on the Cooperative Growth of Pearlite in Carbon Steel: A Phase Field Study

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In this article, effects of carbide on the cooperative growth of pearlite have been explored based on a multi-phase field method. Optical microscopy observations of pearlite structure in eutectoid steels have been taken to validate the accuracy of the developed multi-phase field method. Besides, the simulation results are compared with the experimental results of Ridley and Frye, as well as the phase field simulation results of Steinbach, a good consistency has been found. Comparison results demonstrate that the growth morphology of pearlite is more accurate when the effects of carbide are considered. Influences of the carbide location and size have been considered to further analyze the cooperative growth of pearlite. Results demonstrate that the presence of carbide above the lamellar pearlite changes the growth situation by promoting the growth of cementite lamellar pearlite when it is just above lamellar cementite. Lamellar cementite in the vicinity of carbide easily melts off with the growth of lamellar pearlite, and the lamellar cementite evolves to form islands. Moreover, effects of different carbide diameter on the growth of lamellar pearlite have been explored. The final growth rate and growth morphology are basically the same despite variations in diameter. Relationship, which can be used to control the morphology and performance of lamellar pearlite, between the size of the carbide and the horizontal maximum bending distance have been obtained through statistical analysis of the transverse maximum bending distance of the lamellae.

KEY WORDS: carbon steel; carbide; cooperative growth; phase field.

1. Introduction

Carbon steel is widely used in the machinery industry because it has a simple smelting process and becomes stronger with heat treatment. Ultimately, the performance of carbon steel is determined by microstructure. One of its key microstructures, pearlite, can have either lamellar or spherical compositions, which produce considerable differences in performance. Given this flexibility, carbon steel can meet different technological requirements by controlling the different structures of pearlite. Cooperative growth is the main growth mechanism of lamellar pearlite, and spherical cementite is the main component of its globular structure. In order to understand pearlite’s growth mechanism and control its microstructure, the effect of spherical carbide on the cooperative growth of pearlite would be researched.

Scholars have conducted extensive research\(^1\) on the cooperative growth of lamellar pearlite in recent years. At present, the research methods include CA and phase field. However, description of the phase interface is simple and clear, and Ginzburg Landau theory has been identified as the physical basis of the phase field method. Therefore, the phase field method has become one of the main research methods. The cooperative growth mechanism mainly consists of the eutectic solidification of the phase transition from liquid to solid and the eutectoid growth of the solid phase transformation. Research into eutectic growth has gained a relatively advanced status, and eutectoid growth is derived from the theory and equations that underlie eutectic growth. In 2000, Nestler et al.\(^6\) established the multi-phase field model of eutectic and peritectic process based on the WBM model by extending the single-phase WBM model to the multi-phase field. In 2004, Kim et al.\(^7\) proposed the KKSM multi-phase model based on early stages of the single-phase KKS model\(^8\) and the interface field model of Ingo Steinbach.\(^9\) The equal concentration hypothesis of the WBM phase field model transits to the equal potential hypothesis of the KKS phase field model. Yang et al.\(^10,11\) analyzed the effect of lamellar thickness on two-dimensional and three-dimensional eutectic lamellar growth. In 2011, Choudhury et al.\(^12\) studied three-phase growth of the ternary alloy through theoretical calculation and numerical simulation. In 2015, Catalina et al.\(^13\) proposed a new eutectic growth model combining multi-component and comparative analysis with classical theory.

These studies of eutectic growth laid the foundation for research into eutectoid growth. In 2006, Nakajima et al.\(^14\) simulated eutectoid growth by considering the diffusion of carbon in the austenite and ferrite phases; they compared
their results to simulations that considered diffusion of carbon only in the austenite phase, and found the growth rate of the former to be four times that of the latter. In 2007, Steinbach et al.\textsuperscript{15} proposed an anisotropic facet model and obtained a “staggered growth” form in which the growth of cementite precedes the growth of ferrite. They found that predictions of the growth rate were very close to the experimental results. In 2012, Steinbach et al.\textsuperscript{16} analyzed pearlite in an article incorporating the cooperative growth model of Zener-Hillert, the nucleation model of cementite in hypoeutectoid steel, and pearlite growth under stress action. In 2013, Kumar et al.\textsuperscript{17} extended the Jackson-Hunt-type calculation to eutectoid transformation, considering diffusion in all phases and simulating the formation of tapered cementite through the phase field method. Ankit et al.\textsuperscript{18–20} researched the growth mechanism of austenite transformed into pearlite in the presence of a large amount of pro-eutectoid cementite. Their results shed light on the influence of austenite grain size, pro-eutectoid cementite particles, and the size of the ferrite area on non-cooperative growth. Using the Avrami equation, Caballero et al.\textsuperscript{21} established nucleation and growth models of pearlite-to-austenite transformation through isothermal experiments under different temperatures. Aranda et al.\textsuperscript{22} studied the effect of austenitic grain size on pearlite transformation through experimental analysis.

Previous studies mainly focused on simulating pearlite’s internal growth mechanisms, such as diffusion and lamellar spacing, without considering the effect of carbide on the cooperative growth of pearlite. However, practical applications often involve a great deal of pro-eutectoid cementite, which can promote or inhibit the growth of lamellar pearlite. Because pro-eutectoid cementite is found in compounds, it bears similarities to carbon-containing compounds. Therefore, this paper analyzes the effects of different locations and sizes of carbide on the cooperative growth of pearlite.

2. Experimental Procedure

Metallography experiment specimens, with the size of 15 mm×15 mm×15 mm, were cut from the commercial steel T8, composition of the steel shown in Table 1. The samples were austenitized at 1 273 K for half an hour subsequently isothermally transformed at 970 K for 5 min, 15 min, 30 min or 45 min, and followed by quenching into water at 293 K. 2–3 samples would be treated for each scheme. Specimens were ground and polished using standardized techniques. Then a 4% nitric acid alcohol solution was used to disclose pearlite in specimens.

3. Numerical Simulation

3.1. Multi-phase Field Model

The multi-phase field model\textsuperscript{1,23} has matured through extensive study. Steinbach’s model,\textsuperscript{24} which uses “double obstacle” potential energy, can describe finite interface thickness better than the model that uses “double well” potential energy. Therefore, this paper adopts Steinbach’s model.

The total free energy function is the key point to the multi-phase field model in the simulation area. Expression of the total free energy function used in this paper considers both interfacial energy density \( f_{\text{int}} \) and chemical energy density \( f_{\text{chem}} \), as shown in Eq. (1). The functions of interfacial energy density and chemical energy density are shown in Eqs. (2) and (3).

\[
F\left(\{\phi_\alpha\}, \{\tilde{c}_\alpha\}\right) = \int_\Omega \left[ f_{\text{int}}\left(\{\phi_\alpha\}\right) + f_{\text{chem}}\left(\{\phi_\alpha\}, \{\tilde{c}_\alpha\}\right)\right] d\Omega, \tag{1}
\]

\[
f_{\text{int}}\left(\{\phi_\alpha\}\right) = \sum_{a=1}^{N} \sum_{\beta=1}^{N} \left[ \frac{4\sigma_{\alpha\beta}}{\eta} \nabla \phi_\alpha \cdot \nabla \phi_\beta + \phi_\alpha \phi_\beta \right], \tag{2}
\]

\[
f_{\text{chem}}\left(\{\phi_\alpha\}, \{\tilde{c}_\alpha\}\right) = \sum_{a=1}^{N} \phi_a f_a \left(\tilde{c}_a\right) + \lambda \left[ c - \sum_{a=1}^{N} \left(\phi_a c_a\right) \right], \tag{3}
\]

where \( \phi_a \) and \( \phi_\beta \) are field variables of different phases \( \alpha \) and \( \beta \), or field variables of different orientations but the same phase. They further satisfy the constraints of Eq. (4). \( \eta \) is the interface thickness, \( \sigma_{\alpha\beta} \) is the interface energy between \( \alpha \) and \( \beta \), \( f_a \left(\tilde{c}_a\right) \) is the volume free energy density of phase \( \alpha \), \( \lambda \) is the Lagrange multiplier. Since this paper studies the Fe-C binary system, here \( c \) is the concentration of carbon, \( c_a \) is the concentration of carbon in phase \( \alpha \), and \( N \) is the number of phases.

The total free energy function is substituted into the standard multi-phase equation represented in Eq. (5), which allows for Eqs. (6) and (7) to be obtained.

\[
\dot{\phi}_a = \frac{1}{N} \sum_{b=1}^{N} \nabla \psi_{a\beta} = \sum_{b=1}^{N} \mu_{a\beta} \left[ \frac{\delta}{\delta \phi_\beta} - \frac{\delta}{\delta \phi_\beta} \right] F_{a\beta}, \tag{5}
\]

\[
\psi_{a\beta} = \mu_{a\beta} \left[ \sigma_{a\beta} \left( I_a - I_\beta \right) + \sum_{y=1}^{N} \left( \sigma_{y\beta} - \sigma_{a\gamma} \right) I_y \right] + \frac{\pi^2}{4\eta} \Delta \tilde{\phi}_{a\beta}, \tag{6}
\]

\[
I_a = \left\{ \nabla^2 \phi_a + \frac{\pi^2}{\eta} \delta \phi_a \right\}, \tag{7}
\]

\[
\mu_{a\beta} = \hat{\mu}_{a\beta} \left( 4\eta / \pi^2 \right), \tag{8}
\]

\[
\Delta \tilde{\phi}_{a\beta} = \Delta S_{a\beta} \left[ T' + m_{a\beta} \left( c_a - c'_a \right) - T \right]. \tag{9}
\]

Where \( \psi_{a\beta} \) is the interface field between \( \alpha \) and \( \beta \), and \( \mu_{a\beta} \) is the interface mobility between \( \alpha \) and \( \beta \). \( \Delta S_{a\beta} \) is the transformation entropy, \( T' \) is the reference temperature, \( m_{a\beta} \) is slope of a corresponding line in phase diagram, and \( c'_a \) is the concentration of carbon in phase \( \alpha \).
is the concentration at the reference temperature $T'$. $T$ is transformation temperature. The diffusion equation is used to calculate the concentration field. Incorporating Eqs. (10) and (11), the concentration field equation\footnote{ISIJ International, Vol. 57 (2017), No. 10} shown in Eq. (12) can be obtained:

\[
c = \sum_{a=1}^N \phi_a c_a, \quad \dot{c} = \sum_{a=1}^N \phi_a D_a \nabla c_a.
\]

Where $k_{\alpha\beta}$ represents the ratio of the concentration between $\alpha$ and $\beta$. This value is determined by the concentration ratio at the eutectoid transition temperature. $M$ is the chemical mobility and $D_\alpha$ is the diffusion coefficient in phase $\alpha$.

### 3.2. Computational Conditions and Procedure

In this paper, the Fe–C binary alloy system is studied, and the thermodynamic data for the Eq. (9) can be obtained from the phase diagram. Here, an approximation line is used instead of the transformation line in the phase diagram (such as Fig. 1). The thermodynamic parameters are shown in Table 2.

In this paper, effects of both the locations and sizes of the carbide on the cooperative growth of pearlite have been analyzed, details of the location determination and size determination have been given bellow:

![Fe–C binary phase diagram.](image)

**Fig. 1.** Fe–C binary phase diagram.

![Schematic diagram of different locations for carbide.](image)

**Fig. 2.** Schematic diagram of different locations for carbide (the simulated domain width is four times the lamellar spacing in the actual simulation).

### 3.2.1. Location Determination of Carbide

**Figure 2** shows a schematic diagram of different locations for carbide. It does not set different locations for carbide in the vertical direction. This is mainly due to the fact that greater distances between carbide and the forefront of lamellar pearlite correspond with longer times for independent growth, which may cause the size of carbide to exceed the lamellar thickness. Moreover, pearlite grows vertically, such that setting different height locations on the cooperative growth of pearlite in the vertical direction has very little effect.

The amount of mesh was $180,000$ ($300 \times 600$) in the actual simulation. The mesh size was $4 \times 10^{-5} \text{m}$, and the initial lamellar spacing was $0.30 \times 10^{-6} \text{m}$. Meanwhile, the initial lamellar spacing of ferrite $\alpha$ was $S_{\alpha} = 0.25 \times 10^{-6} \text{m}$, and the initial lamellar spacing of cementite $\beta$ was $S_{\beta} = 0.05 \times 10^{-6} \text{m}$. The initial lamellar height was $4 \times 10^{-8} \text{m}$. The initial radius of carbide was $R = 0.032 \times 10^{-6} \text{m}$. The undercooling was 30°C. The initial concentration of austenite was 3.6 mole%, while that of ferrite was 0.12 mole%. Lamellar and carbide had the same initial concentration of 25 mole%. Carbide also grows during the process of eutectoid growth. Table 3 shows the three kinds of locations.

**Table 2.** Thermodynamic parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\alpha}$</td>
<td>1 185 K</td>
<td>$m_{\beta\gamma}$</td>
<td>– K/mol</td>
</tr>
<tr>
<td>$T_{\beta}$</td>
<td>2 615 K</td>
<td>$m_{\alpha\beta}$</td>
<td>1 500 K/mol</td>
</tr>
<tr>
<td>$T_{\alpha\gamma}$</td>
<td>38 350 K</td>
<td>$\Delta S_{\alpha\gamma}$</td>
<td>$-3.5 \times 10^{8}$ J/Kgm$^3$</td>
</tr>
<tr>
<td>$T_e$</td>
<td>1 000 K</td>
<td>$m_{\alpha\beta}$</td>
<td>0 K/mol</td>
</tr>
<tr>
<td>$m_{\gamma\beta}$</td>
<td>$-53.47$ K/mol</td>
<td>$\Delta S_{\beta\gamma}$</td>
<td>$-11.1 \times 10^{7}$ J/Kgm$^3$</td>
</tr>
<tr>
<td>$m_{\gamma\alpha}$</td>
<td>75.00 K/mol</td>
<td>$c_e$</td>
<td>3.46 mol%</td>
</tr>
</tbody>
</table>

**Table 3.** Locations of carbide.

<table>
<thead>
<tr>
<th>Serial Number</th>
<th>Position</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>$L = 5.76 \times 10^{-7} \text{m}, H = 8 \times 10^{-4} \text{m}$</td>
<td>Nucleus located just above $\beta$</td>
</tr>
<tr>
<td>P2</td>
<td>$L = 6.00 \times 10^{-7} \text{m}, H = 8 \times 10^{-4} \text{m}$</td>
<td>Nucleus located just above interface between $\alpha$ and $\beta$</td>
</tr>
<tr>
<td>P3</td>
<td>$L = 7.24 \times 10^{-7} \text{m}, H = 8 \times 10^{-4} \text{m}$</td>
<td>Nucleus located just above $\alpha$</td>
</tr>
</tbody>
</table>
3.2.2. Size Determination of Carbide

Figure 3 shows a schematic diagram of different diameters for carbide. Because cementite lamellae are thin, the diameters of carbide above them must fall within a small range. Therefore, carbide can only be set right above ferrite lamellae. The three sizes are \( R_1 = 0.016 \times 10^{-6} \text{m} \), \( R_2 = 0.032 \times 10^{-6} \text{m} \), and \( R_3 = 0.048 \times 10^{-6} \text{m} \). The size of \( R_2 \) is equal to the radius of carbide described in Section 3A. Since the actual carbide diameter may be greater than the lamellar spacing, the other three kinds of carbide sizes are added, which are greater than the lamellar spacing. Their sizes are \( R_4 = 0.140 \times 10^{-6} \text{m} \), \( R_5 = 0.180 \times 10^{-6} \text{m} \), and \( R_6 = 0.220 \times 10^{-6} \text{m} \). The amount of meshes was 202,500 (450 × 450). The small-sized carbide’s location was the same as P3 listed in Table 3. The large-sized carbide’s location was following, \( L = 0.9 \times 10^{-6} \text{m} \), \( H = 0.32 \times 10^{-6} \text{m} \). The mesh size and lamellar spacing were the same as described in Section 3A.

This paper uses simulation parameters (shown in Table 4) from the literature, in which \( \alpha \) is ferrite, \( \beta \) is cementite, and \( \gamma \) is austenite.

The simulation process is shown in Fig. 4. Firstly, the initial parameters and initial phase, concentration are needed. The specific values are described above. Then the phase field and the concentration field are calculated, but at each calculation step, boundary conditions need to be rebuilt. When the simulation time reaches the termination time, the simulation will end, otherwise, the boundary will be pro-

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interface energy</td>
<td>( \sigma_{\alpha\beta}, \sigma_{\alpha\gamma}, \sigma_{\beta\gamma} )</td>
<td>1.0 J/m²</td>
</tr>
<tr>
<td>Mobility</td>
<td>( \gamma \beta \mu_{\alpha\beta}/\mu_{\alpha\gamma} )</td>
<td>( 9.0 \times 10^{-16} - 5.0 \times 10^{-15} \text{m}^3/\text{Js} )</td>
</tr>
<tr>
<td></td>
<td>( \gamma \alpha \mu_{\alpha\gamma}/\mu_{\beta\gamma} )</td>
<td>( 9.0 \times 10^{-13} - 2.0 \times 10^{-12} \text{m}^3/\text{Js} )</td>
</tr>
<tr>
<td>Diffusion coefficient</td>
<td>( \beta \sigma_{\alpha\beta} )</td>
<td>Stoichiometry</td>
</tr>
<tr>
<td></td>
<td>( \gamma \sigma_{\alpha\gamma} )</td>
<td>( D_{\alpha\gamma} = 2.2 \times 10^{-4} \text{m}^2/\text{s} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( Q_{\alpha\gamma} = 122.5 \times 10^3 \text{J/mol} )</td>
</tr>
<tr>
<td></td>
<td>( \gamma \sigma_{\beta\gamma} )</td>
<td>( D_{\beta\gamma} = 1.5 \times 10^{-3} \text{m}^2/\text{s} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( Q_{\beta\gamma} = 142.1 \times 10^3 \text{J/mol} )</td>
</tr>
</tbody>
</table>

Fig. 3. Schematic diagram of different diameters for carbide (the simulated domain width is four times the lamellar spacing in the actual simulation).

Fig. 4. Flow chart of simulation.
cessed for the next calculation of phase field and concentration until the time reaches the requirement.

4. Results and Discussions

4.1. Experimental Results and Model Verification

The experimental microstructure at different holding time is shown in Fig. 5. The black and white strip microstructure is pearlite, the gray strip microstructure is martensite. So most of the region in Fig. 5(a) is martensite and the upper small area is pearlite. In Fig. 5(b), most of the area is granular pearlite. In Fig. 5(c), the black material is carbide, but intermittent pearlite is mainly distributed around the carbide and the lamellar structure is not obvious. Moreover, the lamellar spacing varies greatly in Fig. 5(d), this probably leading to grain coarsening due to a long holding time. Simultaneously, it can be found that the transformation volume of microstructure is more and more complete with the extension of the holding time. When the holding time is only 5 minutes, a very small amount of austenite grows into pearlite. The time can be used as a reference on nucleation study. The austenite completely transformed into pearlite in 15–30 minutes from Figs. 5(b) and 5(c). In addition to the study. The austenite completely transformed into pearlite in only 5 minutes, a very small amount of austenite grows into pearlite. The time can be used as a reference on nucleation study. The austenite completely transformed into pearlite in 15–30 minutes from Figs. 5(b) and 5(c). In addition to the

Figure 6 shows the experimental results of pearlite growth in the presence of carbides. When the lamellae encounter carbides, there would be two kinds of phenomena. One is that the lamellae are just below the carbides, it are directly blocked and stopped growing since the diameter of the carbides is larger relative to the lamellar thickness. The other is that the lamellae would bend and bypass the carbide, then continue to grow when the lamellae is at the edge of carbide or at a certain range from the carbide edge. And for the second phenomenon, the thickness of the bent lamellae is slightly thinned and intermittent growth occurs at the maximum diameter of the carbide. In contrast to the lamellar growth of other area, the lamellae show linear growth due to the absence of carbides. And the lamellar spacing is essentially the same as that when the carbide is present.

The stability of the multi-phase field model has been studied by Steinbach et al. In order to verify the accuracy of pearlite’s cooperative growth results considering the effects of carbide, the Young’s equation was used to analyze the triplet point and the extended Jackson Hunt was used to analyze the velocity.

As we all know, the essence of the Young’s equation is force balance at the three-phase junction, for the verification of the proposed model, the interface directions need to be obtained since the force is equal to the interface energy at the three-phase junction. Firstly, three different relations of interface energy are set up and the force relationship at the three-phase junction without carbides is simulated and analyzed. Then the force relationship at the three-phase junction containing carbides is analyzed. Figure 8 shows the interfacial direction at the three-phase junction under three different interface energy at 0.1 s (the interfacial normal are obtained from the program, and then the tangential direction of interface can be calculated, that is, the interfacial direction). The interface relationship of Fig. 8(a) corresponds to the sequence number 1 in Table 5. Fig. 8(b) corresponds to sequence number 2 and Fig. 8(c) corresponds to the sequence number 3. The forces at these three-phase junctions are projected in x and y directions, the resultant force in x and y directions are obtained list in Table 5. It can be found that the error of resultant force in x direction at sequence number 1 is greater than 1%, but the others

![Fig. 5. Microphotograph of sample at different time.](image1)

![Fig. 6. Experimental results of pearlite growth in the presence of carbides.](image2)

### Table 5. The calculation results compared with Young’s equation for the triplet point.

<table>
<thead>
<tr>
<th>Sequence number</th>
<th>The relation of interface energy</th>
<th>Resultant force (the direction of $\sigma_{ij}$ is positive)</th>
<th>Theoretical value (Young’s equation)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\sigma_{xy} = \frac{1}{2} \sigma$, $\sigma_{yz} = \frac{\sqrt{3}}{2} \sigma$, $\sigma_{zz} = \sigma$</td>
<td>$F_x = 0.017$</td>
<td>0</td>
<td>1.7%</td>
</tr>
<tr>
<td>2</td>
<td>$\sigma_{xy} = \frac{1}{2} \sigma$, $\sigma_{yz} = \frac{\sqrt{3}}{2} \sigma$, $\sigma_{zz} = \sigma$</td>
<td>$F_y = -0.003$</td>
<td>0</td>
<td>-0.3%</td>
</tr>
<tr>
<td>3</td>
<td>$\sigma_{xy} = \sigma$, $\sigma_{yz} = \sigma$, $\sigma_{zz} = \sigma$ (no carbide)</td>
<td>$F_x = 0.005$</td>
<td>0</td>
<td>0.5%</td>
</tr>
<tr>
<td>4</td>
<td>$\sigma_{xy} = \sigma$, $\sigma_{yz} = \sigma$, $\sigma_{zz} = \sigma$ (have carbide)</td>
<td>$F_y = -0.002$</td>
<td>0</td>
<td>0.2%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_x = 0.007$</td>
<td>0</td>
<td>0.7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F_y = -0.003$</td>
<td>0</td>
<td>-0.3%</td>
</tr>
</tbody>
</table>
are all less than 1%. The force is in equilibrium within the allowable range of error. And the calculating results can indicate that the model is accurate. The interfacial direction at the three-phase junction containing carbides is shown in Fig. 7 and the resultant force is also listed in Table 5. It can be found the result also meets the Young’s equation within the error range.

According to the extended Jackson Hunt model, 12,17) the relationship17) between lamellar spacing and growth rate in theory can be obtained. The simulation results and the prediction in theory are consistent as shown in Fig. 9. Combined with the previous numerical validation of Young’s equation, the accuracy of the simulation results is valid.

Firstly, the growth morphology would be analyzed. Figure 7 shows the similar growth morphology to that of Fig. 6. The carbide can hinder the growth of lamellae which is underneath it, and it can guide the lamellae which are at the edge of the carbide or at a certain range from the carbide edge bending growth. Simultaneously, the thickness of lamellar cementite in the blending region became narrow, similar to the growth form identified in the experiment.

Secondly, the growth rate would be analyzed. Figure 10 shows the growth rate results of simulation and experiment. As can be seen from the comparison results, the steady growth rate of lamellar pearlite calculated in the simulation show good consistency with the experimental results. The velocity of “no carbide” is different with P1–P3, it indicates that the carbide with different locations have different effects on the growth rate. The carbon content of Ridley's 25) experiment was 0.81 mass%, and the carbon content of Frye’s 26) experiment was 0.78 mass%, both of which were larger than the eutectoid carbon content at 0.77 mass%. So the experiment has closer carbon content to this paper. For Steinbach’s simulation, 14) compared with the velocity of the “no carbide”, the growth rate is low because the mobility and the concentrate are different.

Based on these results, the effect of these factors on the cooperative growth of pearlite was investigated in detail.

4.2 Effect of Location of Carbide

Figure 11 shows that when carbide is at location P1, its diameter exceeds the width of the lamellar cementite. When the two meet, the carbide hinders the lamellar cementite’s growth, preventing \( \beta \) from growing and combining two adjacent lamellar ferrite \( \alpha \) into one lamella. As the pearlite

![Fig. 7. Distribution of concentration and phase field of P1.](image)

![Fig. 8. The interfacial direction at the three-phase junction under three different interface energy at 0.1 s.](image)

![Fig. 9. Comparison of pearlite growth velocities derived from the extended Jackson Hunt calculation with the results of this paper.](image)

![Fig. 10. Comparison of the growth rates of pearlite versus temperature between the Zener-Hillert model, Steinbach's phase-field simulation, and experimental results from Ridley and Frye.](image)
grows, the lamellar spacing adjusts to $0.40 \times 10^{-6}$ m.

When carbide is at location P2, the carbide completely hinders the lamellar cementite’s growth although it is not in right ahead of lamellar cementite $\beta$. Therefore, the growth morphology is identical to that of location P1. Near the lamellar cementite $\beta$, the growth rate of the right-side lamellar ferrite exceeds that of the left-side lamellar ferrite.

When the carbide is at location P3, its diameter is far less than the width of the lamellar ferrite. In this case, the lamellar ferrite bypasses the carbide, and the adjacent lamellar cementite bends correspondingly. Which has been verified above. The width of the lamellar cementite in the bend region becomes narrower as the bending degree increases, until eventually the bending region disconnects, forming an island of cementite at the bottom. If the amount of carbide is very large, the microstructure evolves such that many cementite islands distribute on the ferrite matrix. In this situation, the microstructure is called spheroidal pearlite. Lamellar pearlite grows according to the original lamellar spacing after the lamellae bypass the carbide, and the growth rate obviously exceeds that of the first two locations. The conclusion can be obtained from the forward location of the pearlite at 0.35 s. Essentially, the growth rate is still caused by the magnitude of the lamellar spacing; to a certain extent, the greater the lamellar spacing, the slower the growth rate.

Figure 11 demonstrates that the lamellar morphology near the carbide differs from that of other parts at 0.0125 s, which suggests that carbide has a promoting effect on lamellar ferrite and an inhibitory effect on lamellar cementite. The average growth rates of the three locations from the initial location to the carbide are 1.22 $\mu$m/s, 1.50 $\mu$m/s, and 4.23 $\mu$m/s, respectively.

Figure 12 shows the growth rate curves along Line A, the vertical line that passes through the center of the carbide in Fig. 11. Figure 12 only describes the growth rate curves after the lamellae bypass the carbide. The overall trends of the three curves are approximately the same, with the growth rate falling to a low value from a high value before increasing to a relatively constant value. The time at which the lamellar pearlite bypasses the carbide at locations P1 and P2 is smaller than that at location P3. This is mainly because the lamellar spacing greatly increases when the carbide hinders the growth of lamellar pearlite at locations P1 and P2. However, the relative lamellar spacing decreases at location P3, while the pearlite growth rate is greater. Local lamellar spacing exceeds the initial lamellar spacing after the lamellae hinder the carbide, but the local spacing of the neighbor lamellae is smaller than the initial spacing. The growth rate slows down as the local lamellar spacing increases, and conversely, the growth rate speeds up when the local lamellar spacing is small. Adjustments are made between the two degrees of lamellar spacing, such that volatile growth appears, and finally the growth rate slowly tends toward a constant. In other words, the result is stable growth with average spacing.

Finally, three pairs of lamellae occur at locations P1 and P2, with average lamellar spacing of 0.4 $\mu$m and average growth rates of 3.6 $\mu$m/s and 2.3 $\mu$m/s, respectively. The lamellar spacing is similar between P1 and P2, but the growth rates are very different. This difference derives from the locations of Line A in the three situations. In condition P1, Line A is at the center of the lamellar cementite, but in condition P2 Line A lies at the interface between lamellar ferrite and lamellar cementite. The merged lamellae ferrite is the object of this study, and the location of Line A appears to deviate from the center of the ferrite lamellae. In order to ensure that the total energy of the system remains low, the ferrite lamellae begin growing with the axis of the carbide’s center. This adjustment process accompanies a local change in spacing, then guiding the growth direction of Line A’s deviation from the vertical direction. However, the calculated lamellae growth rate in Fig. 12 is a projection of the vertical direction, such that it will be less than that at P1.
4.3  Effect of the Size of Carbide

Figures 13 and 14 allows for analysis of the different sizes of carbide. Figure 13 shows the lamellar growth state when a small-sized carbide is present. The three different sizes of carbide appear to have fundamentally similar effects on the growth rate and growth morphology of lamellar pearlite, although the smaller the carbide is, the faster its corresponding lamellar spacing stabilizes. This occurs because the lamellae bend slightly in the horizontal direction when the carbide size is small. In other words, when the oscillation amplitude of lamellar growth is small, it is easier to stabilize at the same growth rate. Shorter times required to bypass carbide at the same velocity correspond to smaller carbide sizes in the vertical direction.

Figure 14 shows the lamellar growth state of large-sized carbide. The effects of three large-sized carbides on the carbide sizes in the vertical direction. To bypass carbide at the same velocity correspond to smaller carbide sizes when a small-sized carbide is present. The three different sizes of carbide appear to have fundamentally similar effects when a small-sized carbide is present. Figure 13 shows the lamellar growth state of large-sized carbide (small size). The locations of points A and B listed in the table are shown in Figs. 13 and 14. For larger-sized carbides, the bottom edge is farther away from the lamellae. When the lamellae encounter the carbides, the carbides have grown to larger diameters. But for small-sized carbides, the bottom edge is closer to the lamellae. When the lamellae encounter carbides, the carbide hardly grows. The final diameter of the large-sized carbide in the Table 6 is different from the initial diameter.

Equation (13) describes the relationship between distance of AB and the size of the small-sized carbide. When no carbide exists, or \( R_0 = 0 \), the distance \( D_0 \) between point A and point B is 87, which does not satisfy the Eq. (13). This suggests that carbide changes pearlite’s original growth rule. However, when the size of carbide changes, the transverse maximum bending distance increment of neighboring cementite lamellae equals the increment of the carbide’s diameter. In combination with Eq. (13), this relationship allows the morphology and performance of lamellar pearlite to be controlled well.

Since the diameter of the large-sized carbide exceeds a lamellar spacing, which actually affects the microstructure within three times of lamellar spacing. When the large-sized carbide edge increases from the blocked cementite lamellae to the adjacent two cementite lamellae, the increasing rate of carbide size is greater than that of the distance between the maximum bending points of the lamellae. This is because the location of the carbide edge is less than the center of the adjacent two cementite lamellae, so it shows a different law.

<table>
<thead>
<tr>
<th>Size</th>
<th>Point A</th>
<th>Point B</th>
<th>Distance of AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_0 = 0 )</td>
<td>( X = 138 )</td>
<td>( X = 225 )</td>
<td>( D_0 = 87 )</td>
</tr>
<tr>
<td>( R_1 = 4 )</td>
<td>( X = 124 )</td>
<td>( X = 239 )</td>
<td>( D_1 = 115 )</td>
</tr>
<tr>
<td>( R_2 = 8 )</td>
<td>( Y = 79 )</td>
<td>( Y = 79 )</td>
<td>( D_2 = 123 )</td>
</tr>
<tr>
<td>( R_3 = 12 )</td>
<td>( Y = 102 )</td>
<td>( Y = 102 )</td>
<td>( D_3 = 131 )</td>
</tr>
<tr>
<td>( R_4 = 45 )</td>
<td>( X = 100 )</td>
<td>( X = 350 )</td>
<td>( D_4 = 250 )</td>
</tr>
<tr>
<td>( R_5 = 55 )</td>
<td>( X = 95 )</td>
<td>( X = 355 )</td>
<td>( D_5 = 260 )</td>
</tr>
<tr>
<td>( R_6 = 65 )</td>
<td>( X = 85 )</td>
<td>( X = 365 )</td>
<td>( D_6 = 280 )</td>
</tr>
</tbody>
</table>

Table 6. Locations of maximum bending.
with the small carbide.

\[ D_{i+1} = D_i + 2(R_{i+1} - R_i) \quad i = 1, 2, \ldots, \ldots \quad (13) \]

The growth rate curves for pearlite shown in Fig. 15 can be obtained according to the growth morphology of different sizes. The curves’ change trends are consistent across the three situations of the small-sized carbide, with growth rates falling from a high value to a low value before increasing and fluctuating around a relatively constant value. Ultimately, the growth rate of each situation is basically the same, at 4.0 \( \mu m/s \). This suggests that carbide size exerts little effect on the growth rate of pearlite, provided that the size of carbide varies within a certain range. As the carbide size increases, the amplitude of the growth rate curves also increases, and the size of carbide is inversely proportional to the local lamellar spacing. The curves’ change trends are consistent across the three situations of the large-sized carbide. However, they have slightly differences with the small-sized carbide. The curves are firstly increased to the peak value and then reduced to a stable value. This is the result of the interaction between the bent lamellae and the ferrite lamellae of the central large region when the lamellae grow around the carbide. The final growth rate is stabilized at 2.5 \( \mu m/s \), this mainly due to the fact that the large carbide hinders the growth of the bottom lamellae, resulting in an increase in the lamellar space of overall area around the carbide and a significant reduction in the growth rate.

5. Conclusion

These results suggest that carbide has a significant influence on the cooperative growth of pearlite. So the role of carbide need to be considered when obtain the experimental data of pearlite.

The effect of the location and size of carbide on the cooperative growth of pearlite in carbon-steel has been analyzed by phase-field method. Comparing growth form of pearlite in simulation and experiment, the following conclusions can be draw:

(1) The existence of carbide will result in bend growth of pearlite lamellae, and the bent lamellae will bypass the carbide. The maximum horizontal distance between carbide and the nearest survived lamella follows the relationship of \( D_{i+1} = D_i + 2(R_{i+1} - R_i) \) when the diameter of carbide is smaller than lamellar spacing.

(2) When the diameter of carbide is smaller than lamellar spacing, the location of carbide have a significant influence on the pearlite lamellar. If the carbides are in the front of a cementite lamellae, the cementite lamellae will not survive because of its restrained growth; if the carbide is in the front of a ferrite lamellae, all lamellae near the carbides will bypass the carbide, and the lamellar spacing and growth velocity will not change finally.

(3) When the diameter of carbide is larger than lamellar spacing, the growth of pearlite lamellae below the carbide is restrained by carbide and the lamellae cannot survive to bypass carbide.

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